Neural Network-based Finite-time Control of Nonlinear Systems with Unknown Dead-zones: Application to Quadrotors

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***Abstract*—Over the years, researchers have addressed several control problems of various classes of nonlinear systems. This article considers a class of uncertain strict feedback nonlinear system with unknown external disturbances and asymmetric input dead-zone. Designing a tracking controller for such system is very complex and challenging. This article aims to design a finite-time adaptive neural network backstepping tracking control for the nonlinear system under consideration. In addition, all unknown disturbances and nonlinear functions are lumped together and approximated by radial basis function neural network (RBFNN). Moreover, no prior information about the boundedness of the dead-zone parameters is required in the controller design. With the aid of a Lyapunov candidate function, it has been shown that the tracking errors converge near the origin in finite-time. Simulation results testify that the proposed control approach can force the output to follow the reference trajectory in a short time despite the presence of asymmetric input dead-zone and external disturbances. At last, in order to highlight the effectiveness of the proposed control method, it is applied to a quadrotor unmanned aerial vehicle (UAV).**

Keywords*—Quadrotor; Unmanned aerial vehicle, Backstepping control; Radial basis function neural network; Dead-zone; Nonlinear systems.

I. Introduction

In recent years, the popularity of Nonlinear control techniques is ever increasing. A large number of systems have nonlinear and multivariable characteristics in reality. Moreover, the nonlinear systems usually have time-varying disturbances, unmodelled dynamics, and other uncertainties [1]. Thus, different control methods have been suggested for both practical and theoretical applications including backstepping control [2], Feedback linearization [3], [4], Sliding mode control [5], and Adaptive control [6].

Backstepping techniques have garnered much more interests in the control of complex nonlinear systems such as robotic manipulators [7], chemical processes [2], power systems [8], and multi-agent systems [9]. These techniques break down a complex n-order system into several subsystems, provide a virtual controller for each subsystem and an actual control input for the last subsystem. In addition, a Lyapunov function is built to ensure the stability of each subsystem and the closed loop system [10]. An adaptive backstepping control has been widely used to control nonlinear systems with parametric uncertainties, unmodeled dynamics and external disturbances [11]. An adaptive algorithm can estimate the uncertain parameter of a system online without any prior knowledge of the upper-bound of the parameter [12]–[15]. However, it can only estimate constant or slow time-varying disturbances/uncertainties. This limitation can be tackled by incorporating disturbance observer in the controller. Several types of disturbance observers have been employed to estimate and compensate time-varying disturbances [16]–[19]. Another limitation of adaptive control is that it is only applicable to systems that are linear in the parameters.

Fuzzy logic and Neural networks (RBFNN) are particularly useful for approximating unknown nonlinear functions due to their universal approximation property. Contrary to the adaptive control that estimates each function's parameters, FL and RBFNN estimate the whole function [20], [21]. As such, the requirement for the linearity in parameters is lifted. Adaptive backstepping control of nonlinear systems based on RBFNN has been studied in [22]–[25] and based on FL in [26]–[31]. Successive differentiation of the virtual control inputs in backstepping design leads to an explosion of complexity. This problem is avoided by using the dynamic surface control approach. This problem has been avoided by using the dynamic surface control approach [32]–[38].

The presence of a dead zone, one of the common nonsmooth nonlinearities, hinders control performances and may derive a system to instability. It is mostly encountered in actuators, electromagnetic devices, mechanical systems, pneumatic valves, sensors and so on. Therefore, efficient controllers were designed to mitigate the effects of the dead-zones using Adaptive

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RBFNN in [32], [39], [40] and adaptive FL in [19], [31], [41].

The literature mentioned above guaranteed the asymptotic and exponential stability of the closed-loop systems. This implies that the closed-loop systems converge to the desired performances as the time goes to infinity. On the other hand, infinite-time stability is undesirable in practical applications because it results in long transient states [42]. Furthermore, before the system converges to the equilibrium in infinite time, some parameters might have changed, high-frequency external disturbances might have entered the system and lead to inaccurate control.

To obtain faster convergence speed, stronger disturbance suppression, and high robustness against uncertainties, finite-time stability control schemes must be put forward. Recently, finite-time-based RBFNN or FL control approaches have been applied to different classes of nonlinear systems with input dead zone nonlinearities [42]–[47]. Moreover, the input gains in [31], [35], [39]–[47] were assumed to be constants or known functions. In situations where the input gains are unknown or are subjected to parametric uncertainties, these controllers will not work. In addition, they were based on single-input-single-output (SISO) systems.

