

PDC Control for Mobile Robot Formations with Virtual Reference Based on Separation-Bearing

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Abstract – This paper presents a development of leader-follower formation control using separation-bearing control (SBC) and Parallel Distribution Compensation (PDC) control. The formation control involves tracking of each desired trajectory by leader and follower robots. The follower trajectory is generated using SBC approach with respect to predefined trajectory of the leader. This design is used to improve formation control when initial error is given to leader. In order to maintain the formation and avoid internal collision, the error tracking of each robot must be kept near zero. Each robot is controlled by kinematic and dynamics controller which is designed using PDC and PID. The velocity reference for dynamic robots is limited. The simulation result shows the tracking errors for position and orientation with initial lateral error set at 0.5 m are less than 0.5 m and 1.2 rad which then converges to the desired value. Thus, the good trajectory formation tracking is achieved.

Keywords: SBC, PDC, Error Tracking, Kinematic, Dynamics Controller

I. Introduction

The coordination of multi-robot which performs collective tasks have been widely developed for past decades. These types of implementation can be seen in rescue or military robots [1]. Since it involves multiple robots, the formation control require strategy to implement [2]. The control is used to form or maintain formation in point to points motion or tracking reference trajectory [3],[4].

There are several approaches for formation strategy such as behavior-based [5], [6] virtual-structure [7], and leader-follower [8]. Formation using leader-follower is begin with leader moving along predefined trajectory and the follower maintain its position and orientation with respect to the leader [9]. One of the approaches which often be used is separation-bearing controller (SBC) [10]. Based on leader position and desired separation distance, the follower will maintain its position and orientation. The problem with this approach is the value of linear and angular velocity which generated from the kinematic controller of follower. When leader tracks a circle trajectory, follower

linear and angular velocity have the same value when follow the leader at inner and outer circle reference. Thus, the velocity generated from the SBC can only be used if the variables are distance and bearing angle. However, posture result is accurate. As it stated in [11], SBC can be used when the initial error position is not given to the leader. Especially, if the system involves robot dynamics model. In [12], the problem of formation is transformed into trajectory tracking. This paper examines how to coincide the center of mass and wheel axis despite the motion and reference problems are not stated. For individual robot, parallel distributed compensation (PDC) control can be used to minimize error tracking [13]. This method is also suitable to solve tracking problem when dynamics model is used [14]. However, if the number of rules is quite big, the feasibility of the linear matrix inequality (LMI) solver is difficult to be obtained. In this study, SBC controller is used as posture reference for each follower. The kinematic controller is designed using PDC and auxiliary velocity based on model error. Then, dynamics controller is designed using PID controller and forward gain. Using kinematic model of non-holonomic robot, the linear and angular velocity

reference for each robot can be obtained. Thus, the main problem is minimizing the tracking error.

II. Research Method

The diagram block of the system is shown in Fig. 1. The references for each robot are position (x, y) , orientation (θ) , linear velocity (v) , and angular velocity (ω) . The position and orientation references generated from SBC with predefined distance and bearing angle. For linear and angular velocity reference, the values are generated using kinematic model based on SBC output.

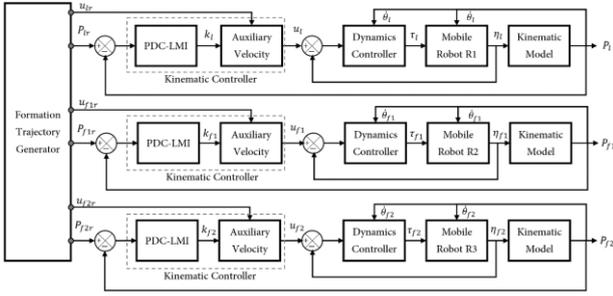


Fig. 1. Formation control based on virtual reference

II.1. Formation trajectory generator

The motion of multi robot in formation need to be coordinated in order to prevent internal collisions between robot. The coordination is arranged in trajectory generator. First, the desired trajectory for leader is defined as follows:

$$\begin{aligned} P_{lr} &= [x_{lr} \quad y_{lr} \quad \theta_{lr}]^T \\ u_{lr} &= [v_{lr} \quad \omega_{lr}]^T \end{aligned} \quad (1)$$

where v_{lr} and ω_{lr} are linear and angular velocities reference for leader. P_{lr} is vector posture of leader virtual reference.

