Design of an Optimal Fractional Complex Order PID Controller for Buck Converter

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***Abstract*—Dynamic and robust controllers are the inherent requirement of power electronic converters, which are subjected to dynamic variations and nonlinearities. The effectiveness of fractional order controllers in non-linear system control has been well-established by studies in the past few decades. Various forms of fractional order controllers have been used in power-electronic control. Recent research indicates that complex order controllers, extensions of fractional controllers, are more robust against uncertainties and non-linearities than their integer and fractional order counterparts. Though complex order controllers have been employed in various nonlinear plants, they have not been extensively tested on power electronic applications. Also, the design and tuning of the controller is difficult. This paper investigates the effectiveness of a complex order PID controller on a typical power electronic DC-DC buck converter for the first time. Two types of complex order controllers of the form $PI^{a+ib}D^c$ and $PI^{a+ib}D^{c+id}$ were designed for a power electronic buck converter. The complex order controllers were implemented in Simulink and the optimal tuning of the complex order controller parameters for various performance indices was performed using different optimization algorithms. The Cohort Intelligence algorithm was found to give the most optimal results. Both the complex controllers showed more robustness towards uncertainties than the linear and fractional PID controllers. The $PI^{a+ib}D^c$ controller gave the smoothest and fastest response under non-linearities. The dynamic performance of the complex order controller is the best and can be expected to be useful for more power electronic applications.**

Keywords*—Fractional order PID controllers; Complex order PID controllers; Buck converters; Performance indices; Optimization; Cohort Intelligence.

I. Introduction

Power Electronic DC-DC converters use switching power devices for power conditioning and conversion [1]. Buck converters are DC-DC converters used in a wide range of applications from switch mode power supplies to renewable energy applications [2]–[4]. These systems have inherent non-linearities due to switching actions, load variations, magnetic saturation etc. [5]. They experience input fluctuations, and load variations exist due to the large packaging densities of onboard chips [6]. Therefore, there is a requirement for robust and stable controllers which give better ripple reduction, disturbance rejection and fast transient response [7]. Conventional power electronic control methods are PI/PID, sliding mode, H_∞, optimal control, predictive control, fuzzy and artificial neural network-based control etc. [8]–[10]. The PID controller is undeniably the most dominant in the industry due to its simplicity of design and ease of auto-tuning process [11]. PID controllers have been used to control DC-DC converters for different applications [5], [12]–[14]. However, the limitations of the PID controller response to uncertainties and parametric variations make them ill−suited for non-linear control applications [15].

In recent years, considerable research has been conducted on the advantages of fractional order (FO) controllers for power electronic control. The fractional controller uses fractional order derivatives and integrals and is the general form of an integer order controller. Hence fractional counterparts of the linear controllers, such as the FO integrator/differentiator, FOPID, FO sliding mode, FO lead/lag compensator etc., have been designed [16]. The main advantage of fractional order controllers is their robustness and flexibility in control. Fractional order PID controllers (FOPID), proposed by Podlubny, have the general form PI^xD^y, where x and y are non-integer orders [17]. FOPID controllers are very robust against parameter variations and uncertainties and performed better than the conventional PID controllers [18]–[23]. At the same time, designing and tuning the fractional order parameters is challenging. Analytical methods are time-consuming and difficult; hence, numerical methods using optimization techniques have become popular for tuning. This involves optimising an objective function such as a performance index or parameters like the overshoot, rise time, gain margin, phase margin etc., chosen according to the design requirements [24]. Various algorithms have been developed for constrained and unconstrained optimization and used for optimal tuning [25], [26].

Fractional orders have been extended to the complex order, where the derivative or integral order can be complex [19]. Love [27] proposed that locally integrable functions could have imaginary order derivatives. It has been shown that complex order impedances can be realized for high pass filters, like the synthesis of fractional order impedances [28]. Complex controllers are the generalized form of fractional controllers, where the fractional order is replaced by a complex order of the form $z = a + ib$, where a and b are the real and imaginary parts of the complex number z. The use of complex order increases the flexibility when both gain and phase characteristics are significant [29]. Complex controllers are robust against all kinds of uncertainties. They have been found to perform better than their integer and fractional counterparts in linear non-minimum phase plants, unstable systems [30], highly chaotic and non-linear plants [31] and time-varying systems. The frequency response of complex order transfer functions was studied, and it was shown that robustness towards uncertainties could be adjusted by variation of the imaginary order [32]. Different transfer functions can be obtained by choosing different pairs of the real and imaginary orders [33].

Oustaloup used complex operators for control applications for the first time in the third-generation CRONE (Commande Robuste d'Ordre Non Entier) controller [34]. The third generation CRONE controller is used when there are other forms of uncertainty like pole and zero misplacement in addition to gain and phase variations. The advantage of the third-generation CRONE controller is that the complex orders of the integrator can be used to independently control the frequency responses of the magnitude and phase.

A complex order controller was proposed for the robust control of a DC motor by Khandani et al., which made the system robust against gain variations [35]. Machado showed the feasibility of optimization of control algorithms by using two new complex conjugate operators to design complex order PIDs for different second order systems with uncertainties [36]. Shahiri et al. proposed a method for the design of a complex order controller with a complex order integrator to control the oxygen excess ratio in a Proton Exchange Membrane Fuel Cell (PEMFC), the behavior of which is highly non-linear and varies with time. This controller showed an improved time response and lesser steady-state error and gave a better performance than the PI, $FOPI$ and H_∞ controllers [37]. Shahiri et al. also proposed a new method to tune parameters of fractional complex order controller with a complex order integrator for a PEMFC, using certain charts called as K-charts, which are obtained from the solution curves to optimize the phase and gain margin requirements and the maximum of sensitivity functions [33]. Guefrachi et al. proposed two new fractional complex PID controllers, $PI^{a+ib}D$ and PID^{a+ib} having complex order integrator and differentiator, respectively, for a Second Order plus time delay resonant system [38], [39]. Five design parameters were fulfilled and frequency and time domain responses were tested. The system was robust against gain variations, output noise and disturbance. Complex order $PI^{x+iy}D$ controller for a Second Order plus time delay system was tuned and performance studied by Hanif et al. [40]. The transfer function was derived for a complex order PID controller with a first order plus delay system, which showed that the design specifications were more accurately attained by the complex controller [41]. Real and complex order PI controllers were optimized and compared for a first-order system with dead time by Moghadam et al. [42]. Complex coefficients were used in PI/PID controllers and tuned for a general plant structure in [43], [44].

A. V. Tare et al. proposed two new complex order PID controllers for controlling fractional order systems using the Genetic algorithm which showed better time responses compared to integer and fractional controllers [45]. Ravi Sekhar et al. designed the complex $PI^{\alpha+j\beta}D^{\gamma+j\theta}$ controller for accurate machining of nanocomposites and showed that the complex order controller could achieve surface roughness specifications more effectively compared to the PID and fractional order PID (FOPID) controllers. Simulation of a third-generation CRONE controller was done to control a wind turbine [46]. Complex order approximation and design of complex order filters and differentiators were studied in [47], [48]. Complex order modelling in biomedical applications and tuning and control have also been studied [49], [50]. All these studies highlight the robustness of the complex order controller, especially in nonlinear systems with parameter variations and uncertainties.

Power electronic systems have been described by complex order equations and analyzed. Complex variables were used to describe induction motor voltage equations, and the complex order transfer function was derived. Here, the fourth-order system could be reduced to a second-order system with complex conjugates [51]. Complex order transfer functions were used for modelling symmetric systems with balanced three-phase impedances, and stability analysis was done [52]. Complex order root locus techniques were defined and used to control three-phase inverter and power systems [53], [54]. Complex controllers for three-phase power converters were designed using pole placement techniques, and complex coefficient filters [55], [56]. Despite these advances, complex order controllers have not been extensively used in power electronics.

Various fractional order controllers have been successfully used for power electronic applications and are more robust than the integer order controllers [57]–[62]. The effectiveness of fractional order controllers in such non-linear systems is the major motivation to extend it to complex order controllers, which are more robust to uncertainties. As the buck converters are non-linear and may have chaotic behaviour, one of the main objectives in design would be to design robust controllers. Even though fractional order PID controllers are robust, their performance can still be improved [63]. Complex order PID controllers have shown better robustness than fractional order controllers and have not been tested on power electronic DC-DC converters, to the best of our knowledge. At the same time, the design and tuning of complex order controllers is a challenge. Hence, this work aims to design an optimal complex order controller for the buck converter. Metaheuristic optimization methods have been used to find the optimal parameters of the controller. The research contributions of this work are:

- The effect of complex order PID controllers was investigated for the first time on a power electronic DC-DC converter.
- Two types of complex order PID controllers have been designed and implemented on SIMULINK.
- The design of the controllers was done using different optimization methods and a performance comparison was done. The effectiveness of the Cohort Intelligence algorithm was observed.

The performance under various test conditions was studied and compared with linear and fractional PID controllers.

II. Fractional and Complex order basics

A. Definitions

Fractional calculus involves the differentiation and integration of arbitrary order, which can be real, complex, variable or distributed in nature. It can be used for more accurate modelling and control of dynamic systems [64], [65]. Many mathematicians have contributed to the basic definitions [66]–[68]. The fractional order operator can be defined as ${}_aD_t^{\beta}$, where a and t are the limits of operation, and β is the fractional order, and belongs to the real plane [16].

$$ {}_aD_t^{\beta} = \begin{cases} \frac{d^{\beta}}{dt^{\beta}} & ;\ R(\beta) > 0 \\ 1 & ;\ \beta = 0 \\ \int_a^t (d\tau)^{-\beta} & ;\ R(\beta) < 0 \end{cases} \tag{1} $$

Valerio and Costa extended the concept of variable order and gave the definitions for the complex case [69]. The fractional order definitions can be extended to the complex order, where β is replaced by the complex number $\alpha = x + iy$, $q \in Z$, Z being the complex plane. The Riemann-Liouville and Caputo definitions were modified for defining complex order derivatives [19].

1) Riemann-Liouville definitions for complex order:

$$ {}_aD_t^{q} f(t) = \begin{cases} \int_a^t \frac{(t-\tau)^{-q-1}}{\Gamma(-q)} f(\tau) d\tau & ;\ x \in R^- \\ f(t) & ;\ \alpha = 0 \\ \frac{d}{dt} {}_aD_t{}^{q-1} f(t) & ; \\ if\ \ x = 0 \wedge y \neq 0 & \\ \frac{d^{\lceil x \rceil}}{dt^{\lceil x \rceil}} {}_aD_t{}^{q-\lceil x \rceil} f(t) & ;\ if\ \ x \in R^+ \end{cases} \tag{2} $$

$$ {}_tD_a^{\alpha} f(t) = \begin{cases} \int_t^a \frac{(t-\tau)^{q-1}}{\Gamma(q)} f(\tau) d\tau & ;\ x \in R^- \\ f(t) & ;\ q = 0 \\ -\frac{d}{dt} {}_tD_a{}^{q-1} f(t) & ; \\ if\ \ x = 0 \wedge y \neq 0 & \\ (-1)^{\lceil x \rceil} \frac{d^{\lceil x \rceil}}{dt^{\lceil x \rceil}} {}_tD_a{}^{q-\lceil x \rceil} f(t) & ;\ if\ \ x \in R^+ \end{cases} \tag{3} $$

In these equations, $\lceil . \rceil$ is the ceiling function and $\Gamma(.)$ stands for the Gamma function.