Inspired by the above challenges, a general finite-time adaptive backstepping controller is designed using RBFNN for uncertain nonlinear systems with unknown external disturbances, unknown input gains and dead-zones. The main contributions of this paper are as follows:

1) One RBFNN is employed to approximate the uncertain nonlinear functions together with the external disturbances and the derivatives of the virtual controllers. Therefore, disturbance observer [16]–[19] and first-order filter based on dynamic surface control [32]–[35], [37], [38] are not needed. As such, the computational cost is significantly reduced.
2) The authors in [31], [35], [39]–[47] imposed a strict assumption that the input gains of the nonlinear systems are constant or known functions. However, the controllers developed for these systems would fail in practical applications if this strict requirement could not be met. In this work, the strict assumption is relaxed, and RBFNN is utilized to identify the unknown input gains.
3) In [31], a parametric dead-zone inverse model was constructed. The parameters were assumed to be piece-wise time-varying and many adaptive rules were constructed to estimate them. On the other hand, the dead-zone inverse and prior knowledge of dead-zone parameters are not required in this work.

The contents of this article are arranged as follows: The problem formulation and preliminaries are presented in Section II. The adaptive controller is designed in Section III. The proposed control scheme is applied to a quadrotor unmanned aerial vehicle (UAV) in Section IV. In Section V, the conclusion of the work is given.

II. Problem statement

Consider a class of uncertain nonlinear system with unknown external disturbances and input dead-zones written as:

$$\begin{cases} \dot{x}_1 = f_1(x) + g_1(x)x_2 + \delta_1 \\ \dot{x}_2 = f_2(x) + g_2(x)x_3 + \delta_2 \\ \dot{x}_3 = f_3(x) + g_3(x)x_4 + \delta_3 \\ \vdots \\ \dot{x}_{n-1} = f_{n-1}(x) + g_{n-1}(x)x_n + \delta_{n-1} \\ \dot{x}_n = f_n(x) + g_n(X)U_z + \delta_n \\ y_1 = x_1 \end{cases} \quad (1)$$

$$U_z(u) = \begin{cases} s_r[u(t) - h_r], & u(t) \geq h_r \\ 0, & -h_l < u(t) < h_r \\ s_l[u(t) + h_l], & u(t) \leq -h_l \end{cases}$$

where $x_i(t) \in \mathbb{R}^n$, $i = 1, 2, \ldots, n$ denotes the state variables, $y \in \mathbb{R}$ is the output, $f_i(x), g_i(x) \in \mathbb{R}^n$, $i = 1, 2, \ldots, n$ are the unknown smooth functions, $\delta_i \in \mathbb{R}^n$, $i = 1, 2, \ldots, n$ contains the unknown parametric & nonparametric uncertainties and external disturbances, $u \in \mathbb{R}$ represents the control input, s_l and s_r stand for the unknown left and right dead-zone slopes, $h_l > 0$ and $h_r > 0$ denote the unknown left and right dead-zone breakpoints, U_z represents the dead-zone output.

The dead-zone output U_z can be transformed into a slowly time-varying input-dependent function as follows [48]:

$$U_z(u) = su(t) + d(t) \quad (2)$$

where

$$s = \begin{cases} s_r, & u(t) > 0 \\ s_l & u(t) \leq 0 \end{cases}$$

$$d(t) = \begin{cases} -sh_r, & u(t) \geq h_r \\ -su(t), & -h_l < u(t) < h_r \\ sh_l, & u(t) \leq -h_l \end{cases}$$

The following assumptions, definitions, and lemmas must be considered for proper analysis and design of appropriate backstepping control.

Definition 1: [45]. Consider the nonlinear system:

$$\dot{\chi} = \mathbf{f}(\chi(\mathbf{t})), \quad \mathbf{f}(\mathbf{0}) = \mathbf{0}, \quad , \chi(\mathbf{0}) = \mathbf{x_0}, \quad \chi \in R^n \quad (3)$$

$\mathbf{f} : \beta \longrightarrow R^n$ is continuous in an open neighborhood β of the origin. The zero solution of (8) is said to be semi-global practical finite-time stable (SGPFS) If there exist $r > 0$ and $0 < T(x_0) < \infty$, $\|\chi(t)\| < r$ holds, for all $t \leq t_0 + T$

Lemma 1: [2]. Suppose a continuous positive definite Lyapunov function $L(\chi)$, satisfies the inequality

$$\dot{L} \leq -\psi_1 L + \psi_2 \tag{4}$$

with $\psi_1 > 0$ and $\psi_2 > 0$, then $L(\chi)$ is SGPFS and converges to the neighbourhood of the origin in a finite settling time $T > 0$.