There are two followers, $F1$ and $F2$, that will be used in this study as it shown in Fig. 2.

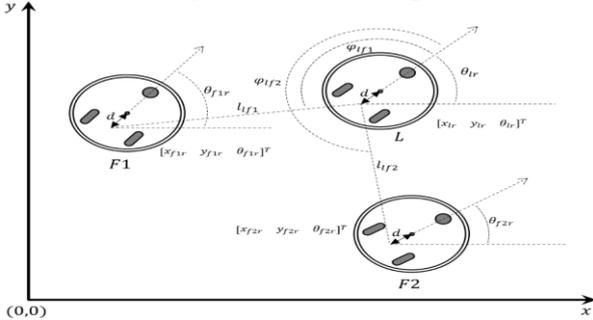


Fig. 2. Formation virtual reference using SBC

Using reference virtual trajectory of leader, the virtual distance (l_{lf}) and bearing angle (φ_{lf}) of formation can be arranged as follows [10]:

$$l_{lf} = \sqrt{(\psi)^2 + (y_{lr} + y_{fr} - d \sin \theta_{fr})^2} \quad (2)$$

$$\varphi_{lf} = \pi - \arctan 2(y_{fr} - y_{lr} + d \sin \theta_{fr}, \psi) - \theta_{lr}$$

where $\psi = x_{lr} - x_{fr} - d \cos \theta_{fr}$ and d is the distance between the midpoint of two wheel and the robot center point.

The SBC kinematic model of the follower virtual reference can be written as:

$$\begin{aligned} \dot{l}_{lf} &= v_{fr} \cos \gamma - v_{lr} \cos \varphi_{lf} + d \omega_{fr} \sin \gamma \\ \dot{\varphi}_{lf} &= \frac{v_{lr} \sin \varphi_{lf} - v_{fr} \sin \gamma - \omega_{lr} l_{lf} + d \omega_{fr} \cos \gamma}{l_{lf}} \end{aligned} \quad (3)$$

$$\dot{\theta}_{fr} = \omega_{fr}$$

where $\gamma = \theta_{lr} - \theta_{fr} + \varphi_{lf}$.

The position and orientation virtual references of each follower can be derived from (3) as follows:

$$\begin{aligned} x_{fr} &= x_{lr} + l_{lf} \cos(\theta_l + \varphi_{lf}) \\ y_{fr} &= y_{lr} - l_{lf} \sin(\theta_l + \varphi_{lf}) \\ \theta_{fr} &= \int \omega_{fr} \end{aligned} \quad (4)$$

The different reference value between two followers in (4) is the angle bearing as it shown in Fig. 2.

II.2. Kinematic and Dynamics Model of the robot

The robots are assumed to have nonholonomic constraints with pure rolling constraint and no lateral slip motion [15]. Let $i=1,2,3$ denote leader, follower 1 (F1), and follower 2 (F2). The forward kinematic model of the robot can be expressed as follows:

$$\dot{p}_i = \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \frac{R}{2} \cos \theta_i & \frac{R}{2} \cos \theta_i \\ \frac{R}{2} \sin \theta_i & \frac{R}{2} \sin \theta_i \\ \frac{R}{2L} & \frac{R}{2L} \end{bmatrix} \begin{bmatrix} \dot{\varphi}_{Ri} \\ \dot{\varphi}_{Li} \end{bmatrix} \quad (5)$$

where p is the robot posture (x, y, θ) , $\dot{\varphi}_R$ and $\dot{\varphi}_L$ are the angular velocities of right and left wheel, R is the wheel radius, and $2L$ is the distance between right and left wheel.

The coordinates configuration of the robot can be expressed as:

$$q_i = [x_i \quad y_i \quad \theta_i \quad \varphi_{Ri} \quad \varphi_{Li}] \quad (6)$$

where φ_R and φ_L are the angle of right and left wheel.