2) Caputo definitions for complex order:

$$ {}_aD_t^{q} f(t) = \begin{cases} \int_a^t \frac{(t-\tau)^{-q-1}}{\Gamma(-q)} f(\tau) d\tau & ;\ x \in R^- \\ f(t) & ;\ q = 0 \\ {}_aD_t{}^{q-1} \frac{d}{dt} f(t) & ; \\ if\ \ x = 0 \wedge y \neq 0 & \\ {}_aD_t{}^{q-\lceil x \rceil} \frac{d^{\lceil x \rceil}}{dt^{\lceil x \rceil}} f(t) & ;\ if\ \ x \in R^+ \end{cases} \tag{4} $$

$$ {}_tD_a^{q} f(t) = \begin{cases} \int_t^a \frac{(\tau-t)^{-q-1}}{\Gamma(-q)} f(\tau) d\tau & ;\ x \in R^- \\ f(t) & ;\ q = 0 \\ -{}_tD_a{}^{q-1} \frac{d}{dt} f(t) & ; \\ if\ \ x = 0 \wedge y \neq 0 & \\ (-1)^{\lceil x \rceil} {}_tD_a{}^{q-\lceil x \rceil} \frac{d^{\lceil x \rceil}}{dt^{\lceil x \rceil}} f(t) & ;\ if\ \ x \in R^+ \end{cases} \tag{5} $$

If set to zero, the imaginary order in these equations will result in the fractional derivatives. Hartley et al. [70] showed that complex conjugates could be used as the order of fractional differintegrals to give real time response and real transfer functions. It was shown that fractional system identification can be generalized for complex-order derivatives to describe real-time behaviors better [71], [72]. Barbosa et al. proposed methods for discretization of complex order differintegrals of the form s^{γ}, where γ is a complex number [73].

B. Approximation of operators

The irrational terms in the fractional and complex order systems are infinite-dimensional, and hence the implementation of such systems require finite dimensional approximations. The fractional operator s^{β} is replaced by an integer order transfer function using analog or digital approximation methods. Many integer approximation methods have been proposed [74],but the most commonly used approximation method is the Oustaloup recursive approximation method or the CRONE approximation [75]. Here,the term s^{β}, where $\beta \in [-1, 1]$ can be approximated as shown in Equation (6). Here, N poles and zeros are recursively placed within a limited frequency range $\{\omega_l, \omega_h\}$. The gain K is adjusted so that the gain of $(j\omega)^{\beta} = 1$ at 1 rad/sec.

Discrete order approximations like the Tustin method is used for implementation of discrete FO systems. The fractional operator s^{β} is approximated by the Oustaloup method as:

$$s^{\beta} \approx K \prod_{i=1}^{N} \frac{1 + (s/\omega_{z,i})}{1 + (s/\omega_{p,i})} \tag{6}$$

$$\omega_{z,i} = \omega_l \left(\frac{\omega_h}{\omega_l}\right)^{(2i-1-\beta)/2N} \tag{7}$$

$$\omega_{p,i} = \omega_l \left(\frac{\omega_h}{\omega_l}\right)^{(2i-1+\beta)/2N} \tag{8}$$

Complex order transfer functions can also be approximated as fractional and integer approximations. The Crone approximation again is the best method, as given in [19].

C. Fractional and Complex order PID Controllers

Fractional order PID (FOPID) controllers are a generalized version of the integer order PID (IOPID) controllers [76], [77]. The general FOPID controller is represented as:

$$C(s) = K_p + K_i s^{-x} + K_d s^{y} \tag{9}$$

Here, K_p, K_i and K_d are the gains of the proportional, fractional integrator, and fractional differentiators respectively. x and y are the fractional integrator and differentiator orders. If $x = y = 1$, the integer order PID controller is obtained. Similarly various combinations of x and y give the PI,PD, FOPI and FOPD controllers [78], [79]. If the orders of the integrator (x) and/or the differentiator (y) are complex, the complex order PID (COPID) controller is obtained.

The general form of a COPID controller would be [80], [81]:

$$C(s) = K_p + K_i s^{-(a+ib)} + K_d s^{(c+id)} \tag{10}$$

If $b = d = 0$, the FOPID controller is obtained. If only $d = 0$, we obtain the COPID controller with a complex order integrator and a fractional order differentiator. Similarly, we can obtain various types of controllers by varying the values of b and d. Hence, the linear and fractional PID controllers are simpler versions of the COPID controller. If both the integrator and differentiator orders are complex, there would be seven parameters to be tuned.

III. System Description

The proposed system is a buck converter with a complex order PID controller realized in SIMULINK. The circuit model is used for simulation instead of a mathematical model for more accuracy [82]. Two types of complex order PID controllers have been used for testing. The first controller has a complex order integrator and fractional order differentiator, and the second controller has complex orders for both the integrator and differentiator. Two buck converter systems, one with a resistive load and the other with a motor load, have been tested with COPID controller, and the performance is compared to that of fractional order controllers.

The buck converter is a switch-mode power converter which converts a DC voltage to a lower magnitude using high-frequency power switching devices. A typical control strategy for a DC-DC buck converter circuit controlled using a COPID controller is shown in Fig. 1.

The output is connected to the input when the switch is on, and disconnected when the switch is off. The output signal from the load is compared with the reference signal and the error signal is given to the COPID controller. The controller's output is used for generating Pulse Width Modulation (PWM) signals to actuate the power MOSFET [1]. For a given input V_g and period T, the output voltage V_o and duty cycle,D are given by (11) and (12) respectively.

$$V_o = DV_g \tag{11}$$

$$D = \frac{T_{on}}{T} \tag{12}$$

The buck converter specifications are given in Table I. The design parameters of the buck converter with resistive load can be formulated based on [83], [84]. A voltage mode control strategy was used for the buck converter with resistive load.

TABLE I. Parameters of the Buck Converter with Resistive Load

Parameter	**Value**
Filter Inductance	70 μH
Filter Capacitance	22 μF
Load resistance, R	10 Ω
Switching frequency	100 KHz
Input voltage	24 V
Output voltage	15 V
Inductor current ripple	20% of i_L

For the buck converter system with motor load, the speed control of the motor was done using armature voltage control strategy [85]. In armature voltage control, speed control below and equal to rated speed can be achieved; hence, the buck converter can be used for speed control. A separately excited DC motor was used for the motor load, with the armature voltage regulated by PWM control of buck converter and the field voltage excited from a fixed DC source. The field flux ϕ is controlled by the field current, which is fixed.

$$V_a = E_b + L_a \frac{dI_a}{dt} + I_a R_a \tag{13}$$

$$E_b = K_b \phi \omega \tag{14}$$

$$T_d = K_b \phi * I_a \tag{15}$$

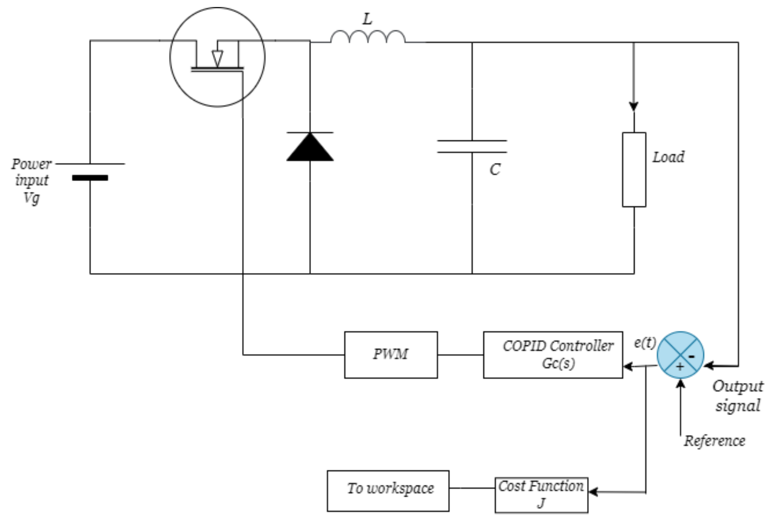

Fig. 1. Buck converter with COPID controller

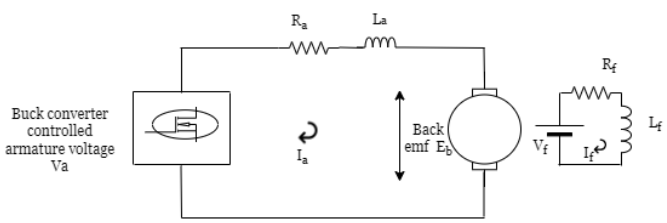

Fig. 2. Separately excited DC Motor equivalent circuit

The armature resistance, armature inductance, back emf, developed torque, armature current and back emf constant of the motor are represented by R_a, L_a, E_b, T_d, I_a and K_b respectively. Equations (13-15) give the relations between armature voltage V_a and angular speed ω. The motor's speed can be controlled by varying the armature voltage, which is the output voltage of buck converter controlled by the COPID controller, in this case. The system specifications with motor load are shown in Table II.