Lemma 2: [49] For arbitrary positive real constants $a > 0$, $b > 0$,$c > 0$, $p > 1$ and $q > 1$ satisfying $1/p + 1/q = 1$, then the following Young's ineaquality is always true

$$ab \leq \frac{a^p}{p} c^p + \frac{b^q}{q} c^{-q} \tag{5}$$

Lemma 3: [2] For the smooth functions $F_i(X)$,$G_i(X) \in \mathbb{R}^m \longrightarrow \mathbb{R}$, a RBFNN can be employed to approximate them over a compact set $\Omega_x \subset \mathbb{R}^m$ as:

$$\begin{cases} F_i(X) = V_i^T \Theta_i(X) + \mu_{f_i} \\ G_i(X) = W_i^T \Theta_i(X) + \mu_{g_i} \end{cases} \tag{6}$$

where $X = [x_1 \dots x_n]^T$ is the input vector, $V_i = [v_1, \dots, v_l]^T$ and $W_i = [w_1, \dots, w_l]^T$ are the RBFNN weight vectors, $\Theta_i = exp\left(-\|X - C_{ij}\|/2\varrho_j^2\right)$ is a Gaussian function, μ_{f_i} and μ_{g_i} $i = (1, 2, \dots, n)$ are RBFNN approximation errors.

Assumption 1: For a given smooth functions $F_i(X)$, $G_i(X)$ and RBFNN approximators (6) , there exist an ideal weight vectors V_i and W_i such that $\mu_{fi} \leq \bar{\mu}_{\mathbf{f_i}}$ and $\mu_{gi} \leq \bar{\mu}_{\mathbf{g_i}}$, with constants $\bar{\mu}_{\mathbf{f_i}} > 0$, $\bar{\mu}_{\mathbf{g_i}} > 0$ for all $X_i \in \Omega_x$.

Generally, the ideal weights V_i and W_i are unknown and have to be estimated. Let $\hat{V}_i$ and $\hat{W}_i$ be the estimates of V_i and W_i respectively, and the weight estimation errors are $\tilde{V}_i = V_i - \hat{V}_i$ and $\tilde{W}_i = W_i - \hat{W}_i$.

Assumption 2: The external disturbances meet $\delta_i \leq \varrho_i$ with $\varrho_i > 0$ is an unknown constant.

III. Control Design

The aim of this note is to design RBFNN adaptive backstepping control laws to achieve the errors $\lim_{t \to T_f} z_i \leq c_q$ in a small compact set in finite-time.

Conventional backstepping design involves n-steps recursive procedures. Using Lyapunov functions to design the control laws at each step is tedious and lengthy and leads to complex algorithms. Therefore, this monotonous procedure is avoided and a systematic approach is used in the design.

Step : 1: Virtual controllers $\alpha_1, \dots, \alpha_{n-2}$ are designed for the states $x_1, \dots, x_{n-1}$. Define the the tracking errors as follows

$$\begin{cases} z_1 = x_1 - x_{1d} \\ z_2 = x_2 - \alpha_1 \\ z_3 = x_3 - \alpha_2 \\ \dots \\ z_{n-1} = x_{n-1} - \alpha_{(n-2)} \end{cases} \tag{7}$$

The time-derivatives of the error variables (7) can be calculated as follows:

$$\begin{cases} \dot{z}_1 = f_1 + G_1 x_2 + \delta_1 - \dot{x}_{1d} \\ \dot{z}_2 = f_2 + G_2 x_3 + \delta_2 - \dot{\alpha}_1 \\ \dot{z}_3 = f_3 + G_3 x_4 + \delta_3 - \dot{\alpha}_2 \\ \vdots \\ \dot{z}_{n-1} = f_{n-1} + G_{n-1} x_n + \delta_{n-1} - \dot{\alpha}_{(n-2)} \end{cases} \tag{8}$$

From (7), the following equation can be obtained:

$$\begin{cases} x_2 = z_2 + \alpha_1 \\ x_3 = z_3 + \alpha_2 \\ x_4 = z_4 + \alpha_3 \\ \dots \\ x_{n-1} = z_{n-1} + \alpha_{(n-2)} \end{cases} \tag{9}$$