Let define wheels angular velocities as:

$$\eta_i = \begin{bmatrix} \dot{\varphi}_{Ri} \\ \dot{\varphi}_{Li} \end{bmatrix} \quad (7)$$

Using Lagrange dynamic approach, the dynamics model of the robot can be written as follows:

$$\bar{M}(q)\dot{\eta}_i + \bar{V}(q, \dot{q})\eta_i = \bar{B}(q)\tau_i \quad (8)$$

with:

$$\bar{M}(q) = \begin{bmatrix} I_w + \frac{R^2}{4L^2}(m_1) & \frac{R^2}{4L^2}(m_2) \\ \frac{R^2}{4L^2}(m_2) & I_w + \frac{R^2}{4L^2}(m_1) \end{bmatrix}$$

$$\bar{V}(q, \dot{q}) = \begin{bmatrix} 0 & \frac{R^2}{2L^2}m_c d \dot{\theta} \\ \frac{R^2}{2L^2}m_c d \dot{\theta} & 0 \end{bmatrix} \quad (9)$$

$$\bar{B}(q) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where τ_i is input torque for each robot, m_c is robot mass without wheel, $m_1 = mL^2 + I$, $m_2 = mL^2 - I$, $I = I_c + m_c d^2 + 2m_w L^2 + 2I_m$, m is the robot total mass, $\bar{M}(q)$ is inertia matrix, $\bar{V}(q, \dot{q})$ is the centripetal and Coriolis matrix, and $\bar{B}(q)$ is identity matrix.

In this study, the kinematic robot is modelled based on posture error $p_e = [e_x \ e_y \ e_\theta]^T$. The model configuration of robot is shown in Fig. 3.

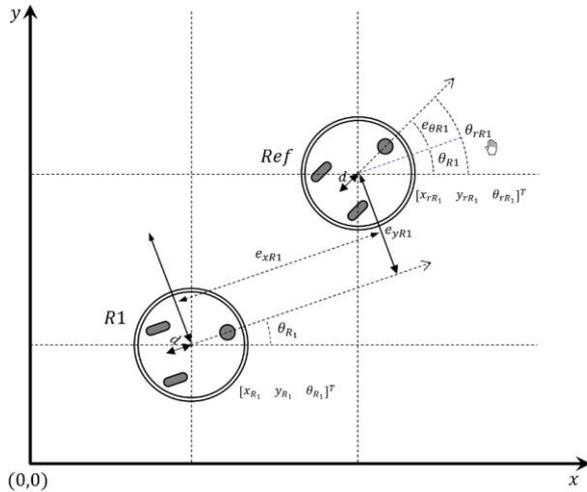


Fig. 3. The model of robot posture error

The posture error p_e can be defined as follows:

$$\begin{bmatrix} e_{xi} \\ e_{yi} \\ e_{\theta i} \end{bmatrix} = \begin{bmatrix} \cos \theta_i & \sin \theta_i & 0 \\ \sin \theta_i & \cos \theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{ri} - x_i \\ y_{ri} - y_i \\ \theta_{ri} - \theta_i \end{bmatrix} \quad (10)$$

where e_x is leading error, e_y is lateral error, and e_θ is orientation error.

Based on the velocity reference of each robot (v_{ri}, ω_{ri}), the kinematic error model can be obtained as follows:

$$\begin{bmatrix} \dot{e}_{xi} \\ \dot{e}_{yi} \\ \dot{e}_{\theta i} \end{bmatrix} = \begin{bmatrix} \cos e_{\theta i} & 0 \\ \sin e_{\theta i} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{ri} \\ \omega_{ri} \end{bmatrix} + \begin{bmatrix} -1 & e_{yi} \\ 0 & -e_{xi} \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v_{di} \\ \omega_{di} \end{bmatrix} \quad (11)$$

where v_{di} and ω_{di} are the auxiliary velocities control.

As in [15], the following form is chosen to be the auxiliary velocities control:

$$u_{di} = [v_{di} \ \omega_{di}]^T$$

$$u_{di} = \begin{bmatrix} v_{ri} \cos e_{\theta i} + k_{xi} e_{xi} \\ \omega_{ri} + k_{yi} v_{ri} e_{yi} + k_{\theta i} v_{ri} \sin e_{\theta i} \end{bmatrix} \quad (12)$$

where k_{xi} , k_{yi} , and $k_{\theta i}$ are parameters to be determined.