TABLE II. PARAMETERS OF THE BUCK CONVERTER WITH MOTOR LOAD

Parameter	Value
Input voltage	300 V
Filter Inductance	200 μH
Filter Capacitance	500 μF
Load resistance, R	10 Ω
Switching frequency	25 KHz
Armature voltage of motor	240 V
Rated speed of motor	1750 rpm
Field voltage	300 V
Armature Resistance	2.581 Ω
Armature Inductance	28 mH

IV. DESIGN OF THE COPID CONTROLLER

A complex order PID controller has complex orders instead of fractional /integer orders for integration and differentiation, and originates from the 3rd generation CRONE control. Here, two cases are considered for C(s): i) COPID controller with a complex order integrator and fractional order differentiator, denoted as the PI^zD^c controller; ii) COPID controller with complex orders for both integrator and differentiator, denoted as the PI^zD^z controller.

i)PI^zD^c controller:

$$C(s) = K_p + K_i\left(\frac{1}{s}\right)^{a+ib} + K_d s^c \tag{16}$$

Here $z = a + ib$ is the complex order of integration, and c is the fractional order of the differentiator. The complex integrator can be written as:

$$\begin{aligned}\left(\frac{1}{s}\right)^{a+ib} &= \left(\frac{1}{s}\right)^{a}\left(\frac{1}{s}\right)^{ib}\\ &= \left(\frac{1}{s}\right)^{a} \times e^{\ln\left(\frac{1}{s}\right)^{ib}} = \left(\frac{1}{s}\right)^{a} \times e^{ib\ln\left(\frac{1}{s}\right)}\\ &= \left(\frac{1}{s}\right)^{a}\left[\cos\left(b\ln\left(\frac{1}{s}\right)\right) + i\sin\left(b\ln\left(\frac{1}{s}\right)\right)\right]\end{aligned} \tag{17}$$

If the complex operator is applied to a real input it will result in a complex response. So, in practice, it is realized by extracting the real part as $\Re[(\frac{1}{s})^{a+ib}]$ [86].

$$\Re\left(\frac{1}{s}\right)^{a+ib} = \left(\frac{1}{s}\right)^{a}\cos\left(b\ln\left(\frac{1}{s}\right)\right) \tag{18}$$

The imaginary part is ignored, for practical implementation, and the real part is implemented, [87]. But the imaginary part has effect on the complex transfer function, as shown in (18). Hence, the PI^zD^c controller can be expressed as:

$$C(s) = K_p + K_i \left[\left(\frac{1}{s}\right)^a \times \cos\left(b \ln\left(\frac{1}{s}\right)\right)\right] + K_d s^c \quad (19)$$

ii)PI^zD^z controller: This controller has complex orders $(a+ib, c+id)$ for the integrator and differentiator respectively.

$$C(s) = K_p + K_i \left(\frac{1}{s}\right)^{a+ib} + K_d (s)^{c+id} \quad (20)$$

Similar to Eqn(19), the complex differentiator can be written, by ignoring the imaginary part, as:

$$\begin{aligned} \Re(s)^{c+id} &= (s)^c (s)^{id} \\ &= s^c \cos\left(d \ln(s)\right) \end{aligned} \quad (21)$$

Hence the PI^zD^z controller can be expressed as:

$$\begin{aligned} C(s) = K_p + K_i &\left[\left(\frac{1}{s}\right)^a \cos\left(b \ln\left(\frac{1}{s}\right)\right)\right] \\ &+ K_d \left[s^c \cos\left(d \ln(s)\right)\right] \end{aligned} \quad (22)$$

The implementation of the PI^zD^z and PI^zD^c controllers described by the equations shown were done on SIMULINK with the help of the FOMCON toolbox of MATLAB, which can be used for fractional order modelling and control [88]. The implementation of the PI^zD^z controller on SIMULINK is shown in Fig. 3.

V. Methods

The objective is to design a complex order PID controller for a buck converter system. Two cases have been considered:

i) Voltage mode control of a Buck converter with resistive load using COPID controller.

ii) Speed control of a buck-converter fed motor load using COPID1 controller.

The plant with the COPID controller is implemented on SIMULINK. The COPID controllers of the form PI^zD^c and PI^zD^z have been implemented according to (19) and (22) and were used individually for the control of the plant. The controller parameters are obtained by optimising a suitable objective function J. Here, the performance indices ISE (Integral Squared Error), IAE (Integral Absolute Error), ITSE(Integral of time multiplied by the squared error), and ITAE (Integral of time multiplied by Absolute error) have been used as objective functions for comparative analysis. These indices are used in optimal control to evaluate the performance of the closed-loop system [89].

The performance indices are calculated from the error signal, which is the difference between the output signal of the plant and the reference signal [90], [91]. The output voltage is compared with the desired reference voltage for the buck converter with resistive load, and the error generated is used to calculate the desired performance index. The error signal is the difference between the actual motor speed and the reference speed for the system with the motor load. Various performance indices are defined in (23)- (26) where the error signal is $e(t)$ [90], [91].

$$J_1 = ISE = \int_0^t e^2(t)dt \quad (23)$$

$$J_2 = ITSE = \int_0^t te^2(t)dt \quad (24)$$

$$J_3 = ITAE = \int_0^t t \mid e(t) \mid dt \quad (25)$$

$$J_4 = IAE = \int_0^t \mid e(t) \mid dt \quad (26)$$

The parameters of the PI^zD^c controller to be tuned are the proportional, integral and derivative gains K_p, K_i, K_d, the complex integrator orders (a, b) and the fractional differentiator real order c. The PI^zD^z controller has a complex differentiator; hence, an additional parameter corresponding to the imaginary order d must be tuned. These are obtained by an optimization algorithm, which calculates the optimal parameters by minimization of the cost function J. Different metaheuristic optimization algorithms like the Cohort Intelligence (CI) algorithm [92]–[95], Particle Swarm Optimization (PSO) [96], Artificial Bee Colony (ABC) optimization [97] and the Genetic Algorithm (GA) [98] were tested on the controller for comparison. The process of tuning the controller is summarized below:

- Plant with complex order controller is implemented on SIMULINK.
- The respective complex order controller block is implemented as shown in Fig. 3.
- The fractional integrator order $\left(\frac{1}{s}\right)^a$ and fractional differentiator order s^c are approximated using Oustaloup's approximation with order five and a frequency band with the limits [0.01,1000000].
- The COPID controller's parameters K_p, K_i and K_d are chosen from a sampling interval of [0,200], and the complex order parameters (a, b, c, d) are chosen from a sampling interval of [0,2] after some trial runs.
- The performance index is calculated by the optimization program, which calls the circuit in SIMULINK for each iteration.
- The parameters are updated for each value of the cost function.
- This process is repeated till the termination condition is reached, and the optimal values of the controller parameters and the performance index are returned.

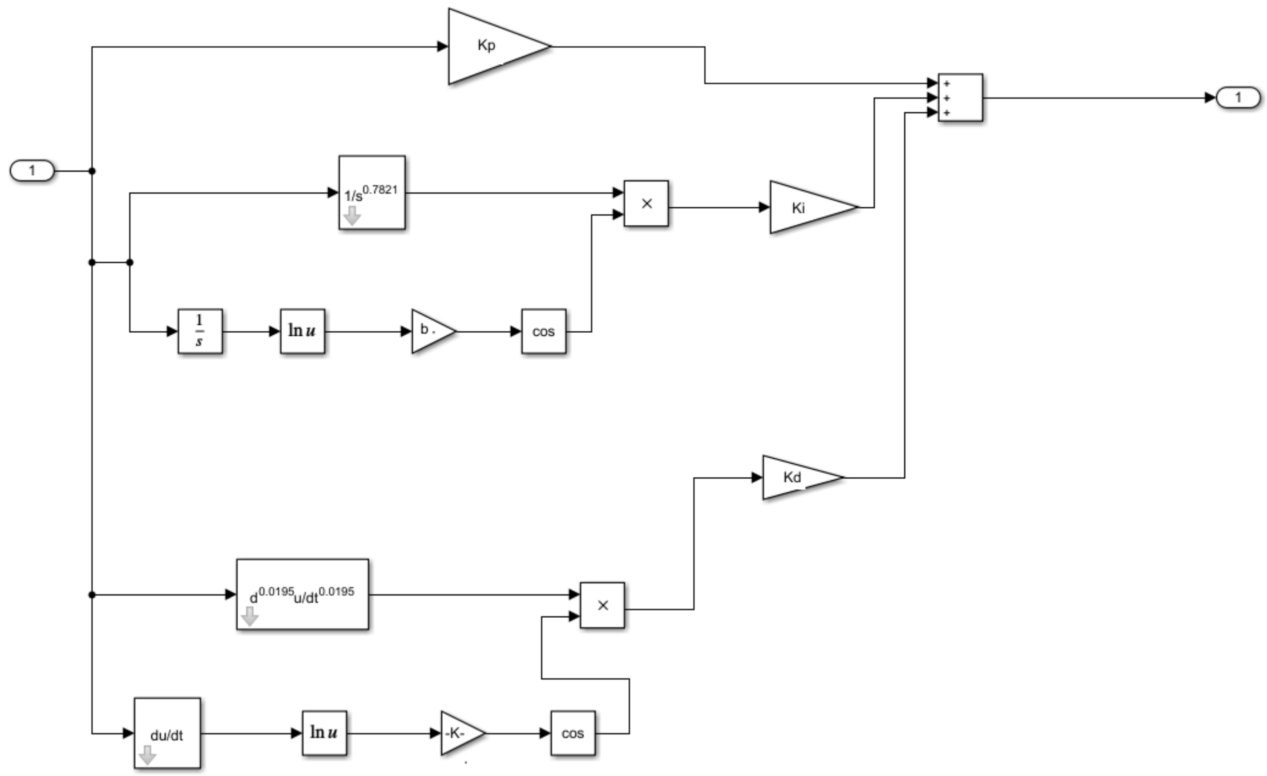

Fig. 3. Implementation of PI^zD^z controller in SIMULINK

Metaheuristic algorithms like the PSO, ABC, GA and CI have been used for optimization and performance was compared. The PSO and ABC are based on the behaviour of a swarm, and the GA method uses natural selection and evolution. The Cohort Intelligence (CI) method is a metaheuristic algorithm, proposed by Kulkarni et al [92]. This method works on the behaviour of a class of candidates, called the cohort, and is useful in constrained and unconstrained problem solutions [99], [100]. A particular behaviour can be imbibed from a set of qualities. Here the candidates of the cohort interact with each other and try to improve their behaviour by observing the others and improving their individual qualities. This leads to an overall improved behaviour for the whole cohort, as each candidate tries to learn from the others [101]. The cohort behaviour reaches saturation when the behaviour remains the same after several attempts. While using the CI algorithm for the complex order controller tuning, the candidate's qualities are the controller parameters randomly chosen from an interval, which is modified using a reduction factor, till the termination condition occurs. The qualities are improved by observing the other candidates to obtain the best behaviour. Here, the controller's parameters are tuned for the best objective function.

The CI algorithm gave faster results; hence, the values obtained have been used for testing various operating conditions. The flow chart of the CI algorithm for optimal tuning of the COPID1 controller is shown in Fig. 4. The COPID2 controller, which has seven parameters, can also be tuned similarly. The COPID1-controlled buck-converter system with motor load is shown in Fig. 5.

VI. Results

A. Buck converter with resistive load: Plant 1

The buck converter system with complex order PID controllers was implemented on SIMULINK. The optimization algorithms were run on MATLAB R2020b On Windows 10 with Intel Core i5 processor. The system was tested with the PI^zD^c and PI^zD^z controllers and the results are discussed.

