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A Robust Fuzzy Fractional Order PID Design Based On Multi-Objective Optimization For Rehabilitation Device Control

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Abstract—**Gait rehabilitation robots show great promise in assisting people to reform their lower limbs with injuries or disabilities. Certainly, providing an accurate and customized aid specifically to children necessitates an effective control strategy. Such strategy should ensure robust and adaptive control.In this context, Fuzzy Fractional Order Proportional Integral Derivative (FOPID-FLC) controllers are emerged as efficient approaches due to their flexibility and ability to handle nonlinearities and uncertainties. This paper proposes the use of a FOPID-FLC controller for a two-degree-of-freedom (2-DOF) lower limb exoskeleton. Our proposal is based on an enhanced control approach that combines fuzzy logic advantages and fractional calculus benefits. Contrary to popular existing methods, that use the FLC to tune the FOPID parameters, the FLC in this work is used to generate the system torque depending on patient morphology. Indeed, our fundamental contribution is to design and implement an enhanced FOPID-FLC that achieves an adequate optimal control based on system rules composed of optimal torques and input data. The fractional calculus is approximated using successive first order filters. Next, a multi-objective optimization is established for the tuning of each FOPID parameters. Finally, the FLC is used to adjust the torque depending on the kid's age. The effectiveness of the proposed controller in various scenarios is validated based on numerical simulations. Extensive analyses prove that the FOPID-FLC outperforms the FOPID with a 90% of improvement in terms of error performance indices and 20% of improvement for the control action. Moreover, the controller exhibits improved robustness against uncertainties and disturbances encountered in rehabilitation environments.**

Keywords*—*Rehabilitation robot; Fractional order PID controller; Fuzzy logic controller; Optimization; Genetic algorithm; Robustness.

I. INTRODUCTION

Lower limb dysfunction refers to impairments, limitations, or abnormalities in the lower limb's function, including the hip, knee, ankle, and foot [1]. It is caused essentially by injuries, diseases affecting the musculoskeletal, neurological, or vascular systems [2]. Such dysfunction has significant implications for public health, healthcare systems, and individuals. Indeed, it causes reduced mobility, increased risk of falls, limitations in daily activities, loss of independence, and decreased overall well-being [3].

These symptoms' severity depends on many factors including age, lifestyle, and health conditions [4]. Clinical studies prove that they are more severe in children. In particular, children affected by Cerebral Palsy (CP) risk to develop musculoskeletal degeneration, reduced muscle strength, joint stiffness, and balance impairments [5]. Rehabilitation and interventions targeting lower limb dysfunction play a crucial role in managing these symptoms, improving functional abilities, and enhancing the quality of life for affected individuals [6].

The physiotherapy treatment helps patients to restore their limited range of motion, restrengthen weak muscles, recover dynamic equilibrium, and thus progressively restore their movement ability [7]-[8]. Such treatment requires a lengthy, repetitive, and strict rehabilitation process [9]. Furthermore, due to time, effort and resource constraints, traditional rehabilitation cannot provide sufficient training frequency and intensity [10].

To substitute conventional training and its long-term process, assistive technologies are proposed as an efficient alternative to the traditional physiotherapy treatment [11]-[12]. Besides, rehabilitation robots aim to address the specific needs of individuals with lower limb impairments [13]-[14]. The latter are machines that are designed to assist people with disabilities in regaining their physical or cognitive abilities [15]-[18].

Indeed, we distinguish robots designed to help children with specific physical tasks, such as walking or grasping objects [19][20]. Another category of robots is used to help children with cognitive tasks, such as memory and attention [21]. In [22], authors design and implement ”ArmeoSpring,” a robotic arm is used to help children with CP to improve their upper limb function.

Another example of rehabilitation robots intended for children affected by CP is the Lokomat [23]. This robotic exoskeleton assists patients to reform their mobility [24]. It uses sensors to detect the child's movements and adjusts the level of assistance. Hence, it allows to child to practice walking in a safe and controlled environment [25]. These robots offer reduced healthcare costs and easy access to care [26]. However, an effective control mechanism is crucial to attain these goals [27]-[28].

In literature, we distinguish two main categories of control systems: open-loop and closed-loop control [29]. Robots of the first type execute predetermined movements. Thus, they do not require feedback from the patient. In contrast, the second type of robot uses the feedback provided by patients to adjust their movements [30]. While the first category of robots is simpler to design, implement and maintain, the second category provides more efficient rehabilitation thanks to their personalized assistance [31].

Nowadays, there is no convergence to a specific type of controllers [32]. Indeed, research studies concur that the control strategy depends on the envisaged therapeutic goals [33]-[34]. However, ensuring an adaptive and robust control for these devices is still a challenging task [35]. Hence, optimizing the control adaptability is the main focus of current research activities in this field [36]. Findings show that Fuzzy Logic Fractional Order Proportional Integral Derivative Controllers (FOPID-FLC) ensure high adaptability [37]-[38].

In this paper, we design an enhanced (FOPID-FLC) based on a hybrid control strategy [39]. Then both FOPID and FLC techniques are used to ensure respectively flexibility and robustness [40]-[41]. The robot's actions are guided by a predetermined motion plan, while also being adaptable to the patient's movements, responses, and requirements [42].

Authors in [43] conduct a simulation study to prove the robustness against disturbances of the FOPID compared to the traditional PID. The strength of these controllers consists on their ability to capture accurately the dynamics of complex systems [44]-[45]. Thanks to the fractional calculus, the controller design allows for more effective modeling of the time-varying systems, improving thus its ability to handle disturbance [46]-[47].

A comprehensive motivation behind using the fractional order PID control in rehabilitation robotics, including gait rehabilitation robots is presented in [48]. This study provides a summary of the existing literature and identifies opportunities for future research.

Authors in [49]-[54] propose fractional order PID controllers for rehabilitation robots. They prove the effectiveness of their proposals in improving the gait of post-stroke patients based on extensive experiments. However, the robustness of a fractional order PID controller against disturbances is strongly related to the used control parameters [55]. FOPID controllers present thus the advantage of offering more degrees of freedom in their parameters tuning [56]. Allowing so more flexibility in the controller design and better adaptation to changing system conditions [57].

The current challenge in the design of FOPID controllers is the choice of the most appropriate parameters tuning approach [58]-[60]. Researchers demonstrate the efficiency of the Fuzzy control approach in assisting patients with different physical morphology's [61]. This approach uses fuzzy logic to make decisions based on imprecise or uncertain data [62]-[63].