Substituting (9) into (8) gives:

$$\begin{cases} \dot{z}_1 = f_1 + G_1 z_2 + G_1 \alpha_1 + \delta_1 - \dot{x}_1 \\ \dot{z}_2 = f_2 + G_2 z_3 + G_2 \alpha_2 + \delta_2 - \dot{\alpha}_1 \\ \dot{z}_3 = f_3 + G_3 z_4 + G_3 \alpha_3 + \delta_3 - \dot{\alpha}_2 \\ \vdots \\ \dot{z}_{n-1} = f_{n-1} + G_{n-1} z_n \\ \qquad + G_{n-1} \alpha_{n-1} + \delta_{n-1} - \dot{\alpha}_{(n-2)} \end{cases} \tag{10}$$

In order to avoid: (1) using disturbance observer to estimate δ_i; (2) using first-order filter to estimate $\dot{\alpha}_{i-1}$, we let

Remark 1: Instead of using a disturbance observer to estimate δ_i $(i = 1, 2, \dots, n)$ as in [16]–[19] or using a first-order filter to estimate the derivatives of the virtual control inputs $\dot{\alpha}_{i-1}$ $(i = 1, 2, \dots, n-1)(\dot{\alpha}_0 = \dot{x}_{1d})$, a single RBFNN is used to estimate δ_i together with $\dot{\alpha}_{i-1}$ and the unknown nonlinear function $f_i(x)$.

Let $F_i = f_i + \delta_i - \dot{\alpha}_{i-1}$, $(i = 1, \dots, n-1)$. Then, the virtual controllers are designed as follows:

$$\begin{cases} \alpha_1 = \frac{1}{\hat{G}_1}[-\hat{F}_1 - \hat{G}_1 z_2 - K_1 z_1] \\ \alpha_2 = \frac{1}{\hat{G}_2}[-\hat{F}_2 - \hat{G}_2 z_3 - K_2 z_2] \\ \alpha_3 = \frac{1}{\hat{G}_3}[-\hat{F}_3 - \hat{G}_3 z_4 - K_3 z_3] \\ \dots \\ \alpha_{n-1} = \frac{1}{\hat{G}_{n-1}}[-\hat{F}_{n-1} - \hat{G}_{n-1} z_n - K_{n-1} z_{n-1}] \end{cases} \tag{11}$$

where $\hat{F}_i$ and $\hat{G}_i$ $i = 1, \ldots, n-1$ are the estimates of F_i and G_i respectively. Adding $\hat{G}_i\alpha_i - \hat{G}_i\alpha_i$ (i=1, …n-1) to (10) yields:

$$\begin{cases} \dot{z}_1 = F_1 + G_1 z_2 + \tilde{G}_1\alpha_1 + \hat{G}_1\alpha_1 \\ \dot{z}_2 = F_2 + G_2 z_3 + \tilde{G}_2\alpha_2 + \hat{G}_2\alpha_2 \\ \dot{z}_3 = F_3 + G_3 z_4 + \tilde{G}_3\alpha_3 + \hat{G}_3\alpha_3 \\ \vdots \\ \dot{z}_{n-1} = F_{n-1} + G_{n-1}z_n + \tilde{G}_{(n-1)}\alpha_{(n-1)} \\ \qquad + \hat{G}_{(n-1)}\alpha_{(n-1)} \end{cases} \tag{12}$$

where $\tilde{G}_i = G_i - \hat{G}_i$ (i = 1, …, n-1). Substituting the virtual controllers (11) into (12) yields:

$$\begin{cases} \dot{z}_1 = \tilde{F}_1 + \tilde{G}_1 z_2 + \tilde{G}_1\alpha_1 - K_1 z_1 \\ \quad = \tilde{F}_1 + \tilde{G}_1 x_2 - K_1 z_1 \\ \dot{z}_2 = \tilde{F}_2 + \tilde{G}_2 z_3 + \tilde{G}_2\alpha_2 - K_2 z_2 \\ \quad = \tilde{F}_2 + \tilde{G}_2 x_3 - K_2 z_2 \\ \dot{z}_3 = \tilde{F}_3 + \tilde{G}_3 z_4 + \tilde{G}_3\alpha_3 - K_3 z_3 \\ \quad = \tilde{F}_3 + \tilde{G}_3 x_4 - K_3 z_3 \\ \vdots \\ \dot{z}_{n-1} = \tilde{F}_{n-1} + \tilde{G}_{n-1}z_n + \tilde{G}_{n-1}\alpha_{n-1} \\ \qquad - K_{n-1}z_{n-1} \\ \qquad = \tilde{F}_{n-1} + \tilde{G}_{n-1}x_n - K_{n-1}z_{n-1} \end{cases} \tag{13}$$

where $\tilde{F}_i = F_i - \hat{F}_i$ $i = 1, \ldots, n-1$.