Substituting (12) into (11), the kinematic model can be rearranged as follows:

$$\dot{p}_{ei} = A_i p_{ei} + B_i u_{ki} \quad (13)$$

where:

$$u_{ki} = [k_{xi} \ k_{yi} \ k_{\theta i}]^T$$

$$A_i = \begin{bmatrix} 0 & \omega_{ri} & 0 \\ -\omega_{ri} & 0 & v_{ri} \sin e_{\theta i} \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_i = \begin{bmatrix} -e_{xi} & v_{ri} & v_{ri} e_{yi} \sin e_{\theta i} \\ 0 & -v_{ri} e_{xi} e_{yi} & -v_{ri} e_{xi} \sin e_{\theta i} \\ 0 & -v_{ri} e_{yi} & -v_{ri} \sin e_{\theta i} \end{bmatrix}$$

III. 3. Kinematic and Dynamics Controller

The kinematic controller of each robot is designed and stabilized using PDC control law based on Takagi-Sugeno (T-S) fuzzy model. There are some parameters which need to be defined for the design such as fuzzy set (M_{mp}), the number of rules (r), and the premise ($z_p(t)$). Knowing the parameters, the T-S fuzzy model can be defined as the following closed loop form [15]:

$$\dot{e}_i(t) = h_{z1} h_{z2} \{A_m - B_m F_n\} e_i(t) \quad (14)$$

where:

$$h_{z1} = \sum_{m=1}^r h_m(z(t)), h_{z2} = \sum_{n=1}^r h_n(z(t))$$

$$m = 1, 2, 3, \dots, r$$

$$z(t) = [z_1(t) \ z_2(t) \ z_3(t) \ \dots \ z_p(t)]$$

$$F_n = k_1, k_2, \dots, k_n$$

Let define the number of rules (r) is 3 and the premise variables as follows:

$$z(t) = [e_{xi} \ e_{yi} \ \sin e_{\theta i}] \quad (15)$$

Using (15), the matrix A_m and B_m can be expressed as:

$$A_m = \begin{bmatrix} 0 & \omega_{ri} & 0 \\ -\omega_{ri} & 0 & v_{ri} z_3 \\ 0 & 0 & 0 \end{bmatrix} \quad (16)$$

$$B_m = \begin{bmatrix} -z_1 & v_{ri} & v_{ri} z_2 z_3 \\ 0 & -v_{ri} z_1 z_2 & -v_{ri} z_1 z_3 \\ 0 & -v_{ri} z_2 & -v_{ri} z_3 \end{bmatrix}$$

The premises are constructed using maximum and minimum value as well as the membership function in order to obtained the PDC control law as follows:

$$u_{ki} = -h_{z1} F_m \quad (17)$$

where F_m is feedback gain.

In order to obtained the stability of the controller, the LMI equations (18)-(21) is used to calculate the feedback gain.

$$-X_i A_{mi}^T - A_{mi} X_i + M_{ni}^T B_{mi}^T + B_{mi} M_{ni} > 0 \quad (18)$$

$$\begin{bmatrix} 1 & e(0)^T \\ e(0) & X_i \end{bmatrix} \geq 0 \quad (19)$$

$$\begin{bmatrix} X_i & M_{mi}^T \\ M_{mi} & \mu^2 I \end{bmatrix} \geq 0 \quad (20)$$

$$\begin{bmatrix} X_i & X_i C_m^T \\ C_m X_i & \lambda^2 I \end{bmatrix} \geq 0 \quad (21)$$

where:

$$m = 1, 2, 3, \dots, r$$

$$i = 1, 2, 3$$

$$X_i = P_i^{-1}$$

$$F_{mi} = M_m X_i^{-1}$$

The result of input velocities control in (12) will be used as reference for the dynamics robot system. Because the dynamics model (8) uses the right and left wheel angular velocities instead of linear and angular velocities, the input velocities (12) will be converted using the following form:

$$u_{ri} = \begin{bmatrix} \dot{\phi}_{Rri} \\ \dot{\phi}_{Lri} \end{bmatrix} = \begin{bmatrix} \frac{v_{di} + b\omega_{di}}{R} \\ \frac{v_{di} - b\omega_{di}}{R} \end{bmatrix} \quad (22)$$