Fuzzy fractional order PID controllers show significant improvements in trajectory tracking accuracy and stability in various fields of engineering [64]. However, the design of this controller is challenging and requires a good understanding of the system dynamics [65].

In this context, many research activities have focused on the design, implementation and evaluation of FOPID-FLC control-based exoskeleton. In [66]-[69] authors proposed fuzzy fractional PID controllers for different lower limb exoskeleton models. These controllers permit to adjust the assistive force based on the user's muscle activity.

Authors in [70]-[71] developed a fuzzy fractional PID controller for a limb exoskeleton to assist patients in rehabilitation. However, this controller exploits the user's phase movement to adjust the exoskeleton's torque. A comparative study of different fuzzy fractional PID controllers for a lower limb exoskeleton is conducted in [72]. Findings prove that this controller outperforms the traditional PID controller in terms of tracking accuracy.

In [73], authors designed a fuzzy fractional order PID controller for the trajectory tracking of a robotic gait rehabilitation system. A simulation-based study is further conducted to prove the efficiency of the proposed controller. In [74], authors focused on optimizing the system stability. Thus, they defined more control rules to resist to uncertainties and disturbances. Experimental results showed the efficiency of the proposed controller in improving the walking ability of stroke patients.

Currently, the essential research focus is to develop innovative control approaches that use both fuzzy logic and fractional PID controllers to enhance the rehabilitation process. These approaches are applied in various areas of rehabilitation such as personalized programs, assistive and adaptive control, sensory feedback integration, and game-based rehabilitation [75]-[77].

In this research work, we design an adaptive control strategy for lower limb exoskeleton intended for children aged from two to twelve years old. This control strategy presents the advantage of using multi-FOPID tuned with genetic algorithm process. Each of these FOPIDs is characterized by nine parameters (gains, frequencies and orders).

Tuning all of these FOPID parameters with the fuzzy logic controller is a challenging task since it requires accurate experimentation. Hence, a multi-objective optimization approach is adopted to adjust them to fit with the specificities of the intended age. Besides, The (Fuzzy Logic) FL controller is deployed to guarantee an adaptive and robust torque. Such a controller uses fuzzy logic-based rules to ensure a dynamic switching between the optimal FOPID controllers providing that high versatility and accuracy.

It is worthwhile to note that our proposal is not applied only to patients affected by cerebral palsy but it can be applied also to other cases of lower limb impairments (stroke, spinal cord injury, and epilepsy). The proposed controller is tested on a 2-DOF (degrees of freedom) gait rehabilitation robot, where the joint angle trajectories are tracked in real-time. The proposal is further compared with a traditional FOPID controller. Simulation results show that the FOPID-FLC controller outperforms the legacy FOPID controller in terms of tracking accuracy, adaptability and robustness.

The followings are the fundamental contributions of this research work:

- The design of an efficient and robust fuzzy logic-based control strategy namely the FOPID-FLC for the lower limb exoskeleton for rehabilitation applications. This control strategy combines both the flexibility of the fractional FOPID and the adaptability of the FLC. It depends on a set of rules basically depending on the patient age. The decision is an optimized controller.
- A mathematical model of 2-degrees of freedom (DOF) nonlinear and coupled lower-limb exoskeleton is developed with Simulink/MATLAB, taking into account both the parameters of the patients and the robot.
- Oustaloup approximation is used for implementing the set of controllers. Some frequency conditions are detailed for satisfying the fractional calculus.
- Since the performance of a genetic algorithm can be highly dependent on the choice of the fitness function, a multi-objective function for the optimization process is proposed to ameliorate the convergence to the minimum error rate and energy.
- A comparative study with a highly optimized FOPID controller is conducted. The FOPID-FLC admits better performances than this controller.
- Finally, the robustness of the proposed control strategy is tested vs random disturbances.

The mathematical model of the exoskeleton is elaborated in section 2. Section 3 focuses on the design of the control strategy: the explication of the fractional calculus background and the fuzzy strategy, and finally the implementation of the controller. Simulation results are presented and discussed in section 4. The paper is concluded in section 5.

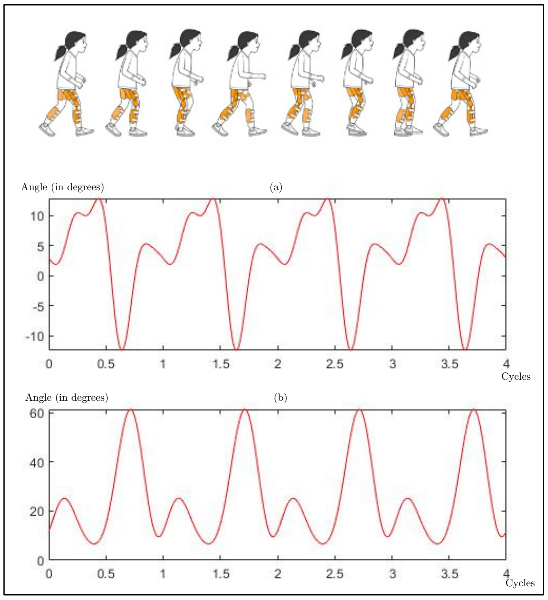

Fig. 1. Children gait cycle: (a) hip movement (b) knee movement

II. THE LAGRANGIAN DYNAMIC EQUATION OF THE SYSTEM

The main goal of using an exoskeleton is to aid in the rehabilitation process for subjects suffering from the lower limb impairment due to a neurological injury, by strengthening their neuro-plasticity [78]. The therapist defines the activities to be performed by the exoskeleton and notes the trajectories to be performed in passive or active exercises. These trajectories are presented in Fig. 1, in which the desired movements for both the hip joint and the knee joint are recorded during the walking cycle of kids. In fact, it shows the degrees of two angles variation until the movement process.