1) Large Signal Response of the System for Plant 1: The transient response of the system with the PI^zD^c (COPID1) and PI^zD^z (COPID2) controller was studied by using different optimization algorithms for the design of the system parameters. The Cohort Intelligence, PSO, ABC and GA algorithms were used respectively for the optimization of the system parameters for the minimization of the cost function J, and the transient response performance was compared. The ISE, IAE and ITAE performance indices were used individually as the cost functions. The lower and upper limits for the controller parameters

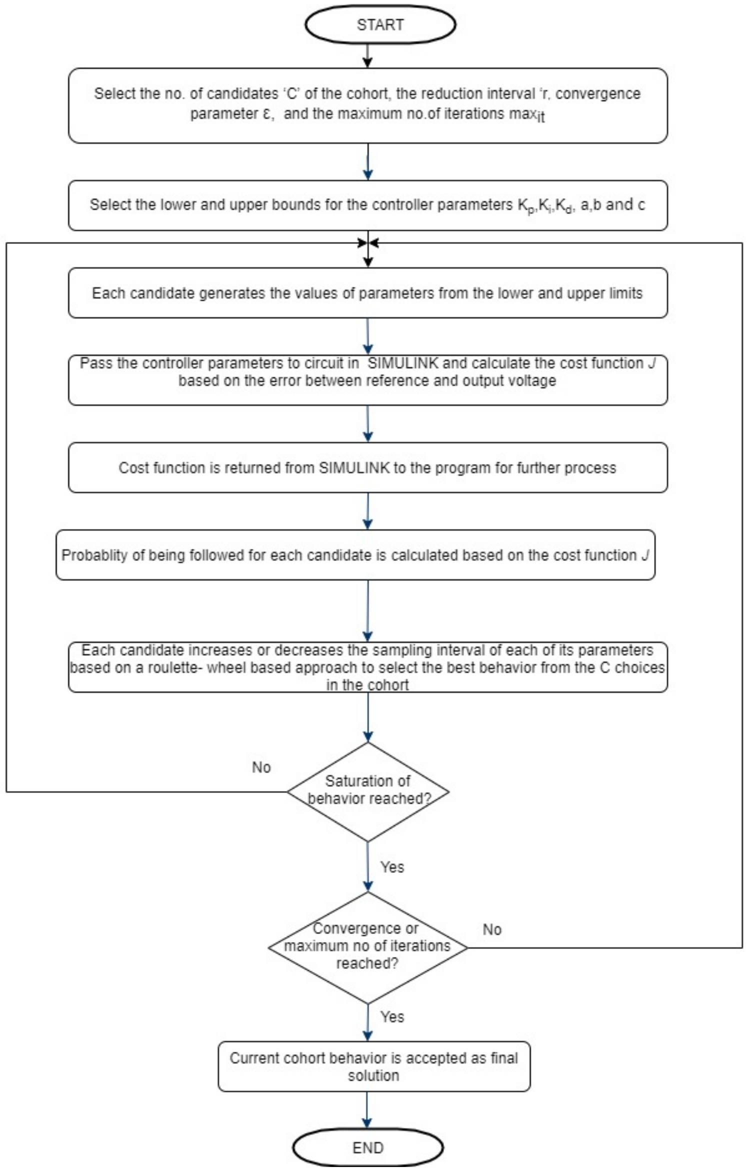

Fig. 4. Flow Chart of the CI Algorithm for the optimization of COPID1 controller

K_p, K_i, K_d were kept at [0,200] for all the algorithms for comparison. The limits were selected after some initial trial runs of the algorithm. After comparison of different algorithms, the CI algorithm was chosen for testing, as it gave the fastest results. Tables III - V shows the results of various algorithms with the COPID1 controller.

The CI algorithm gave the most optimal results as it required the least number of iterations and minimum function count. The cost function values and the transient response parameters were comparable with the other algorithms. But the CI results were the fastest, and hence the CI results were chosen for testing various operating conditions. It was observed that the system with COPID1 controller gave considerable overshoot. To reduce the overshoot, the system was tested with the optimal fractional order controller parameters (K_p, K_i, K_d, a, b), and further optimized by variation of the imaginary order b of the integrator. There was no overshoot and the transient response characteristics were quite similar to the FOPID-based system. The system response with optimal FOPID parameters for a particular value of imaginary order b is shown in Fig. 6.

Table VI shows the various time-domain parameters of the system by variation of the imaginary order 'b'. Negative values of the imaginary order were also tested. The variation of the imaginary order b in the range $(-1.8, 1.5)$ gave good results. The value of b = 1.5 gave the most optimal response with zero overshoot, least steady state error, and less settling time

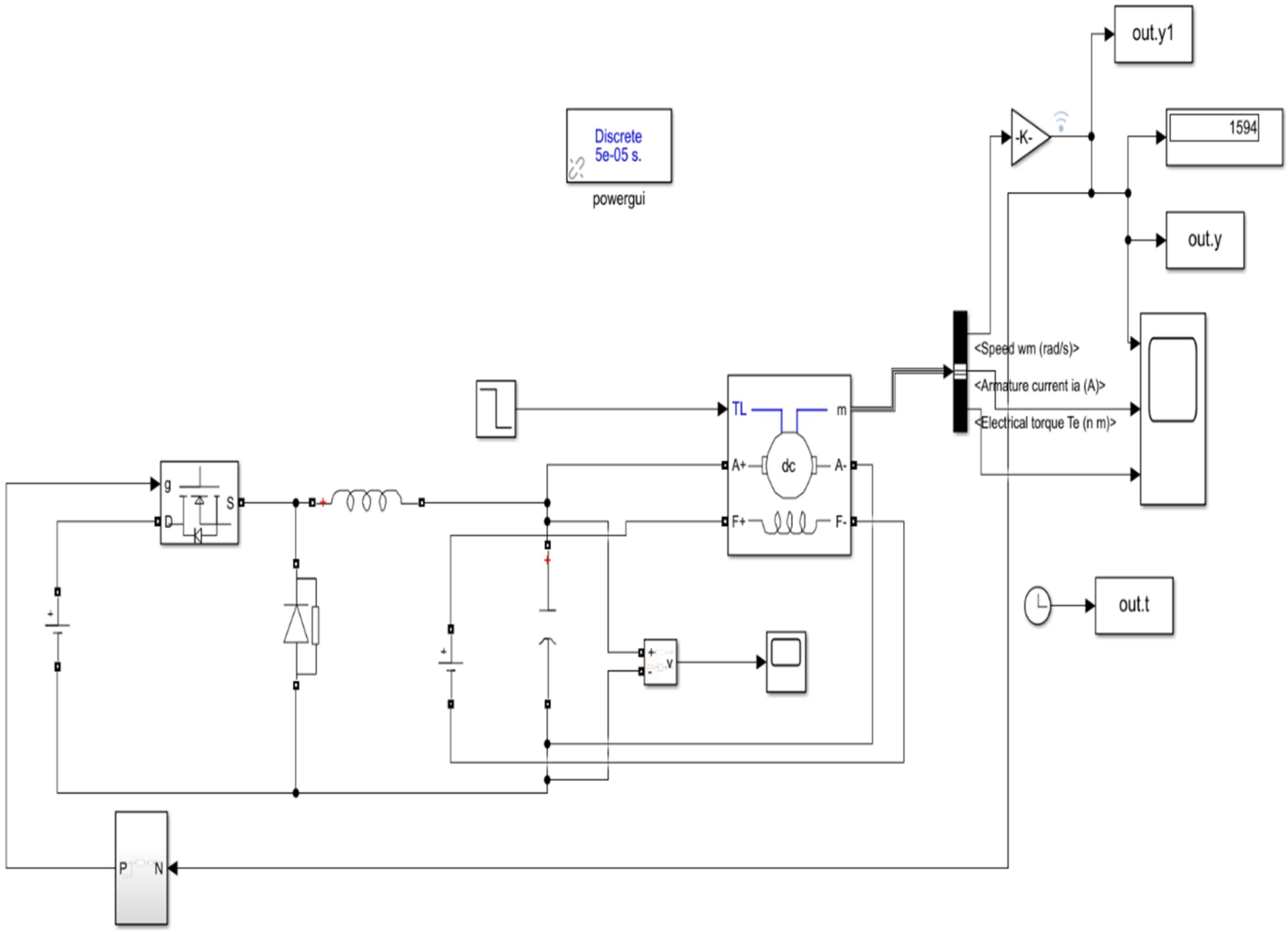

Fig. 5. Implementation of the COPID1-controlled buck-converter system with motor load

TABLE III. Comparison of different algorithms for the COPID1 controlled Plant 1 for the cost function ISE

Algorithm	J=ISE					
	Max. Overshoot (%)	**Rise Time (ms)**	**Settling time (ms)**	**No. of iterations**	**Function count**	J_{ISE}
CI	6.20	0.039	0.34	21	84	0.0092
PSO	7.60	0.05	0.39	33	136	0.0090
ABC	8.86	0.05	0.46	50	200	0.0092
GA	12.20	0.036	0.90	71	284	0.0114

TABLE IV. Comparison of different algorithms for the COPID1 controlled Plant 1 for the cost function IAE

Algorithm	J=IAE					
	Max. Overshoot (%)	**Rise Time (ms)**	**Settling time (ms)**	**No. of iterations**	**Function count**	J_{IAE}
CI	6	0.05	0.37	21	84	0.0045
PSO	6	0.05	0.39	21	84	0.0019
ABC	6	0.05	0.46	50	200	0.0023
GA	12	0.05	0.42	61	244	0.0020

TABLE V. COMPARISON OF DIFFERENT ALGORITHMS FOR THE COPID1 CONTROLLED PLANT 1 FOR THE COST FUNCTION ITAE

Algorithm	J=ITAE					
	Max. Overshoot (%)	Rise Time (ms)	Settling time (ms)	No. of iterations	Function count	J_{ITAE}
CI	6.9	0.055	0.37	21	84	0.00009
PSO	8.2	0.05	0.39	33	136	0.00004
ABC	6.7	0.05	0.46	50	200	0.00002
GA	12	0.05	0.42	62	248	0.0006