Let form the error left and right wheel angular velocities of dynamics model as follows:

$$e_{di} = u_{ri} - \eta_i \quad (23)$$

$$e_{di} = \begin{bmatrix} \dot{\phi}_{Rri} \\ \dot{\phi}_{Lri} \end{bmatrix} - \begin{bmatrix} \dot{\phi}_{Ri} \\ \dot{\phi}_{Li} \end{bmatrix}$$

Using (8), (22), and (23), the dynamics input control can be expressed as follows:

$$\tau_i = \bar{B}^{-1}(q) (\bar{M}(q) u_{dyn} + \bar{V}(q, \dot{q}) \eta) \quad (24)$$

$$u_{dyn} = K_r \dot{\eta} + K_p e + K_d \dot{e} + K_i \int e$$

where K_r is feedforward gain, K_p is proportional gain, K_d is derivative gain, and K_i is integral gain.

III. Result and Discussion

The simulation of the designed controller is implemented using MATLAB and Simulink. In order to verify the proposed method, the simulation sets are conducted using single and formation robot with and without initial error. Trajectory reference for formation is a circle. The linear velocity limitation is 15 rad/s while the angular velocity is ± 15 rad/s. The predefined formation is arranged with $l_{f1} = 1.0954$ m, $\phi_{f1} = 225^\circ$, and $\phi_{f2} = 135^\circ$.

The trajectory tracking of a robot without initial error position is shown in Fig. 4 with motion in clockwise direction. Robot figure is plotted in several point in order to show the robot orientations.

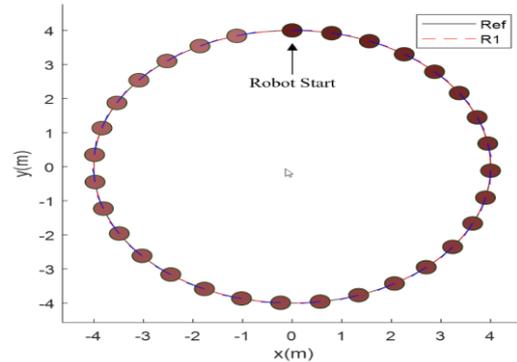


Fig. 4. Circle trajectory tracking without initial error

Using (12) and (24), the result of posture and velocity errors are quite small. The posture error is shown in Fig. 5. The position error is within

$\pm 16 \times 10^{-5}$ m and the orientation is within $\pm 4 \times 10^{-5}$ rad/s.

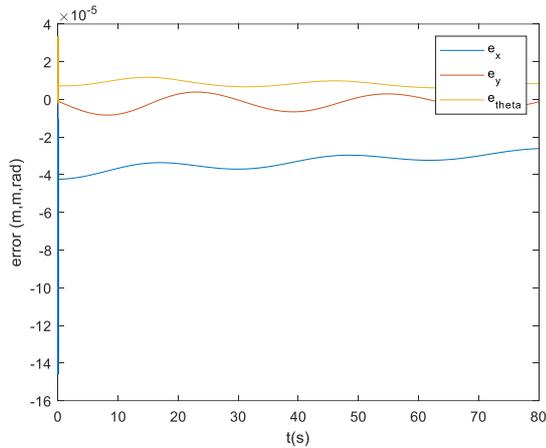
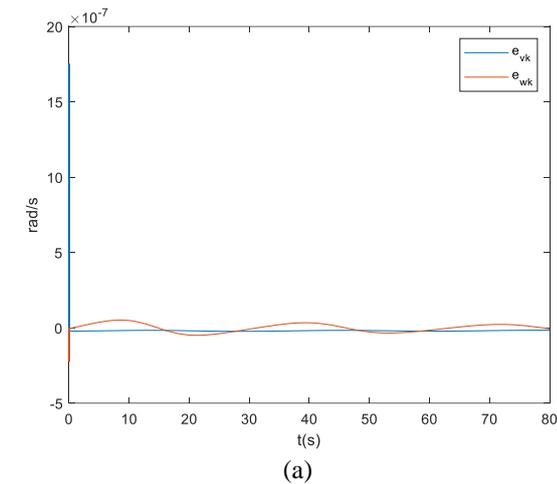
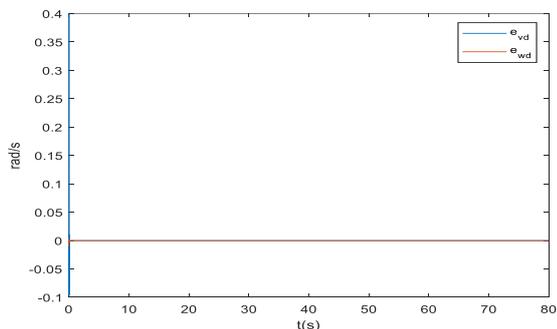