An exoskeleton is defined as an external wearable mechanism that is worn and moves parallel to the human body. In general, exoskeletons of lower extremities have the mechanical structure shown in Fig. 2 where all the parameters of both kid and exoskeleton of the two degrees of freedom (2 DOF) model are presented. It is composed of three vectors:

- $q = [q_1 \; q_2]^T \in \mathbb{R}^2$ denotes the position vector,
- $\dot{q} = [\dot{q}_1 \; \dot{q}_2]^T \in \mathbb{R}^2$ presents the speed vector,
- $\ddot{q} = [\ddot{q}_1 \; \ddot{q}_2]^T \in \mathbb{R}^2$ corresponds to the acceleration vector,

The dynamic model relates both the manipulator and the patient leg parameters. It is presented as follows:

$$I(q)\ddot{q} + N(\dot{q}, q)\dot{q} + G(q) = u \tag{1}$$

It is composed by the following matrices and vectors:

- $I(q) \in \mathbb{R}^{2\times 2}$ presents the inertia matrix

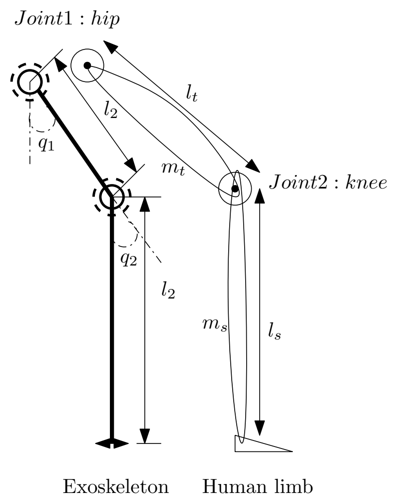

Fig. 2. Dynamic presentation of the proposed exoskeleton

- $N(\dot{q}, q) \in \mathbb{R}^{2\times 2}$ corresponds to the coriolis, centrifugal forces and torques
- $G(q) \in \mathbb{R}^2$ represents the gravity torque vector
- $u \in \mathbb{R}^2$ is the vector of torques

The system characteristics used for the dynamic representation are designed as follows:

- m_1, m_2, l_1, l_2 are respectively the masses and the lengths dealing with thigh and shank segments of the exoskeleton respectively,
- m_t, m_s, l_t, l_s represent the thigh and the shank masses and lengths of the human limb respectively. These parameters depend on the kid's age,
- g is the gravity acceleration.

III. CONTROL STRATEGY

Rehabilitation robots are designed to assist individuals with physical disabilities or injuries in improving their motor function and mobility. These robots typically use various sensors and actuators to interact with the user and provide feedback on their movements. Control systems, such as fuzzy fractional order PID controllers, can be used to improve the accuracy and stability of the robot's movements and to ensure a safe and an effective rehabilitation process.

A. Overview about Fractional Calculus

In terms of sensitivity and process improvement in response to variations dealing with perturbations or parameters, fractional order controllers exceed integer-order controllers [79]. The generalization of the integration and differentiation operations to the non-integer order operator ${}_aD_t^\alpha$ where a and t denote the lower and the upper terminals of the operations respectively, and α is the fractional order as it is presented in [80]-[81]:

$${}_aD_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & \text{if } \alpha > 0 \\ 1 & \text{if } \alpha = 0 \\ \int_t^a (dt)^{-\alpha} & \text{if } \alpha < 0 \end{cases} \tag{2}$$

Oustaloup approximation, also known as the continued fraction expansion method, is a mathematical technique used to approximate the transfer function of a system in the frequency domain. This approach was developed by the french engineer Alain Oustaloup in the 1990s, and it is widely used in the control system engineering [82]-[83].

The advantage of the Oustaloup approximation is that it provides a very accurate approximation of the fractional system with only the requirement of a small number of filter coefficients. This makes it a very efficient and computationally inexpensive technique [84].

The Oustaloup approximation is a useful tool for implementing fractional order PID controllers, as it provides a simple and systematic way to approximate fractional order transfer functions [85]. It approximates the fractional term in a frequency band $[\omega_l, \omega_h]$ generally chosen as:

$$\frac{\omega_h}{\omega_l} = 10^3 \tag{3}$$

This approach offers a practical way to approximate the fractional order derivative operators using a series of simpler first order filters. It allows for more manageable implementation and analysis of complex systems obey to (4):

$$D^\alpha = \prod_{i=-N}^{N} \frac{1 + \frac{s}{\omega_i^{'}}}{1 + \frac{s}{\omega_i}} \tag{4}$$

For N = 3, seven successive serial rational filters are chosen in this case as it is illustrated in Fig. 3. The roots presented in this figure (zeros and poles) of each rational filter are $(-\omega_i^{'})$ and $(-\omega_i)$ admit respectively (5) and (6):

$$\omega_i^{'} = \omega_l \left(\frac{\omega_h}{\omega_l}\right)^{\frac{i+N+0.5(1-\alpha)}{2N+1}} \tag{5}$$

$$\omega_i = \omega_l \left(\frac{\omega_h}{\omega_l}\right)^{\frac{i+N+0.5(1+\alpha)}{2N+1}} \tag{6}$$

In this case, two frequency bands are considered: ($\omega_l^{'}$ and $\omega_h^{'}$) for the fractional integral and ($\omega_l^{''}$ and $\omega_h^{''}$) for the fractional derivative terms ensuring (7) and (8):

$$\omega_h^{'} = 10^{(\frac{3}{2})}\omega_l^{'} \tag{7}$$

$$\omega_h^{''} = 10^{(\frac{3}{2})}\omega_l^{''} \tag{8}$$

For more clarity, the repatriation of the frequency band is presented in Fig. 4 where the behavior of the fractional filters

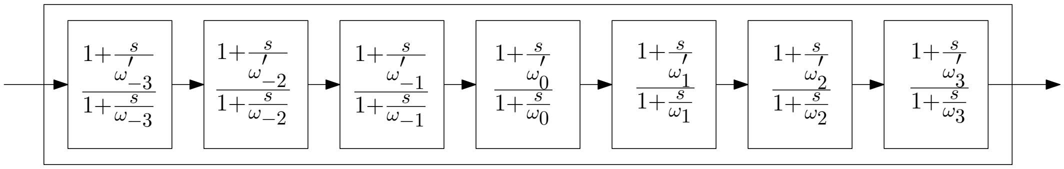

Fig. 3. Fractional derivative operator approximation

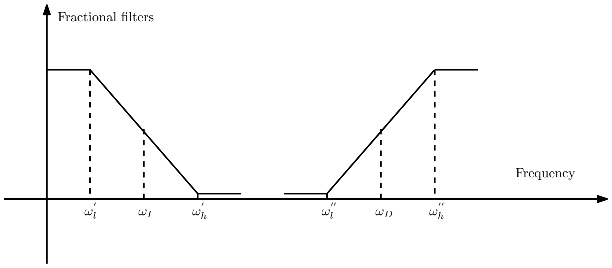

Fig. 4. Frequency band of the FOPID

is described. It shows the location of the frequency parameters of the used FOPID controller. By this way, the FOPID behavior helps in designing and tuning an efficient controller in order to achieve the desired performance and stability.