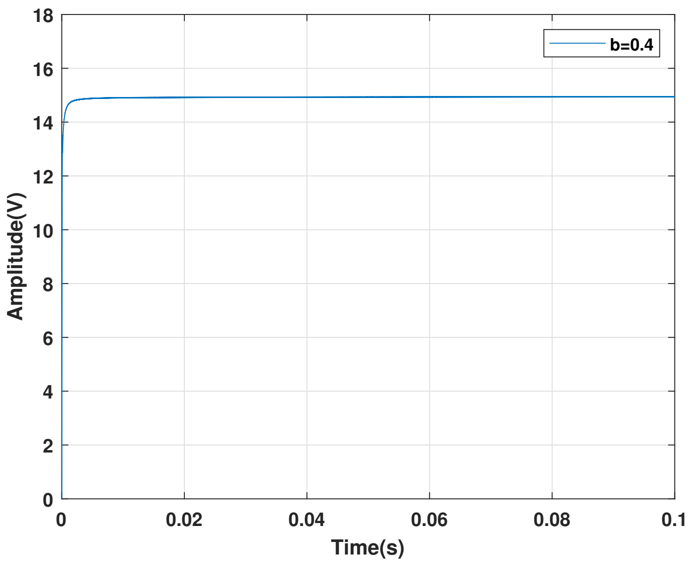

Fig. 6. Start-up response of PI^zD^c system

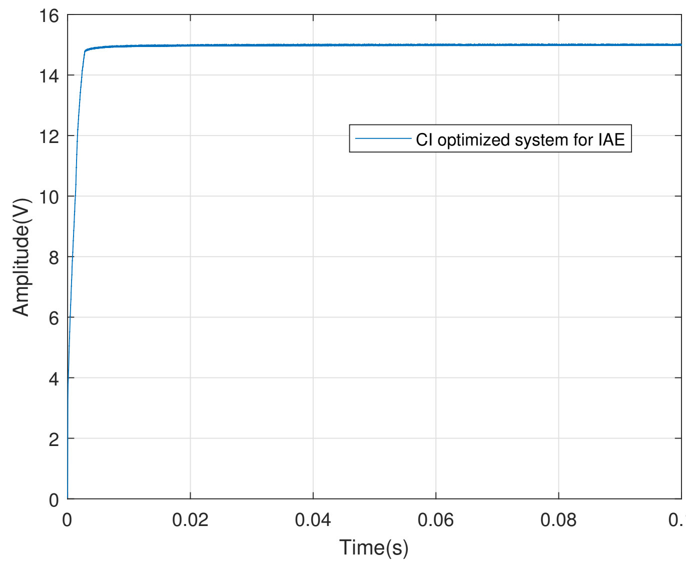

Fig. 7. Start-up response of the COPID2 controlled Plant 1 optimized for IAE

but the rise time was slightly higher. Thus an optimal value of the design specifications can be obtained by adjustment of the imaginary order.

The buck converter plant1 with the PI^zD^z (COPID2) controller was also tested with different optimization algorithms and using different cost functions. Here the parameters b and d are the imaginary orders of the complex order integrator and differentiator respectively. The complex orders a,b,c,d were chosen from the limits between 0 and 2. In comparison to the COPID1 controller start-up response which showed overshoot, the PI^zD^z controller gave no overshoot. The performance indices and transient response parameters obtained with the CI algorithm has been tabulated in Table VII. The results with other optimization algorithms are similar. The response of the COPID2 system optimized with the CI method for IAE is shown in Fig. 7. There is no overshoot, but the rise time and settling time are higher than the COPID1 system response.

The system with PI^zD^c and PI^zD^z controllers was compared with the PID and fractional order PID controllers. The optimal values obtained with the CI algorithm were used for testing all the controller performances. The various time domain parameters obtained with the different controllers using the same values for K_p, K_i and K_d for comparison have been tabulated for the cost function ISE as shown in the Table VIII.

Fig. 8 shows the Plant 1 response with different controllers with the same values of K_p, K_i and K_d. The COPID and FOPID controllers showed no overshoot, whereas the PID controlled system gave 50% overshoot. It is observed that the response of the COPID1 controller was similar to the FOPID controller response. The COPID2 system rise time and settling time is the highest. The buck converter with the COPID1 controller gave continuous conduction mode (CCM) for the optimization of all the cost functions. But the other controllers failed to give CCM for the optimization of the cost function IAE. This is an added advantage of the COPID1 controller.

The control signals for the various controllers are shown in Fig. 9. The control effort for the complex order controllers is lesser than that of the FOPID controller.

2) COPID system response to gain variations: Fractional order controllers exhibit isodamping, which is robustness to gain variations. This property was tested for the COPID controllers by varying the DC gain of the system upto 45.83%. The response with COPID1 controller is shown in Fig. 10. The

TABLE VI. EFFECT OF VARIATION OF THE IMAGINARY ORDER OF INTEGRATOR OF THE COPID1 CONTROLLER FOR PLANT 1

Value of b	Max Overshoot (%)	Steady state error	Rise time (ms)	Settling time (ms)
0.41	0	0.10	0.0019	1.33
0.5	0	0.10	0.0009	1.565
0.9	7.3	0.016	0.0058	0.30
1.5	0	0.024	0.0056	0.90
-0.2	0	0.03	0.0070	0.42
-0.5	0	0.07	0.0095	2.00
-1.8	6.7	0.1	0.0056	0.30

TABLE VII. PERFORMANCE INDICES AND TIME DOMAIN PARAMETERS FOR PI^zD^z CONTROLLER FOR PLANT 1

Index	Parameters							
	M_p (%)	t_r (ms)	t_{ss} (ms)	a	b	c	d	J
ISE	0	2.1	3	0.782	0.019	0.44	1.59	0.0910
IAE	0	2	2.7	0.746	0.054	0.589	0.127	0.0220
ITAE	0	2.2	0.46	0.2961	0.1454	0.501	1.25	0.0004
ITSE	0	2.2	3.29	0.069	0.087	0.361	1.36	0.0001

TABLE VIII. COMPARISON OF PID, FOPID AND COPID FOR PLANT 1

Parameters	PID	FOPID	COPID1	COPID2
Kp	162.08	162.08	162.08	162.08
Ki	133.84	133.84	133.84	133.84
Kd	0.58	0.58	0.58	0.58
Integrator real order 'a'	1	0.067	0.546	0.78
Integrator imaginary order 'b'	0	0	0.407	0.44
Differentiator real order 'c'	1	0.611	0.319	0.0195
Differentiator imaginary order 'd'	0	0	0	1.59
Percentage max. overshoot (M_p)	50%	0	0	0
Rise time (t_r) (ms)	0.096	0.038	0.12	1.19
Settling time (t_s) (ms)	0.25	0.265	0.6	2.2

COPID1 controller responded well and showed robustness to a range of gain variations, similar to the FOPID response. The system settles in approximately 0.18 ms for each case, with a maximum overshoot of 19% for a gain increase upto 45.83%. The rise time is also small, around 0.38 ms for each case. The system behaved similar to the FOPID controller, even though the overshoot was slightly higher.

The response of the COPID2 controlled system subjected to a range of variable DC gains from 8% decrease to 67% increase is shown in Fig. 11. There was no overshoot and the system showed robustness for a wide range. The settling time is more than the COPID1 system, as discussed in the preceding section. But the range of system robustness is much more for the COPID2 controller than for the COPID1 controller, and with no overshoot.

Table IX shows the transient response parameters of the plant 1 with various controllers. Fig. 12 shows the responses of various controllers for an increase in the DC gain upto 45.83%. The PID controller gave the maximum overshoot, with an oscillatory response. The COPID2 controller did not give any overshoot but had a higher settling time. The COPID1 controller had lesser overshoot, and rise time than the FOPID controller, but with a slightly higher settling time.

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3) COPID system response to set-point variations: The response of the COPID system was analyzed for different set-point conditions. The COPID1 and COPID2 controller responses were compared with the FOPID and PID controller responses under the same conditions. The PID-controlled circuit had a much higher overshoot than the FOPID and COPID circuit. A change in the reference value from 15V to 16V at 0.03 s is shown in Fig. 13. The PID controller had large

TABLE IX. CONTROLLER RESPONSE PARAMETERS OF PLANT1 UNDER DC GAIN VARIATION TO 45.83%

Controller	Max Overshoot (%)	Steady state error	Rise Time (ms)	Settling time (ms)
PID	50	0.13	0.7	0.83
FOPID	11	0.07	0.038	0.12
COPID1	2	0.12	0.036	0.24
COPID2	0	0.12	0.38	0.53

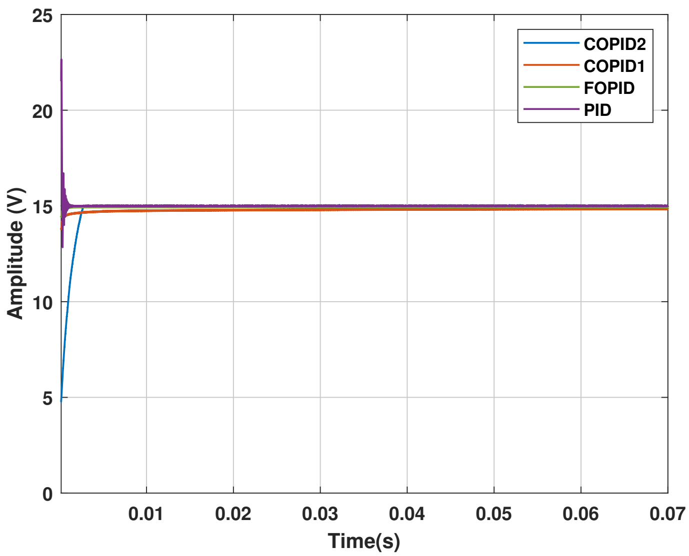


Fig. 8. Comparison of the system start-up response for Plant 1

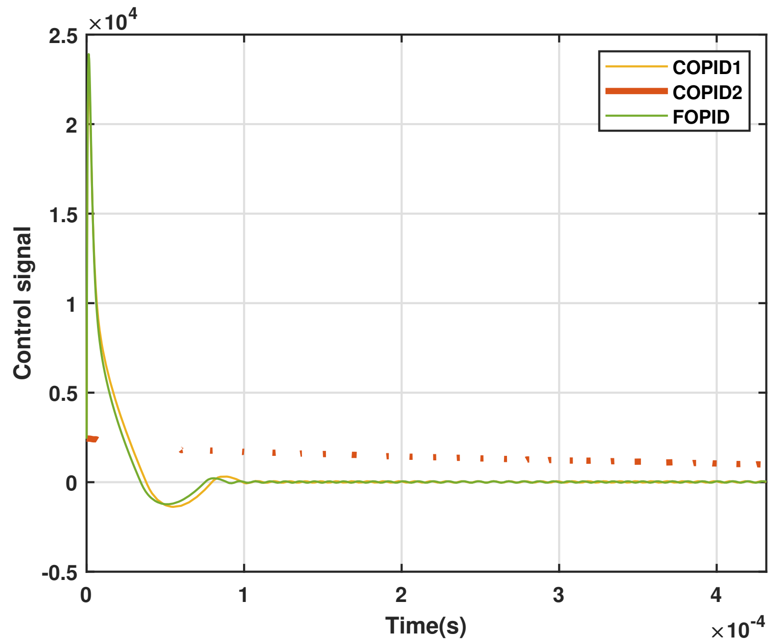


Fig. 9. Control signals of various controllers for the Buck converter

overshoot and took time to settle, whereas the FOPID and COPID1 system responses were smooth. The COPID2 system did not have an overshoot but took much more time to settle. It was observed that the COPID1 system response was the fastest and the smoothest without any overshoot.