Step : 2: The actual control input u that can guarantee the overall stability of the system is computed. The error variable is given by $z_n = x_n - \alpha_{(n-1)}$. The derivative of z_n with respect to time is obtained as:

$$\dot{z}_n = \dot{x}_n - \dot{\alpha}_{(n-1)} = f_n + \delta_n + G_n U_z - \dot{\alpha}_{n-1} \tag{14}$$

Considering the dead-zone expression (2), one has:

$$\dot{z}_n = F_n + G_n^* u \tag{15}$$

where $F_n = f_n + \delta_n + G_n d - \dot{\alpha}_{n-1}$, $G_n^* = G_n s$. Adding $\hat{G}_n^* u - \hat{G}_n^* u$ to (15), we get

$$\dot{z}_n = F_n + \tilde{G}_n^* u + \hat{G}_n^* u \tag{16}$$

The actual controller that can neutralize the dead zone effect and ensures tracking is given by:

$$u = \frac{1}{\hat{G}_n^*}[-\hat{F}_n - K_n z_n] \tag{17}$$

Substituting (17) into (16) yields:

$$\dot{z}_n = \tilde{F}_n + \tilde{G}_n^* u - K_n z_n \tag{18}$$

The RBFNN functional estimates of F_i and G_i in (6) are given as:

$$\begin{cases} \hat{F}_i(X_i) = \hat{V}_i^T \Theta_i(X_i) \\ \hat{G}_i(X_i) = \hat{W}_i^T \Theta_i(X_i) \quad i = 1, 2, \ldots, n \end{cases} \tag{19}$$

The RBFNN functional approximation errors are thus:

$$\begin{cases} \tilde{F}_i(X_i) = F_i(X_i) - \hat{F}_i(X_i) = \tilde{V}_i^T \Theta_i(X_i) + \mu_{f_i} \\ \tilde{G}_i(X) = G_i(X) - \hat{G}_i(X) = \tilde{W}_i^T \Theta_i(X_i) + \mu_{g_i} \end{cases} \tag{20}$$

Using (20), the general error dynamics can be written as:

$$\begin{cases} \dot{z}_i = \tilde{V}_i^T \Theta_i + \tilde{W}_i^T \Theta_i x_{i+1} + \vartheta_i - k_i z_i, (i = 1, \ldots, n-1) \\ \dot{z}_n = \tilde{V}_n^T \Theta_n + \tilde{W}_n^T \Theta_n u + \vartheta_n - K_n z_n \end{cases} \tag{21}$$

where $\vartheta_i = (\mu_{q_I} + \mu_{G_i} x_{i+1})$, $\quad \vartheta_n = \mu_{F_n} + \mu_{G_n} u$.

Theorem 4: Suppose the uncertain nonlinear system (1), the closed-loop system (21) satisfied the assumptions 1 and 2. Take the virtual controllers (11), the actual controller (17), the RBFNN weight tuning laws (22),

$$\begin{cases} \dot{\hat{W}}_i = \beta_i[\Theta_i x_{i+1} z_i - \eta_i \hat{W}_i], \quad (i = 1, \ldots, n-1) \\ \dot{\hat{W}}_n = \beta_n[\Theta_n u z_n - \eta_n \hat{W}_n] \\ \dot{\hat{V}}_i = \xi_i[\Theta_i z_i - \eta_i \hat{V}_i], \quad (i = 1, \ldots, n) \end{cases} \tag{22}$$

with constant $\beta_i > 0$, $\xi > 0$, $\eta > 0$. Then, the errors $z_i, i = 1, 2, \ldots, n$, and RBFNN weight estimates $\hat{W}$ and $\hat{V}$ are semi-globally uniformly ultimately bounded in the compact set $\Omega_q \equiv \{q : \|q\| \leq c_q\}$ with c_q a positive constant.

Proof 1: Let the RBFNN approximation property holds for all q in the compact set $\Omega_q \equiv \{q : \|q\| \leq c_q\}$. Select the following Lyapunov function candidate

$$L = \frac{1}{2}\sum_{i=1}^{n} z_i^2 + \frac{1}{2}\sum_{i=1}^{n} \tilde{V}_i^T \xi_i^{-1} \tilde{V}_i + \sum_{i=1}^{n} \frac{1}{2}\tilde{W}_i^T \beta_i^{-1} \tilde{W}_i \tag{23}$$