B. The Fuzzy Fractional Order Controller

In the last decade, the extension of the application of the fractional order systems (FOS) determined a relevant increase in the control techniques [86]. Conventional controllers, including the fractional-order controllers, operate on error inputs with fixed parameters value of proportional, integral, and derivative terms. As a result, the controller performance is insufficient for a nonlinear and complex system. Instead of a fixed controller, the idea can be made to incorporate dynamic system switch between fractional order controllers with optimal parameters values. This dynamic behaviour improves the system's structure, performs the control process and ensures a rapid convergence of the system output [87].

The basic structure of a fractional order PID controller consists of three components: the proportional, the integral, and the derivative components. The proportional component produces an output that is proportional to the error signal, while the integral component produces an output that is proportional to the fractional integral of the error signal. The derivative component produces an output proportional to the error signal's with fractional derivative action [88].

In 1994, I. Podlubny presented the Fractional Order PID (FOPID) for controlling automatic systems. In fact, this controller is more flexible, offers a better chance to adjust the dynamic properties and provides robustness against variations of the parameters of the non-linear systems [89].

A fuzzy fractional order PID controller that combines the advantages of fuzzy logic and fractional calculus is used in this control system. It is designed to improve the performance of the control system by adjusting the output of the controller based on the error signal and the input age of the child. It allows us to obtain an adequate and optimal torque depending on the morphology of the patients (kids) in order to achieve a good control approach.

The variable masses of the thigh and the shank of the kid's lower limb, are presented in Table I depending on the age. The mean values of the masses are taken at seven years old and the others are calculated with analogy [90]. The fuzzy logic rules define the relationship between the input variables (the age of the patient) and the output variables (the adequate torque generated for the robot). These rules are based on an expert knowledge of the rehabilitation process and the specific characteristics of the children with different ages.

Multiple Gaussian membership functions are used to define the fuzzy variables, each of them represents a different linguistic term (s: small, sm: small-meduim, m:meduim, mh: meduim-high, h: high). Five rules are concluded for the fuzzy control process:

- **Rule 1**: If $(patient\ is\ s)$ Then $u = u_s$
- **Rule 2**: If $(patient\ is\ sm)$ Then $u = u_{sm}$
- **Rule 3**: If $(patient\ is\ m)$ Then $u = u_m$
- **Rule 4**: If $(patient\ is\ mh)$ Then $u = u_{mh}$
- **Rule 5**: If $(patient\ is\ h)$ Then $u = u_h$

Accordingly, for each age j of s, sm, m, mh, h and for every joint k of $1, 2$, each FOPID computes the torque $u(t)$ through the following differential (9):

$$u_{kj}(t) = \left(K_{pkj} e_k(t) + K_{ikj} I^{\lambda_j} e_k(t) + K_{dkj} D^{\mu_j} e_k(t)\right) \quad (9)$$

with

- $e_k(t) = q_{dk}(t) - q_k(t)$ represents the error between the reference and the actual position.
- λ_j and $\mu_j \in]0, 1[$ are respectively the fractional integrator and derivator orders.
- K_{pkj}, K_{ikj} and K_{dkj} are the gains of the $FOPID_{kj}$ controllers.
- I and D are respectively the fractional integrator and derivator operators.

TABLE I
DIFFERENT MASS MEAN VALUES OF KIDS LIMB PROPRIETIES

age	2 years (s)	5 years (sm)	7 years (m)	9 years (mh)	12 years (h)
Shank mass (kg)	0.541	0.812	1.082	1.623	2.164
Thigh mass (kg)	1.7675	2.65125	3.535	5.3025	7.07

To improve the control signal of the FOPID$_{kj}$, the transfer function can be presented by the following equation:

$$u_{kj}(s) = \left(K_{pkj} + K_{ikj}\left(\frac{\omega_{I_j}}{s}\right)^{\lambda} + K_{dkj}\left(\frac{s}{\omega_{D_j}}\right)^{\mu}\right)e_k(s) \tag{10}$$

with

$$\omega_{I_j}\epsilon[\omega_l^{'}, \omega_h^{'}] \tag{11}$$

and

$$\omega_{D_j}\epsilon[\omega_l^{''}, \omega_h^{''}] \tag{12}$$

All the parameters presented in (10) are optimized via the genetic algorithm process. Generating these flexible parameters, the fuzzy logic system is used to handle the linguistic variables and uncertainties. The general control structure of the proposed system is illustrated in Fig. 5. It presents the closed loop of the control strategy, with the non-linear dynamics, the input and the output of the FOPID controllers, the optimization method, and the principles of FLC. It is a general presentation of the system working process.

The membership function defines the degree to which each input variable belongs to a particular fuzzy set. These functions are defined based on the physical capabilities of children of different ages. Five Gaussian membership functions centered at these different age values represent each linguistic term and Fig 6 illustrates the graphic presentation of these functions:

$$F_j(x) = e^{\frac{-(x-x_j)^2}{2.5}} \tag{13}$$

With $x_j \in \{2, 5, 7, 9, 12\}$. These equations describe the fuzzy sets reflecting the degrees of membership ranging from 0 to 1.

C. Combined Optimization of Controllers Parameters

Frequently the objective function minimization based on gradient traditional methodologies confuses local minima. This can be solved by using a dynamic algorithm, such as the Genetic Algorithm (GA), which is employed in this work to obtain the optimal set of controller parameters [91].

Genetic Algorithm is one of the most important evolutionary algorithms. It was introduced by Holland in the 1960s with three important operators which are the crossover, mutation, and selection [92]. The implementation of these operators is highly dependent of the way of encoding. The structure of the genetic algorithm can be described in the pseudocode (**Algorithm 1**).

Three famous minimization objective functions are used to evaluate the fitness of the controllers:

- $O_1 = IAE_1 + IAE_2$
- $O_2 = IAU_1 + IAU_2$
- $O_3 = IATE_1 + IATE_2$

dealing with:

- The integral of the absolute error is

$$IAE_k = \int_0^{\infty} |e_k(t)|dt \tag{14}$$

- The integral of the control signal is

$$IAU_k = \int_0^{\infty} u_k(t)dt \tag{15}$$

- The integral of the time absolute error is

$$ITAE_k = \int_0^{\infty} t|e_k(t)|dt \tag{16}$$

with $k \in \{1, 2\}$ and reffering to hip and the knee respectively.