4) Response of COPID system to dynamic load variations: The dynamic response of the optimized COPID1 system was studied with step changes in the load, and compared with the FOPID controller-based system under the same conditions. The system was subjected to a step load change from 3Ω to 10Ω at 0.005s. The response is shown in Fig. 14. Fig. 15 shows the response for a step load change from 3Ω to 20Ω in 0.005s and from 20Ω to 10Ω in 0.01s. The COPID1 controller responds faster with lesser overshoot. The change is much smoother for the COPID1 system than for the FOPID-controlled system.

The COPID2 controller also gave similar response, with smooth variation for step changes in load. The response for a step-change in load from 40Ω to 50Ω at 0.05s is shown in Fig. 16. It was observed that there was hardly any variation in the system response at the chosen operating point. But the COPID2 controller did not respond well to lower value of loads.

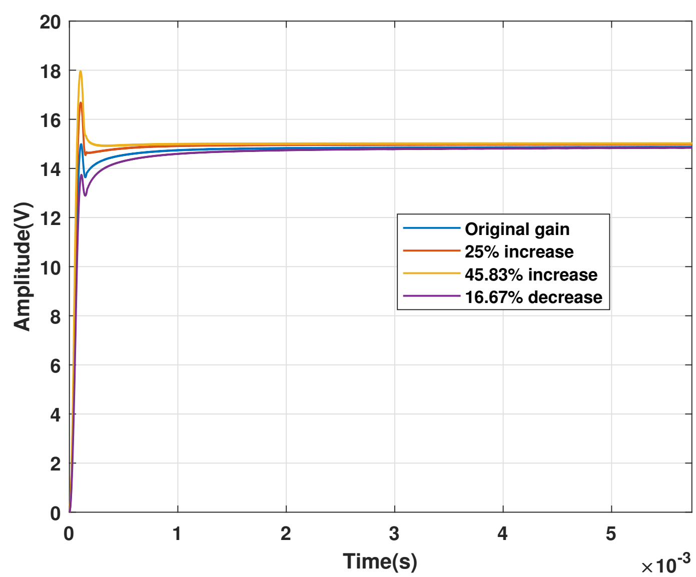


Fig. 10. COPID1 system response to gain variations for Plant 1

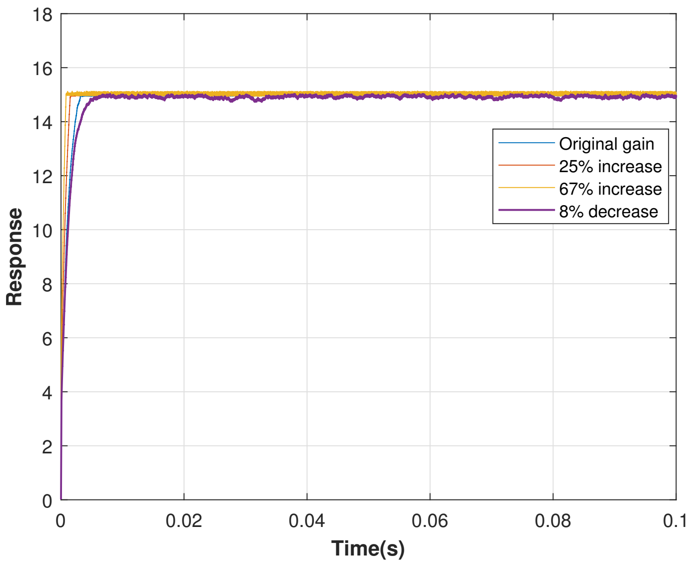


Fig. 11. COPID2 system response to gain variations for Plant 1

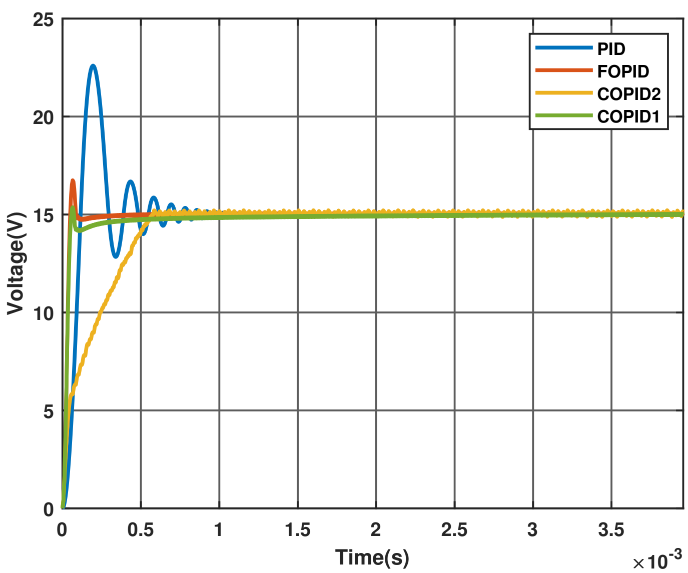


Fig. 12. Plant 1 Response to DC gain variations for various controllers

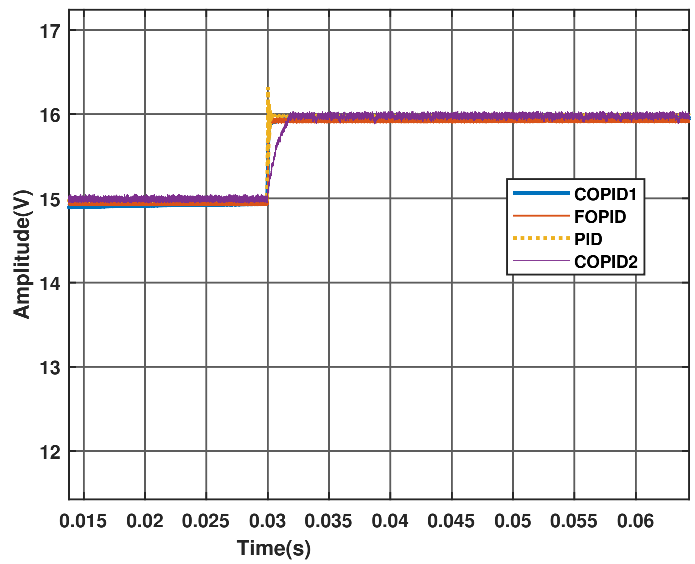


Fig. 13. Comparison of various controller responses of Plant 1 for set-point variation from 15V to 16V.

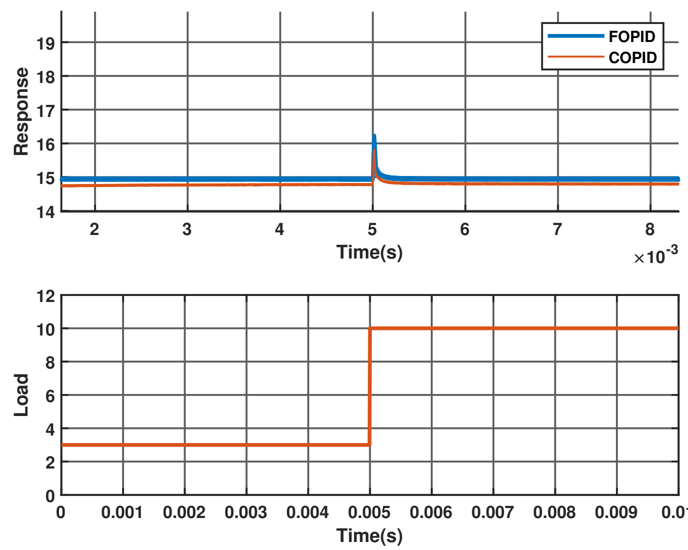


Fig. 14. COPID1 and FOPID system response for a step-change in load from 3Ω to 10Ω

5) Response of COPID controlled system to parametric variations: The buck converter system with the optimized COPID1 controller was subjected to parametric variations. The COPID2 controller also handled parametric variations well, but the response of the COPID1 controller was still better and smoother. The COPID system response to filter parameter variations is shown in Fig. 17.

Fig. 18 shows the system response to load variations. It is seen that the same COPID1 controller can handle parametric variations quite efficiently without retuning. This is compared to the response of the FOPID controller, and it is seen that these variations are handled more smoothly by the COPID1 controller. The steady-state error for the COPID2 system is high for smaller loads. The COPID2 system responds well for loads greater than 40Ω.

For a large range of load variation from 3Ω to 60Ω, the maximum overshoot shown was around 14% for the COPID1 controller, and 24.5% for the FOPID controller, whereas there was no overshoot for the COPID2 controller. The COPID1 response was the best.

A comparison of the different controller responses for Plant1 is shown in Table X.

TABLE X. COMPARISON OF DIFFERENT CONTROLLER RESPONSES FOR PLANT1

Parameters	**Type of Controller**			
	PID	**FOPID**	**COPID1**	**COPID2**
Avg. Overshoot	Highest	Low	Low	No overshoot
Rise Time	Low	Low	Medium	High
Settling time (ms)	Low	Low	Medium	High
Cost function	Low	Lowest	Low	Medium
Robustness to gain variations	Low	Good	High	Highest
Robustness to set-point variations	Low	Good	Highest	High
Robustness to dynamic load variations	Low	Good	Highest	High

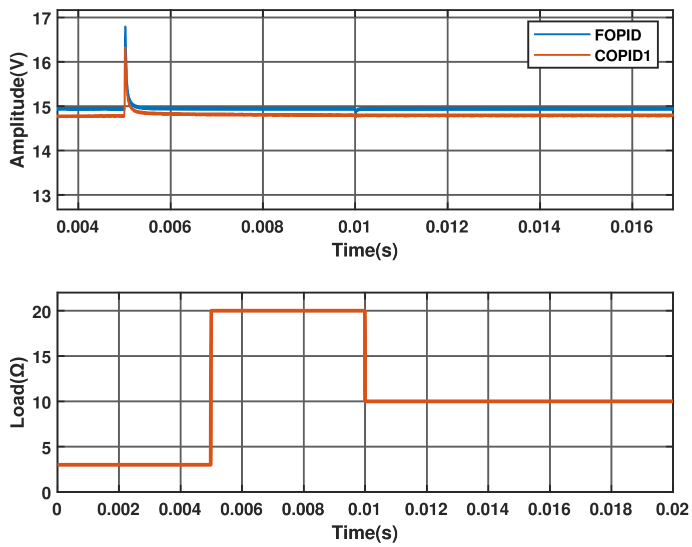


Fig. 15. COPID1 and FOPID system response for step-changes in load from 3Ω to 20Ω and from 20Ω to 10Ω