Differentiating (23) with respect to time yields:

$$\dot{L} = \frac{1}{2}\sum_{i=1}^{n-1} z_i \dot{z}_i - \sum_{i=1}^{n-1} \tilde{V}_i^T \xi_i^{-1} \dot{\hat{V}}_i - \sum_{i=1}^{n-1} \tilde{W}_i^T \beta_i^{-1} \dot{\hat{W}}_i + z_n \dot{z}_n - \tilde{V}_n^T \xi_n^{-1} \dot{\hat{V}}_n - \tilde{W}_n^T \beta_n^{-1} \dot{\hat{W}}_n \tag{24}$$

Substituting (21) and (22) into (24), one obtains:

$$\dot{L} = -\sum_{i=1}^{n} k_i z_i^2 + \sum_{i=1}^{n} \vartheta_i z_i + \sum_{i=1}^{n} \eta_i \tilde{W}_i \hat{W}_i + \sum_{i=1}^{n} \eta_i \tilde{V}_i \hat{V}_i \tag{25}$$

Consider the following Young's inequalities:

$$\begin{cases} z_i\vartheta_i \leq \frac{z_i^2}{2} + \frac{\vartheta_i^2}{2} \\ \eta_i\tilde{V}_i\hat{V}_i = \eta_i\tilde{V}_i[V_i - \tilde{V}_i] \leq \eta_i\frac{V_i^2}{2} - \eta_i\frac{\tilde{V}_i^2}{2} \\ \eta_i\tilde{W}_i\hat{W}_i = \eta_i\tilde{W}_i[W_i - \tilde{W}_i] \leq \eta_i\frac{W_i^2}{2} - \eta_i\frac{\tilde{W}_i^2}{2} \end{cases} \tag{26}$$

Substituting the inequalities into (25) gives:

$$\dot{L} \leq -\sum_{i=1}^{n}\left(k_i - \frac{1}{2}\right)z_i^2 - \sum_{i=1}^{n}\eta_i\beta_i\frac{\tilde{W}_i^2}{2\beta_i} - \sum_{i=1}^{n}\eta_i\xi_i\frac{\tilde{V}_i^2}{2\xi_i} + \sum_{i=1}^{n}\frac{\vartheta_i^2}{2} + \sum_{i=1}^{n}\eta_i\frac{W_i^2}{2} + \sum_{i=1}^{n}\eta_i\frac{V_i^2}{2} \tag{27}$$

Equation (27) can be rewritten as:

$$\dot{L} \leq -\psi_1 L + \psi_2 \tag{28}$$

where

$$\psi_1 = min\{\left(k_i - \frac{1}{2}\right),\ \eta_i\beta_i,\ \eta_i\xi_i\}$$

$$\psi_2 = \sum_{i=1}^{n}\frac{\vartheta_i^2}{2} + \sum_{i=1}^{n}\eta_i\frac{W_i^2}{2} + \sum_{i=1}^{n}\eta_i\frac{V_i^2}{2}$$

Multiplying both sides of (28) by $e^{\psi_1 t}$ and integrating the resulting equation over [0 t], we achieve:

$$L \leq \left(L(0) - \frac{\psi_2}{\psi_1}\right)e^{-\psi_1 t} + \frac{\psi_2}{\psi_1} \leq L(0) - \frac{\psi_2}{\psi_1} \tag{29}$$

Taking into account the Lyapunov function (23), one can gets:

$$\frac{1}{2}\|q\|^2 \leq L(0) - \frac{\psi_2}{\psi_1} \implies \|q\| \leq \sqrt{2\left(L(0) - \frac{\psi_2}{\psi_1}\right)} \tag{30}$$

In view of (30), all the closed-loop signals e, $\tilde{W}$ and $\tilde{V}$ are semi-globally uniformly ultimately bounded in a compact set defined by $\Omega_q \equiv \{q : \|q\| \leq c_q\}$, with $c_q \equiv \sqrt{(L(0) - \psi_2/\psi_1)}$ $\forall t \leq t_0 + T$.

IV. Simulation Study

In this section, the effectiveness of the proposed control approach is demonstrated by implementing it on a quadrotor dynamic model.