By repeating the above process over multiple generations, the genetic algorithm explores the solution space and converges toward the optimal solution. The optimal values of the different types of FOPID controllers obtained after iterations are collected in Table II.

The flowchart of the system is described in Fig. 7 providing more details about the control strategy. Basically, the necessary parameters of each FOPID controller are optimized via GA method. The developed process uses three objective functions to manipulate three sets (mutation, selection, and crossover). These fitness functions are based on performance criteria (IAE, IATE, and IAU).

In the beginning, the reference signals (q_{d1}, q_{d2}) are inputted into the control system. Sensors provide feedback on the actual state of the lower limb exoskeleton's position. This feedback is used to determine the current positions and velocities of joints (q_1, q_2, $\dot{q}_1$, $\dot{q}_2$).

Then, the error (e) between the desired and the real trajectories is loaded in the optimal FOPID controllers to generate five optimal torques. These torques are calculated using the parameter blocks presented in the diagram. On another side, the inputted kid's (patient) age (x_j) is executed by the five membership functions.

Next, the collected data $\big($the outputted torques (U_j) and the outputted functions $F_j(x)\big)$ are treated by the make-decision process based on the Fuzzy Logic (FL) rules. The result is an outputted torque (U_{FLC}) that is optimal, adaptive and robust.

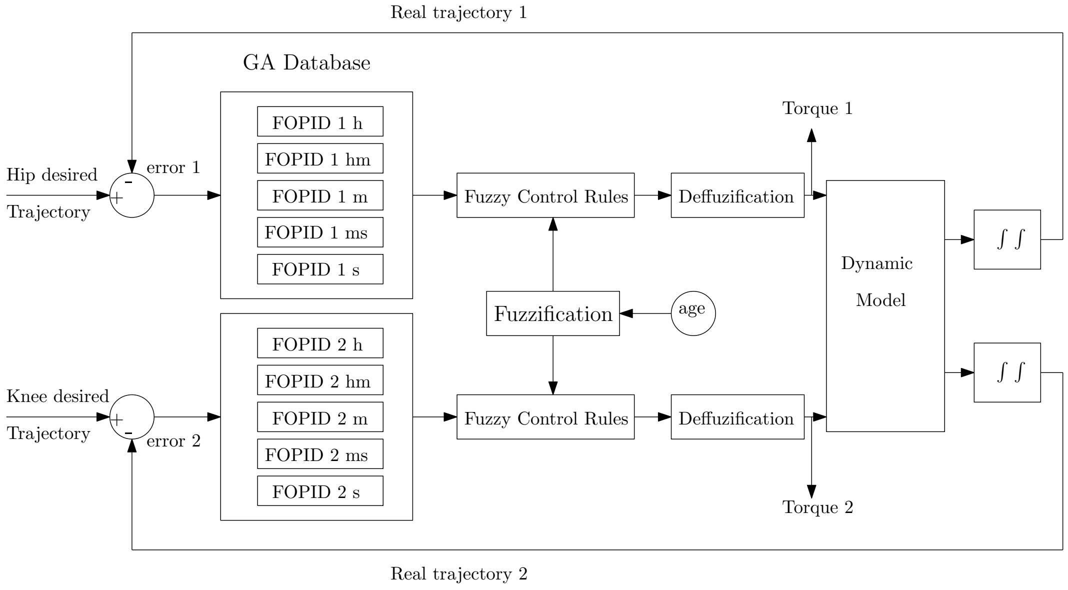

Fig. 5. Description of the control system algorithm

Algorithm 1 The genetic algorithm process

```
Begin
1. Population initialization: start with a randomly set of values of chromosomes which
are the candidate solutions of the first population formulated from 12 chromosomes
(gains, frequencies and orders)
2. Innovate the limits of the roots (gains [1 500], frequencies [0 20] and orders [0 1])
which are the specific bounds or the searching spaces.
3. While (i < 20 iterations)
 3.1 Evaluate the fitness of the individuals using $O1*O2*O3$
 3.2 Perform the selection: choose the most fittest two chromosomes as parents
 3.3 while (j < 12)
      3.1.1 Perform crossover randomly with 0.5 probability
      3.2.2 Perform mutation randomly with 0.5 probability
     end while ( next individual test)
   end while ( next generation test)
End
```

IV. NUMERICAL EXPERIMENTS

From the research results, two scenarios are discussed in further detail. The first is dealing with testing the efficiency of the controller of the nominal case with the different ages. The second study is the influence of the distributions on the preferences of the robot. In addition to the performance used in the GA process, ISE, ITSE, and MSE are also the three performance criteria that are used in control systems engineering to evaluate the performance of the feedback control systems.

ISE quantifies the cumulative error between the desired input and the actual output of the system over a specified time interval. it is calculated via (17):

$$ISE = \int_0^{\infty} e^2(t)dt \tag{17}$$

ITSE stands for Integral of Time Squared Error and is defined by (18). It is used to evaluate the performance of a control system in terms of its ability to track a reference signal over time.

$$ITSE = \int_0^{\infty} te^2(t)dt \tag{18}$$

While, MSE determines the Mean Squared Error presented by (19) and it is defined as the average of the squared error over a specified time period. It is used to evaluate the overall accuracy

TABLE II
THE OPTIMAL VALUES OBTAINED WITH GA PROCESS

Tuned parameters	**Designation**	$FOPID_h$	$FOPID_{hm}$	$FOPID_m$	$FOPID_{ms}$	$FOPID_s$
Hip controller gains	$K1_p$	400	250	300	350	200
	$K1_i$	450	100	300	200	250
	$K1_d$	200	235	200	250	200
Knee controller gains	$K2_p$	300	175	150	400	200
	$K2_i$	300	50	100	75	150
	$K2_d$	200	100	70	150	100
Fractional orders	λ	0.8	0.5	0.75	0.7	0.7
	μ	0.8	0.8	0.75	0.8	0.7
Integrator frequency parameters	$\omega_l^{'}$	0.8	0.7	0.860	0.9	0.826
	ω_I	1.102	1.25	1	1.5	1.014
	$\omega_h^{'}$	25,29	22.135	27,19	28.460	26,12
Derivative frequency parameters	$\omega_l^{''}$	3.8	5.87	3	10.2	3.825
	ω_D	5.1	7.5	5	15.3	5.102
	$\omega_h^{''}$	120,166	185.62	94.86	322	120,957