Fig. 5. The posture error without initial error

Similarly, the errors velocities from both kinematic and dynamics are also quite small which is shown in Fig 6.



(a)



(b)

Fig. 6. Linear and angular errors of tracking with initial error (a). Kinematic (b). Dynamics

For error tracking with initial error $(0, -0.5, 0)$ is shown in Fig. 7.

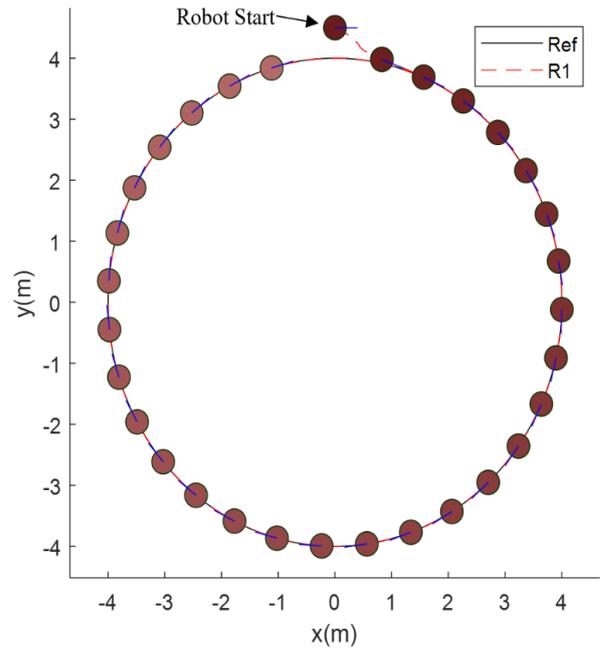


Fig. 7. Circle trajectory tracking without initial error

The posture error sustains undershoot and overshoot but gradually converge near zero around $\pm 4 \times 10^{-3}$ m.

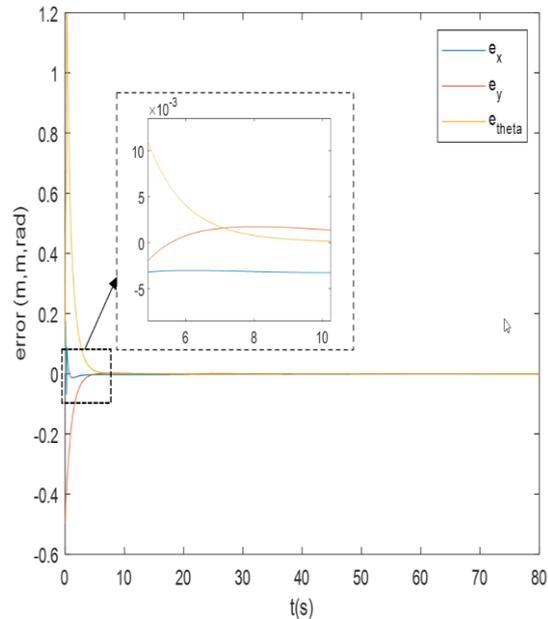


Fig. 8. The posture error of tracking with initial error

The kinematic error is quite big around ± 6 rad/s but the steady state is achieved within $\pm 2 \times 10^{-7}$ rad/s.