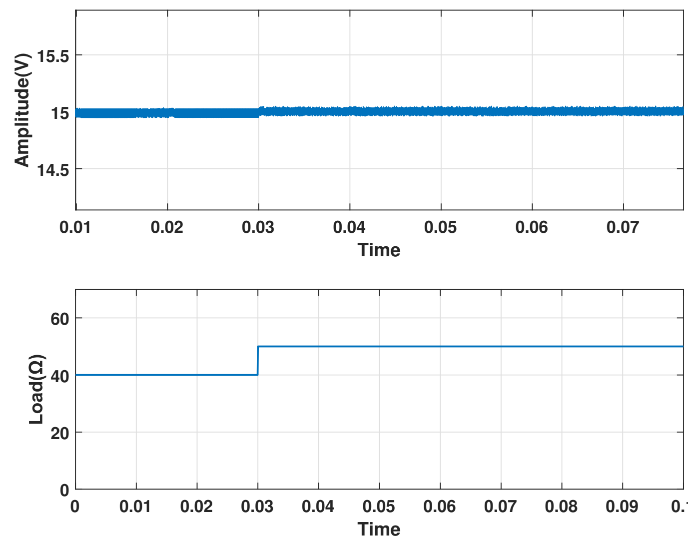


Fig. 16. COPID2 with Plant1 response for step-change in load

6) Performance Comparison of FOPID and COPID1 controller responses with different plant parameters: The Plant 1 parameters were varied using the circuit parameters of [63] and used to compare the COPID1 and FOPID controller responses. The circuit parameters used for testing are shown in Table XI. The proportional, integrator and differentiator gains were also set to the same values of the optimal FOPID controller used in [63]. The transient response parameters of the FOPID and COPID1-controlled systems were observed as shown in Fig. 19. The COPID1 controller responded more smoothly as shown with lesser steady-state error. The FOPID response had a more oscillatory response. The dynamic performance of both systems was studied and shown in Table XII. The overshoot (M_p), settling time (t_{ss}) and peak-to-peak (pp) ripple are given for both systems. For a wide range of gain variations up to 52%, the COPID controller showed a maximum overshoot of only 2.02%. The FOPID controller showed a maximum overshoot of 4.16%. The settling time and peak-to-peak ripple were comparable for both systems. The values show that the COPID1-controlled plant has better robustness to variations than the FOPID-controlled system.

TABLE XI. PLANT1 PARAMETERS OF REF. [63] FOR COMPARISON

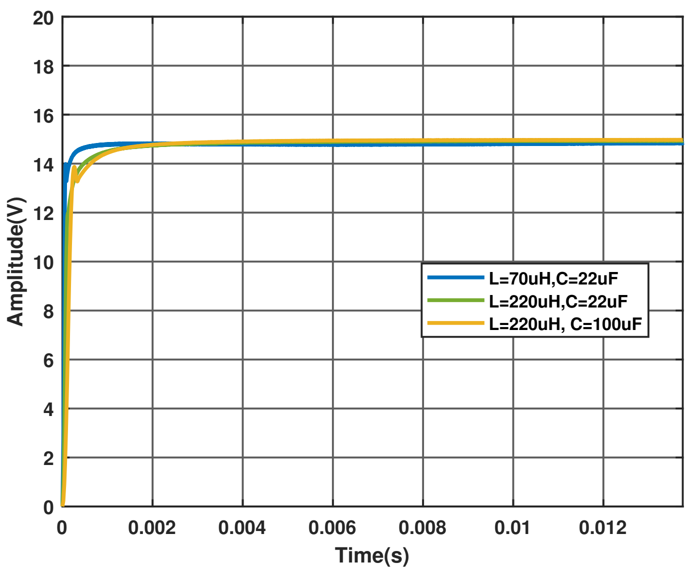
Parameter	**Value**
Filter Inductance	1 mH
Filter Capacitance	100 μF
Load resistance, R	6 Ω
Switching frequency	40 KHz
Input voltage	36 V
Output voltage	12 V

B. Buck converter with motor load: Plant 2

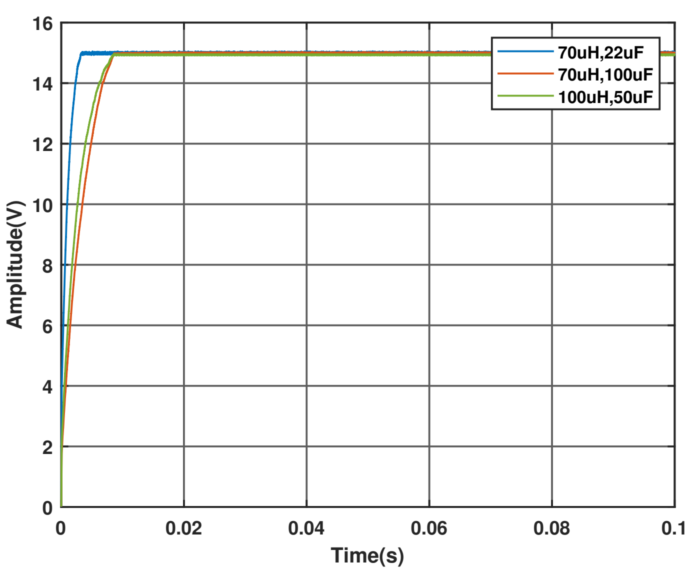
The second system is a buck converter system designed for the speed control of a 5HP, 240V, 1750 rpm separately excited DC motor. It was seen that the COPID2-controlled system response to the motor load was not good, and gave

TABLE XII. PERFORMANCE COMPARISON OF THE DYNAMIC RESPONSE OF FOPID AND COPID1 CONTROLLERS WITH DIFFERENT CIRCUIT PARAMETERS

Dynamic Response	**COPID1**			**FOPID1**		
	M_p (%)	t_{ss}	Ripple (pp)	M_p (%)	t_{ss}	Ripple (pp)
Setpoint variation	1.33	2e-3	0.04	1.33	3e-3	0.04
Gain Variation upto 52%	2.02	4e-3	0.02	4.16	4e-3	0.03
Load current variation from 0.24A to 2A	2.3	4e-3	0.05	2.6	5e-3	0.19

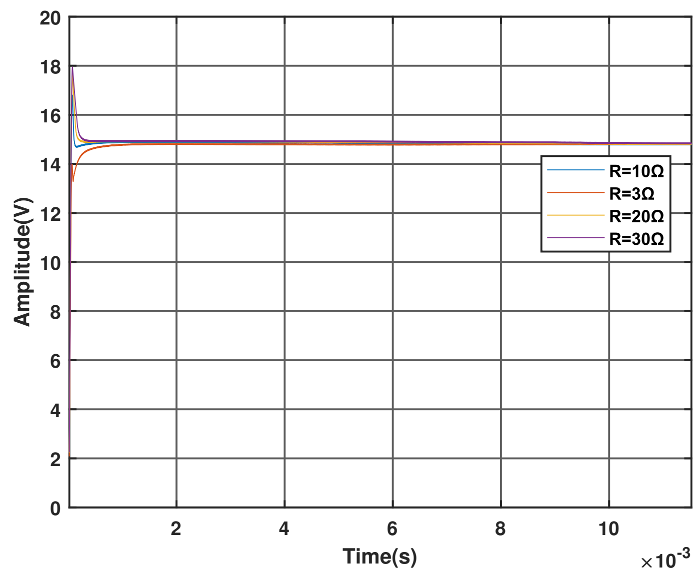


(a) COPID1 system

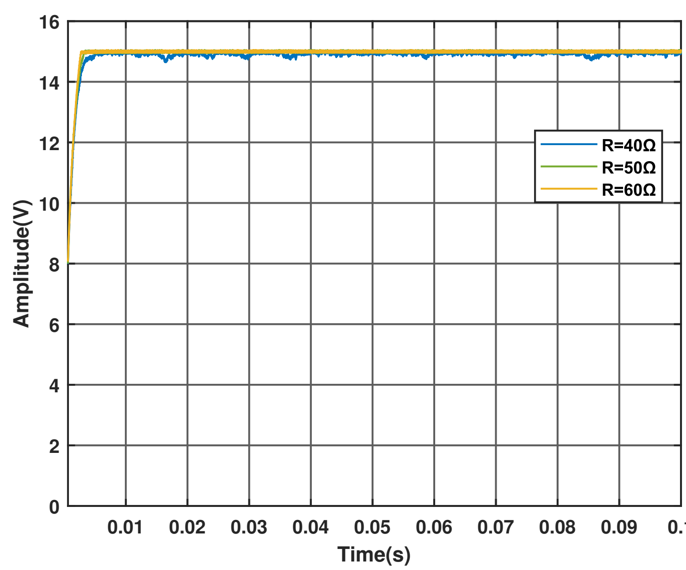


(b) COPID2 system

Fig. 17. COPID system response of Plant 1 to filter parameter variations



(a) COPID1 system



(b) COPID2 system

Fig. 18. COPID response of Plant1 to load variations

a large steady-state error. Hence the system was studied with the COPID1 controller. The COPID1 system response for a set reference speed sp1=1600 rpm was compared with the

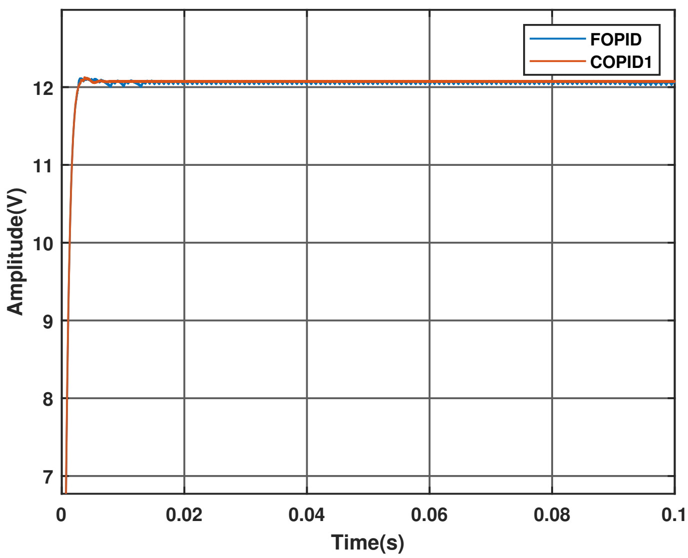


Fig. 19. Transient response of FOPID and COPID1 systems with circuit parameters of [63]

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FOPID system response and shown in Fig. 20. Both the systems showed similar rise times, but the COPID1 system showed lesser overshoot and settled faster than the FOPID system. The system was tested for various set-points and showed similar response.