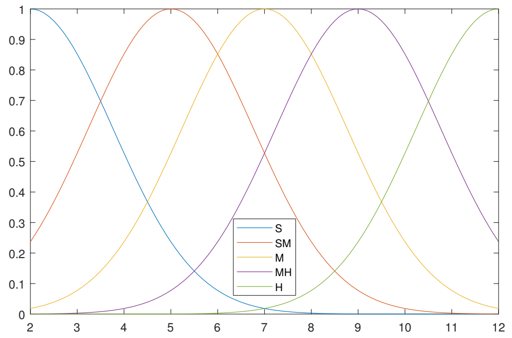

Fig. 6. Membership functions plot

of a control system in terms of its ability to maintain the desired output.

$$MSE = \frac{1}{t}\int_0^{\infty} te^2(t)dt \tag{19}$$

ISE, ITSE and MSE are important performance criteria that are used to assess the effectiveness of control systems in achieving their intended goals. They provide a quantitative measure of the system's performance and can be used to compare different control strategies or to optimize control system parameters. These measures are used to evaluate the accuracy and effectiveness of simulation results.

A. Experiments with Different Ages

The main difference between the FOPID and the FOPID-FLC controllers stands in their tuned parameters. The FOPID controller uses also the genetic optimization algorithm to determine its fixed values (including orders, gains and frequencies). On the other hand, the FOPID-FLC controller uses FLC to automatically switch between optimized fractional order controllers FOPIDs with different sets based on the input age.

The large gap of 10 years (2-12) leads to a significant difference in the anatomy and morphology between children. This difference can sometimes seem between children of the same age. Therefore, rehabilitation robots should adapt to the needs of different ages and provide individualized care for patients. In this case, simulation results are presented with different ages included in the selected band to prove the performance.

For the age of eight, Fig. 8 and Fig. 9 show the positions, errors, speeds, and torques of joints 1 and 2 using respectively the FOPID-FLC and the FOPID controllers. For more clarification, Tables III, IV, V, and VI illustrate the selected performance criteria values and improvements of the two controllers for the age of three, four, eight and eleven years old respectively.

The first curves in the presented figures illustrate the path of the desired and the real trajectories of both joints. Through these curves, we can conclude that using FOPID-FLC controller, the system tracks more accurately the referenced data than using FOPID. The proposed controller allows the adaptation of control parameters and the specific requirements of the robot model. Consequently, it enables flexible movement and effective tracking of the desired trajectories.

Besides, referred to error curves depicted in Fig. 8 and Fig. 9, FOPID-FLC is able to achieve a better performance, in terms of overshoot, than FOPID. It minimizes the discrepancy between the desired and actual positions in the startup gait cycle. Thus,