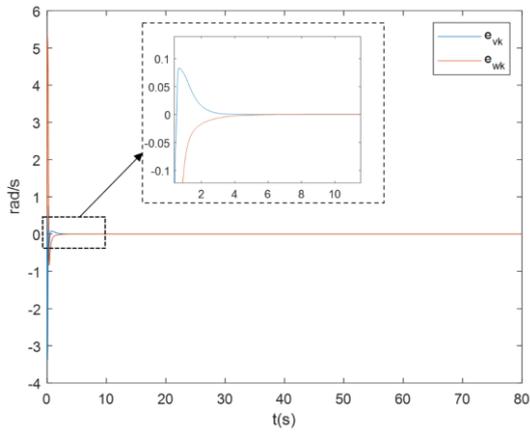


Fig. 9. Kinematic velocity errors of tracking with initial error

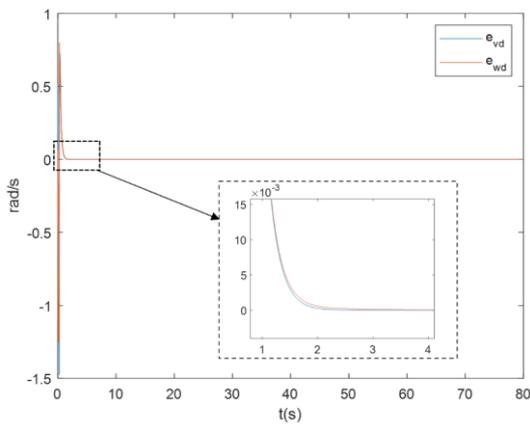


Fig. 10. Dynamics velocity errors of tracking with initial error

For formation trajectory tracking, the result of single robot above is used as the leader reference. The formation trajectory tracking with and without initial error is shown in Fig. 11.

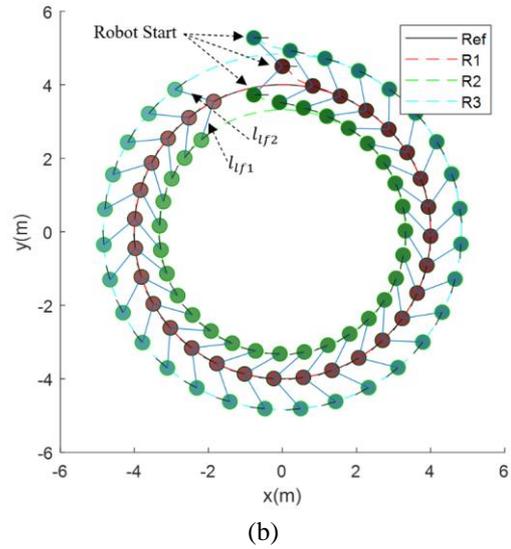
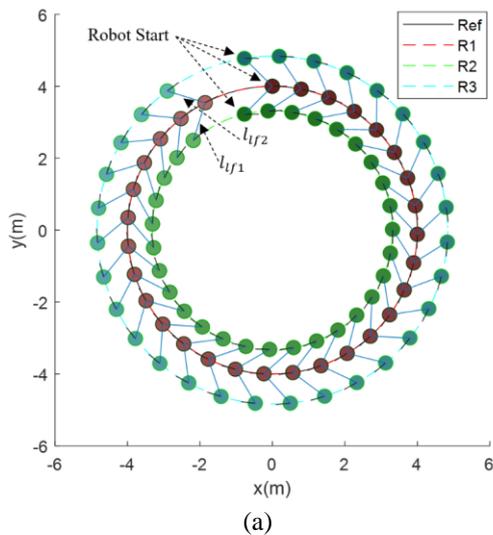


Fig. 11. Formation trajectory tracking (a) without initial error (b) with initial error
The posture errors of F1 and F2 in trajectory tracking without initial error are shown in Fig. 12.