The schematic diagram of the quadrotor is shown in Fig. 1. The fixed body frame $B(0_b; x_b; y_b; z_b)$ and the earth fixed frame $E(0_e; x_e; y_e; z_e)$ of the quadrotor are described in this figure. The position of the quadrotor in the E-frame is represented by the vector $\zeta = [x,\ y,\ z]T$ and the attitude is denoted by $A = [\phi,\ \theta,\ \psi]T$, with ϕ, θ and ψ standing for the roll, the pitch, and the yaw angles, respectively. Let $x_1 = z$, $x_2 = \dot{z}$, $x_3 = \phi$, $x_4 = \dot{\phi}$, $x_5 = \theta$, $x_6 = \dot{\theta}$, $x_7 = \psi$, and $x_8 = \dot{\psi}$. Then, the fully actuated nonlinear state-space model of the quadrotor is given by [50]:

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1 + g_1 U_{z1} + \delta_1 \\ y_1 &= x_1 \end{cases} \tag{31}$$

$$\begin{cases} \dot{x}_3 &= x_4 \\ \ddot{x}_4 &= f_2 + g_2 U_{z2} + \delta_2 \\ y_2 &= x_3 \end{cases} \tag{32}$$

$$\begin{cases} \dot{x}_5 &= x_6 \\ \dot{x}_6 &= f_3 + g_3 U_{z3} + \delta_3 \\ y_3 &= x_5 \end{cases} \tag{33}$$

$$\begin{cases} \dot{x}_7 &= x_8 \\ \ddot{x}_7 &= f_4 + g_4 U_{z4} + \delta_4 \\ y_4 &= x_7 \end{cases} \tag{34}$$

where x_1, x_3, x_5 and x_7 denote the altitude, the roll angle, the pitch angle, and the yaw angle, respectively. $f_1 = a_1x_2 - g$, $f_2 = a_2x_4^2 + a_3x_8x_6 + a_4x_6$, $f_3 = a_5x_6^2 + a_6x_8x_4 + a_7x_6$, $f_4 = a_8x_8^2 + a_9x_4x_6$, $g_1 = \frac{1}{m}(cosx_3\ cosx_5)$, $g_2 = \frac{l}{I_x}$, $g_3 = \frac{l}{I_y}$, $g_4 = \frac{l}{I_z}$, $a_1 = -\frac{K_{az}}{m}$ $a_2 = \frac{K_{ax3}}{I_x}$, $a_3 = \frac{(I_y - I_z)}{I_x}$, $a_4 = \frac{-J_r\Omega_r}{I_x}$, $a_5 = \frac{K_{ax5}}{I_y}$, $a_6 = \frac{(I_z - I_x)}{I_y}$, $a_7 = \frac{J_r\omega_r}{I_y}$, $a_8 = \frac{K_{ax7}}{I_z}$, $a_9 = \frac{(I_y - I_z)}{I_z}$. The physical meaning and values of the parameters are available in [50].

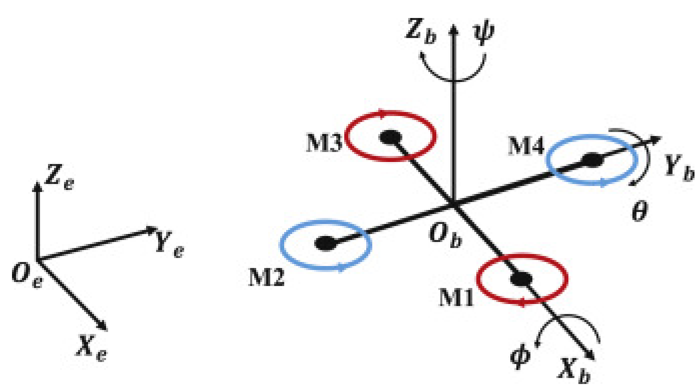

Fig. 1. Schematic of a quadrotor UAV [51]

The external disturbances are set as: $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 3sin(2t)$. The controller parameters are: $K_1 = k_3 = k_5 = k_7 = 10$, $k_2 = k_4 = k_6 = k_8 = 8$. The RBFNN parameters are: $\xi_i = \zeta_i = 0.2$, $\eta_i = 0.01$ (i=1,2, 3, 4). The dead-zone parameters are: $s_{r1} = 1$, $s_{r2} = 0.9$, $s_{r3} = 1.1$, $s_{r4} = 2.8$, $s_{l1} = -2$, $s_{l2} = -0.5$, $s_{l3} = -3$ $h_{ri} = 0.001$, $h_{li} = -0.4$ (i=1,2,3,4). Fig. 2 shows that each of the outputs has successfully followed its desired trajectory with reasonable accuracy. The tracking errors are given in Fig. 3. The control inputs are presented in Fig. 4. The control signals are able to mitigate the impacts of the external disturbances and input dead-zones.