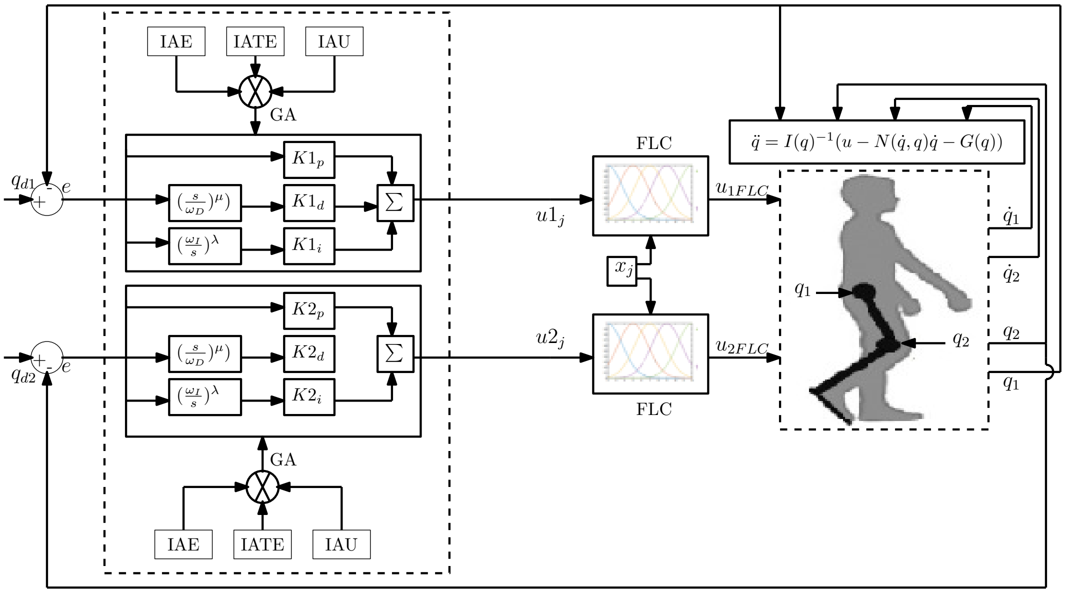

Fig. 7. The flowchart of the whole system

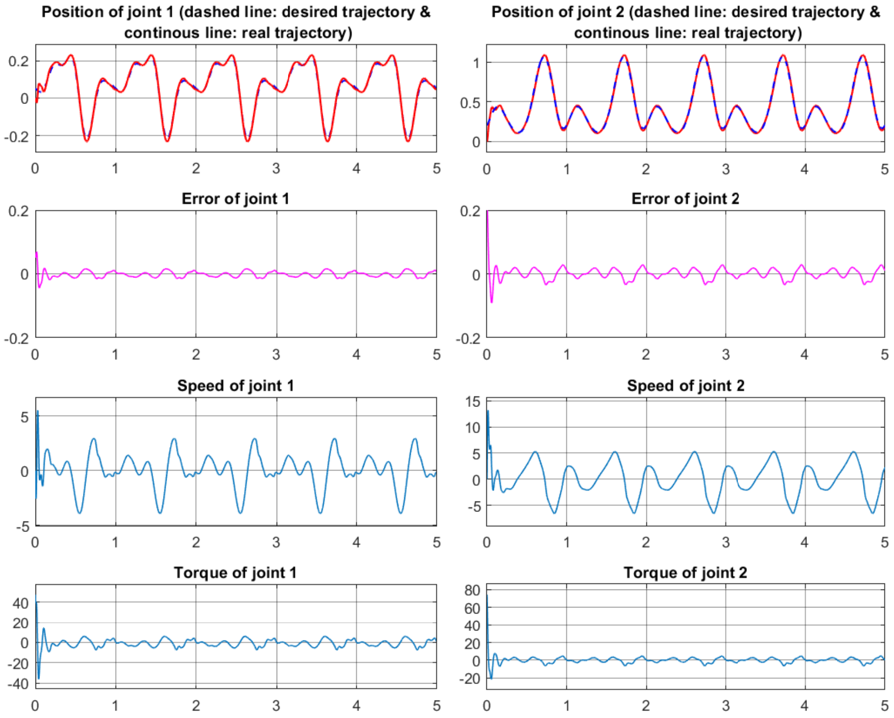

Fig. 8. FOPID-FLC results for the joint 1 and 2 with the age of eight years old

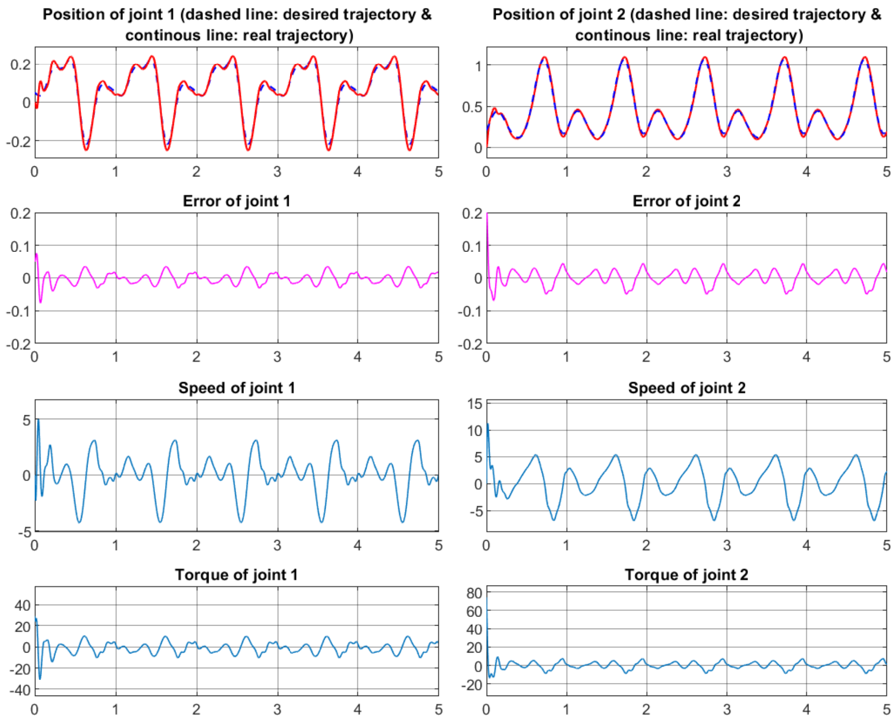

Fig. 9. FOPID results for the joint 1 and 2 with the age of eight years old

TABLE III
PERFORMANCE VALUES AND IMPROVEMENTS USING FOPID-FLC CONTROLLER WITH A PATIENT AGED OF 3 YEARS OLD

Performance criteria	**Joint1**			**Joint2**		
	FOPID	**FOPID-FLC**	**Imp in %**	**FOPID**	**FOPID-FLC**	**Imp in %**
IAE	0.030	0.010	66%	0.100	0.082	18%
ISE	0.003	0.002	33%	0.002	0.001	50%
IAU	15.5	13.59	12%	10.7	8.8	17%
IATE	0.051	0.035	31%	0.052	0.031	40%
ITSE	2 e^{-3}	1 e^{-3}	50%	4 e^{-3}	1 e^{-3}	75%
MSE	6 e^{-5}	3 e^{-5}	50%	6.6 e^{-5}	2 e^{-5}	69%

TABLE IV
PERFORMANCE VALUES AND IMPROVEMENTS USING FOPID-FLC CONTROLLER WITH A PATIENT AGED OF 4 YEARS OLD

Performance criteria	**Joint1**			**Joint2**		
	FOPID	**FOPID-FLC**	**Imp in %**	**FOPID**	**FOPID-FLC**	**Imp in %**
IAE	0.027	0.018	33%	0.096	0.016	83%
ISE	0.002	0.001	50%	0.002	0.001	50%
IAU	12.5	11.59	7%	9.7	7.8	19%
IATE	0.061	0.038	37%	0.042	0.033	22%
ITSE	4 e^{-3}	1 e^{-3}	75%	10 e^{-3}	1 e^{-3}	90%
MSE	8 e^{-5}	3 e^{-5}	60%	22 e^{-5}	2 e^{-5}	90%

it reduces the IAE performance. Values collected in Table V prove this deduction.

Energy efficiency is crucial for prolonged operation and patient comfort. The FOPID-FLC optimizes the control action based on the patient's age. Torque curves are illustrated in the given figures. By adapting the control parameters and optimizing the control effort, the proposed controller can minimize unnecessary energy and reduce the IAU value. This is demonstrated for all the selected ages in Tables III, IV, V, and VI. Thus, using the proposed controller leads to more efficient

TABLE V
PERFORMANCE VALUES AND IMPROVEMENTS USING FOPID-FLC CONTROLLER WITH A PATIENT AGED OF 8 YEARS OLD

Performance criteria	**Joint1**			**Joint2**		
	FOPID	**FOPID-FLC**	**Imp in %**	**FOPID**	**FOPID-FLC**	**Imp in %**
IAE	0.040	0.035	33%	0.158	0.057	54%
ISE	$0.5e^{-3}$	$0.4e^{-3}$	20%	$1.8e^{-3}$	$1.4e^{-3}$	22%
IAU	16.6	16.3	1.8%	11.66	11.57	0.7%
IATE	0.093	0.081	12%	0.97	0.133	86%
ITSE	$9.5\ e^{-4}$	$7\ e^{-4}$	26%	$3\ e^{-3}$	$2\ e^{-3}$	33%
MSE	$1.9\ e^{-5}$	$1.4\ e^{-5}$	22%	$6.1\ e^{-5}$	$4.2\ e^{-5}$	30%