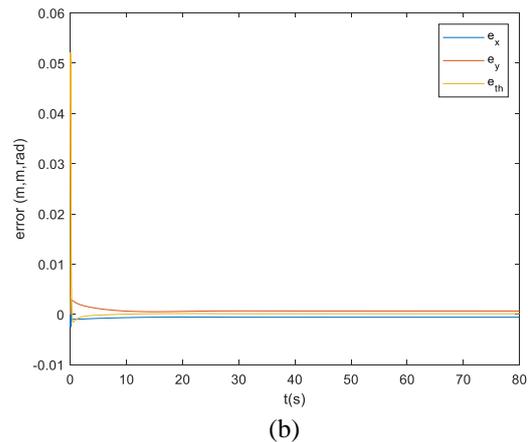
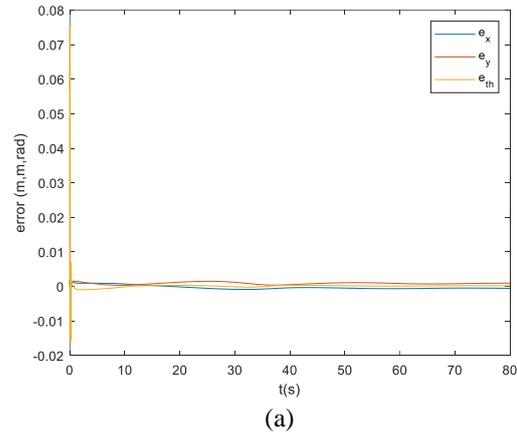


Fig. 12. The posture errors of the formation trajectory tracking without initial error (a). F1 (b). F2

The steady state of each error is achieved for both F1 and F2. The orientation errors for both robots are bit higher but still within target around

$\pm 8 \times 10^{-2}$ rad and the steady state value is within $\pm 1 \times 10^{-3}$ rad. The posture error for tracking with initial error is show in Fig. 13 and Fig. 14.

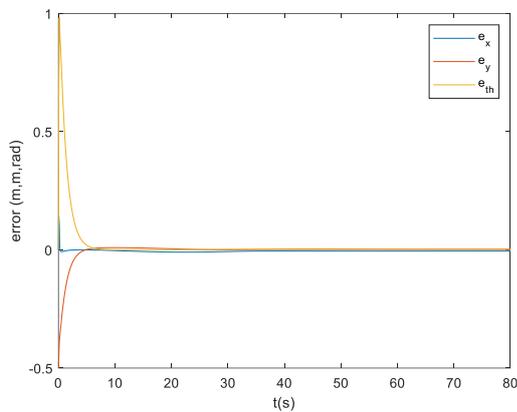


Fig. 13. The posture errors of the follower 1 for tracking with initial error

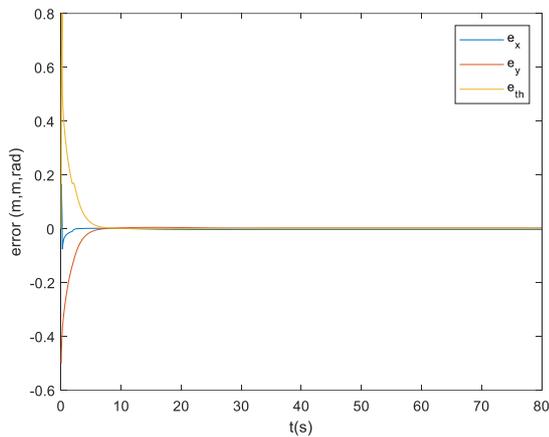


Fig. 14. The posture errors of the follower 2 for tracking with initial error

IV. Conclusion

The development of formation control by utilizing the separation-bearing control as trajectory reference generator for follower robot and parallel distribution compensation as control law for kinematic controller of each robot is presented. The separation-bearing control used the desired trajectory which is given to leader robot as reference in order to virtually form and maintain the predefined formation throughout the tracking process. From the simulation results, the good formation trajectory tracking can be achieved with small errors both without and with initial error. Particularly, the value is not exceeding the given initial error which is 0.5 m (lateral error). In spite of

the rise and settling time are slower than the result of tracking without initial error, the steady state of the posture and velocity errors is obtained. Thus, it can be stated that the proposed method can be used to solve initial error problem in formation control when the robot dynamics model is involved.

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