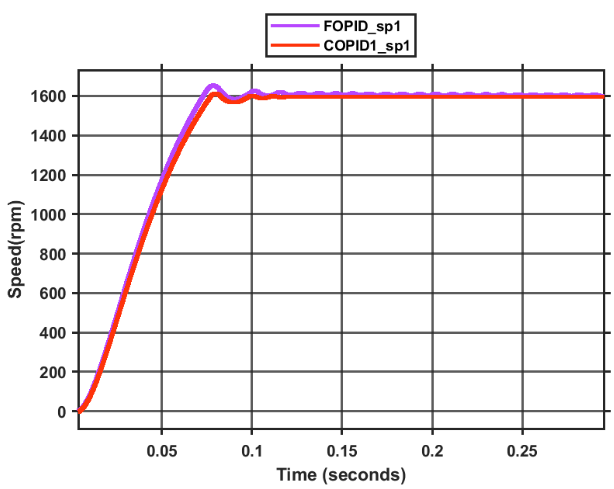

Fig. 20. Plant 2: COPID1 and FOPID System responses with reference speed of 1600rpm

The system with COPID1 controller was optimized using the CI algorithm for the performance indices of ISE and ITSE , and the results are summarized in Table XIII.

1) System response to Setpoint variations: The reference speed was changed from 1750 rpm to 1600 rpm at 1 second, and the system responses with COPID1 and FOPID controllers were compared. The FOPID-controlled system had more oscillations and took more time to settle. The COPID system settled faster than the FOPID-controlled plant and had a lesser steady-state error, as seen in Fig. 21.

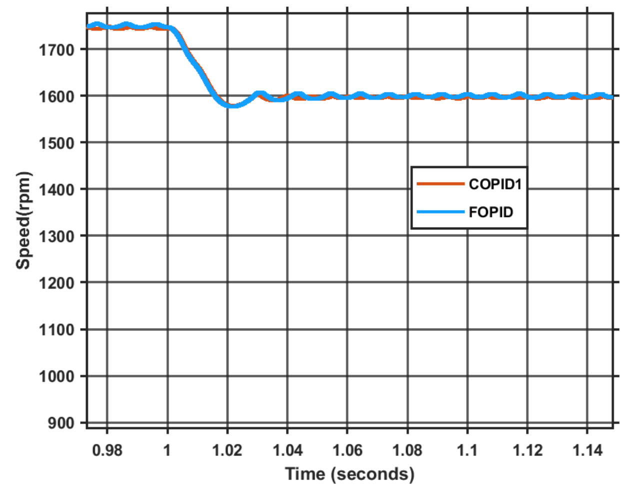

Fig. 21. System response with step-variation in reference speed

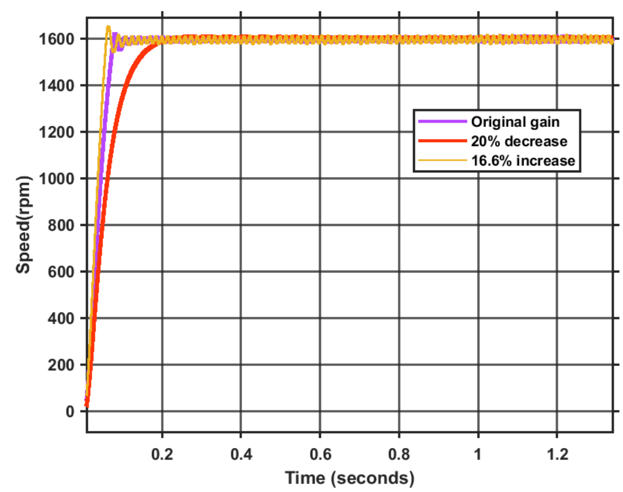

Fig. 22. Plant 2: COPID1-controlled Plant2 response with gain variations

2) System Response to Gain variations: The COPID1-controlled system was subjected to gain variations and the response was observed. It was seen that the system is robust to gain variations like the FOPID system, as shown in Figs. 22 and 23. But the COPID system showed lesser overshoot and settled faster.

3) System subjected to step changes in Load torque: The COPID1 system was subjected to a step-change in load torque at 1 second, and the response was compared with the FOPID system response, as shown in Fig. 24. Both systems responded

TABLE XIII. PARAMETERS OF THE COPID1 SYSTEM OPTIMIZED FOR ISE AND ITSE FOR PLANT 2

Index	K_p	K_i	K_d	a	b	c	$t_r(s)$	$t_s(s)$	J
ISE	84.12	32.38	59.88	0.48	0.49	0.003	0.058	0.163	0.026
ITSE	144.57	20.47	36.30	0.43	0.59	0.35	0.06	0.69	0.0008

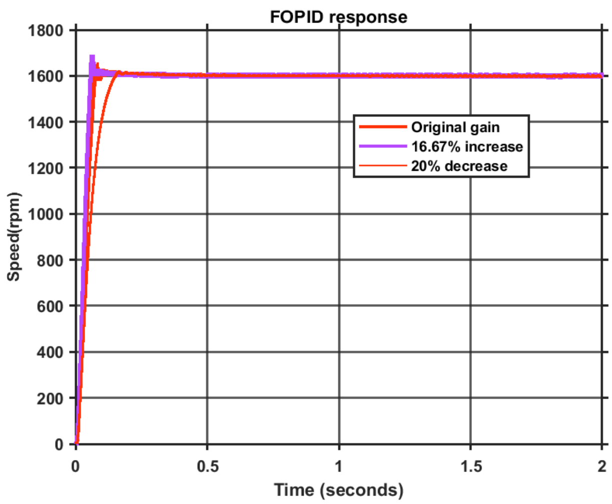

Fig. 23. FOPID-controlled Plant2 response with gain variations

quite smoothly. The overshoot was slightly greater for the COPID system, but it settled faster.

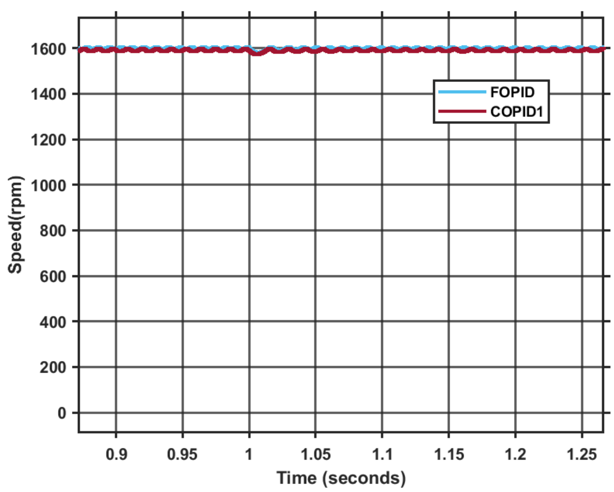

Fig. 24. Plant 2: Comparison of COPID1 and FOPID system response with step-change in load torque

VII. DISCUSSION

The buck converter system with resistive and motor loads was tested with two complex order PID controllers. Both the COPID1 and COPID2 controlled systems responded well to the system with resistive load (Plant1). It was seen that the COPID1 system had similar characteristics to the FOPID-controlled system but had a larger overshoot. But when the parameters (K_p, K_i, K_d, a, c) of the COPID1 system were chosen to be the same as the optimal FOPID parameters, and the imaginary order $'b'$ was optimized, it was seen that there was no overshoot and the system performance was better. By choosing different values of the imaginary order $'b'$, more flexibility was seen, as some values gave lesser overshoot but higher rise time, and some gave more overshoot, but lesser rise time. The optimal value can be chosen from a range of values of b to fine-tune the transient response parameters. The COPID2 system did not give overshoot, but the rise time and settling time were found to be much higher compared to all the other controller responses, as discussed in Section 4.1.1.

The COPID1 and COPID2 systems exhibited the isodamping property of fractional order controllers, i.e, robustness to gain variations. The COPID1 system performance was similar to that of the FOPID system, with slightly higher overshoot. The COPID2 system had no overshoot and lesser steady-state error and showed robustness for larger gain variations. Both controllers responded smoothly to step changes in the set-point and load, but the COPID1 controller response was the best. Both the complex-order controllers could handle parametric variations well. The COPID2 system response to filter parameters and load variations did not have any overshoot, but the response time was higher compared to the COPID1 controller.

The performance of the buck converter system with the motor load (Plant2) was studied with the COPID1 controller. The COPID1 controller was compared with the FOPID performance and was seen to give better robustness to all disturbances and dynamic load changes, compared to the FOPID controller, as expected. The advantages of the COPID controllers are in their dynamic performances.

VIII. CONCLUSIONS

This study aimed to investigate the effectiveness of the complex order PID (COPID) controller for the first time on a power electronic converter. Two complex order PID controllers were designed using optimization techniques and simulated using MATLAB and Simulink, and the time-domain performance of the power electronic buck converter was evaluated. The COPID1 controller had a complex order integrator and fractional order differentiator, and the COPID2 controller used complex orders for both the integrator and differentiator. The voltage mode control of a buck converter with resistive load was tested with both the COPID1 and COPID2 controllers. The speed control of a buck-converter-fed motor was tested with the COPID1 controller. Different optimization methods, such as Cohort Intelligence, Particle Swarm Optimization, Artificial Bee colony optimization etc., were used for comparison. The CI method gave lesser computation time and function count than the other methods and was chosen for testing the controllers under various conditions. Various performance indices like the ISE, IAE, ITAE and ITSE were used as the cost functions for optimizing the system.

The COPID1 controller was the most effective for both resistive and motor load. Both the COPID controllers gave good robustness to load and set-point variations and were compared with fractional and linear order PID controllers. The COPID1 controller showed similar response times as the FOPID controller but showed better robustness and smoother response to variations and non-linearities. The variation of the imaginary order of the integrator affected the transient response parameters and can be used for fine-tuning the system characteristics. The COPID1 controller was seen to be the most optimal and gave a smooth and fast dynamic response. The addition of the imaginary order helps in handling uncertainties. The COPID2 controller showed no overshoot and was robust for wider gain variations, but the rise time and settling time were much higher. The COPID2 controller response was seen to be better for higher load values and hence would be suited for lower current applications. Further analysis needs to be done for its application to different types of loads. To summarize, the COPID1 controller is the most optimum with the best dynamic performance. The limitations of the COPID2 controller are its higher rise time and settling time, and poor response at higher load currents.The dynamic response of the COPID2 controller is good for lower load currents.

The COPID controllers are seen to be better than the fractional order controllers when subjected to dynamic load variations and non-linearities. Hence, the complex order PID controllers can be very effective in controlling power converters used in complex systems such as renewable energy conversion systems, grids and microgrids, and electric vehicles and can be used as an alternative to PID and FOPID controllers in the future. The DC-DC boost and buck-boost converters, DC-AC inverters, etc., are more complicated to control; hence, the complex PID controllers can be more effective. Also, complex order controllers can be tested on fractional-order and complex-order plant models of power electronic systems. Analog realizations of the complex controllers may be designed by realization using complex impedances and tested. The Cohort intelligence method can be used for multiobjective optimization to meet more specifications. Further research on the frequency domain analysis of the complex order controllers and the study of COPID controllers with more complex power electronic plants is being carried out as the next phase of work.

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