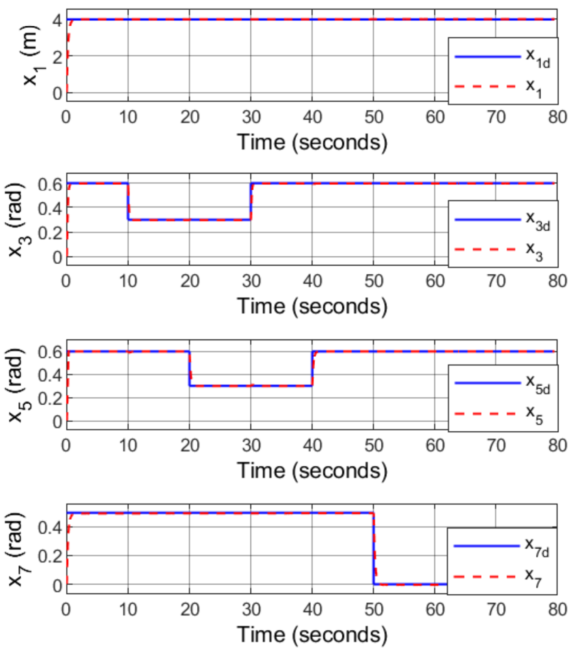

Fig. 2. The trajectory tracking of the outputs

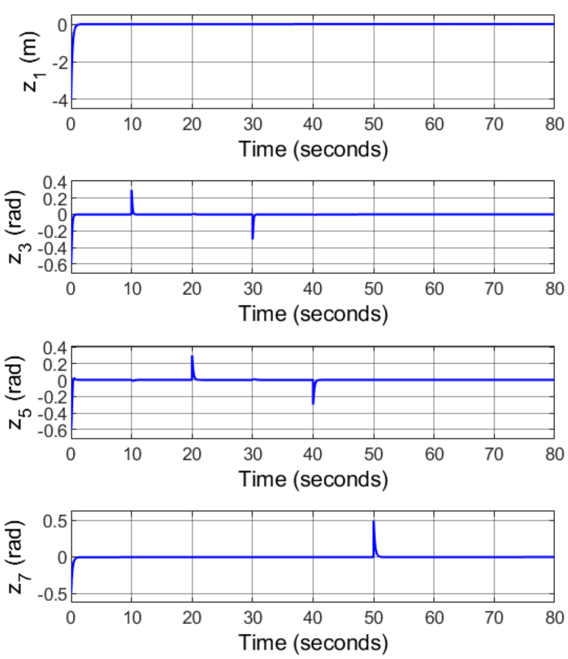

Fig. 3. The tracking errors

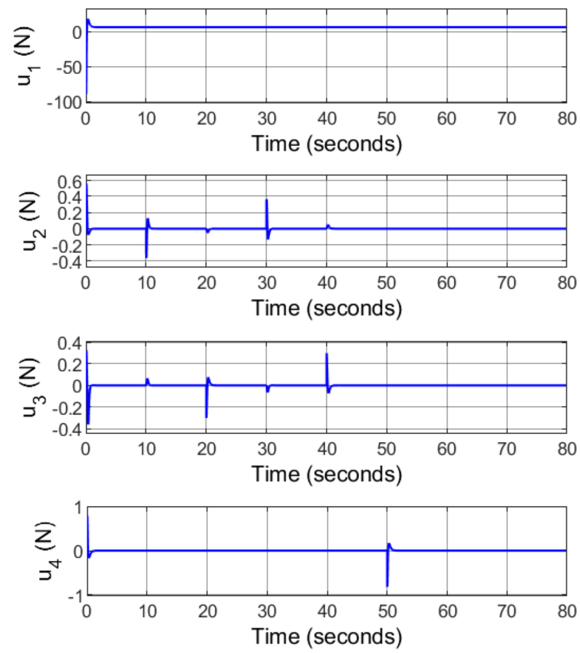

Fig. 4. The control inputs

V. Conclusions

This paper has presented a finite-time RBFNN backstepping control of uncertain nonlinear systems with unknown dynamics and dead-zones subjected to external disturbances. The proposed controller is entirely independent of the system dynamics as it can approximate any unknown function in the system. One RBFNN has been used to estimate the system dynamics, time-varying disturbances, and derivatives of the virtual control laws. As a result, the controller has less computational cost and is easy to implement in practice. Moreover, a positive definite Lyapunov function suggested that all the error signals are semi-globally uniformly ultimately bounded in finite-time near the origin. In order to validate the performance of the proposed controller, it is applied to the dynamic model of a quadrotor with actuator dead-zones and external disturbances. Simulation results show that the proposed controller can achieve excellent tracking in a finite time. In future research, we will investigate how the controller can work for nonlinear systems with unknown time delays and output constraints.

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