TABLE VI
PERFORMANCE VALUES AND IMPROVEMENTS USING FOPID-FLC CONTROLLER WITH A PATIENT AGED OF 11 YEARS OLD

Performance criteria	**Joint1**			**Joint2**		
	FOPID	**FOPID-FLC**	**Imp in %**	**FOPID**	**FOPID-FLC**	**Imp in %**
IAE	0.067	0.060	10%	0.3128	0.101	67%
ISE	0.002	0.001	50%	0.005	0.003	40%
IAU	27.13	26.83	3%	21.5	21.01	2%
IATE	0.165	0.144	12%	0.128	0.124	3%
ITSE	0.003	0.002	33%	0.011	0.007	36%
MSE	$6.3\ e^{-5}$	$4\ e^{-5}$	36%	0.002	0.001	50%

TABLE VII
PERFORMANCE VALUES AND IMPROVEMENTS USING FOPID-FLC WITH DISTURBANCES CONTROLLER WITH PATIENT AGED 3 YEARS OLD

Performance criteria	**Joint1**			**Joint2**		
	FOPID	**FOPID-FLC**	**Imp in %**	**FOPID**	**FOPID-FLC**	**Imp in %**
IAE	0.120	0.035	70%	0.145	0.049	94%
ISE	0.021	0.006	71 %	0.022	0.013	66%
IAU	37.05	33.13	10%	24.5	23.73	3%
IATE	0.154	0.006	93%	0.012	0.008	80%
ITSE	0.029	0.0002	99%	0.007	0.0007	90%
MSE	$4e^{-3}$	$2\ e^{-4}$	95%	$6.5\ e^{-3}$	$7\ e^{-4}$	89%

TABLE VIII
PERFORMANCE VALUES AND IMPROVEMENTS USING FOPID-FLC WITH DISTURBANCES CONTROLLER WITH PATIENT AGED 8 YEARS OLD

Performance criteria	**Joint1**			**Joint2**		
	FOPID	**FOPID-FLC**	**Imp in %**	**FOPID**	**FOPID-FLC**	**Imp in %**
IAE	0.138	0.013	90%	0.435	0.022	94%
ISE	0.010	0.001	9 %	0.023	0.005	78%
IAU	27.13	26.83	1%	21.5	21.01	2%
IATE	0.323	0.022	93%	0.182	0.035	80%
ITSE	0.022	0.0003	95%	0.043	0.003	97%
MSE	$4.5\ e^{-3}$	$1.9\ e^{-4}$	95%	$8.6\ e^{-3}$	$7.3\ e^{-4}$	91%

energy consumption and safety of mechanical system structure than the legacy FOPID.

Simulation results show that the FOPID-FLC achieves higher tracking accuracy and limited energy compared to the FOPID controller. The referenced tables record the improvements of the performance indices of joints 1 and 2 until using FOPID-FLC instead of FOPID for different chosen ages of children. It achieves 83% of improvements in terms of IAE, 50% of improvements in terms of ISE, 19% of improvements in terms of IAU, 86% of improvements in terms of IATE, 75% of improvements in terms of IATSE and 90% of improvements in terms of MSE.

The incorporation of fuzzy logic with FOPID controllers allows more precise control actions, minimizes unwanted oscillatory behavior and provides a good system response. Despite this, the fuzzy FOPID controller presents more challenges in the design and implementation compared to the FOPID. The proposed implementation requires trial and error for the membership presentation, the searching space of optimal solutions, and the rules making.

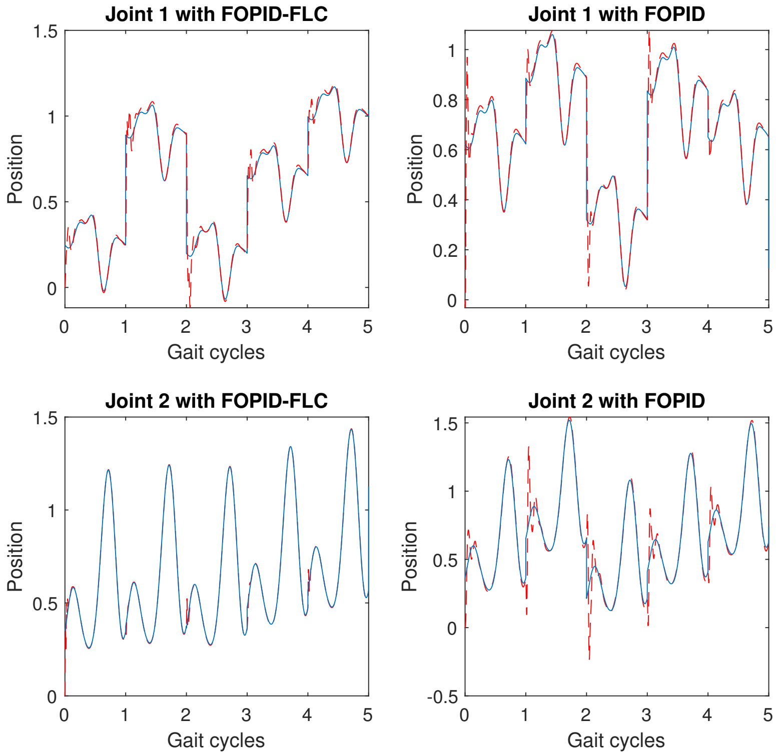

Fig. 10. Controllers response against disturbance (continuous line: disturbance and dashed line: controller response)

B. Robustness Against Disturbances

It is particularly important in rehabilitation settings that the system should be robust to many variations in the environment. For this purpose, FOPID-FLC is emerged as a great controller delivering more safety for children and good functioning of the robot.

A random disturbance is used to test this efficiency. Fig. 10 presents the response of the two controllers versus this disturbance for both joints 1 and 2. Regarding this response, we can conclude that the FOPID-FLC shows its robustness against noise more effectively than FOPID.

Based on the selected performance criteria, a comparison between FOPID-FLC and FOPID is established in Tables VII and VIII for the age of three and eight years respectively. As it is revealed that the system with FOPID-FLC indicates more stability and well-controlled response than FOPID.

The improvement in % is also illustrated in these tables. The FOPID-FLC achieves 90% of improvements for the error indices and 10% of improvements in terms of the control action. Using the Fuzzy FOPID controller enables the robot to be more adequate and less sensitive to disturbances and uncertainties. Thus, it potentially reduces the overshoot thanks to its dynamic behavior.

Our previous work [93] conducts an extensive study of using PID controller optimized by GA process to control the proposed dynamic model. The finding proves that the IAU performs 29.04 and 20.05 for joint 1 and joint 2 respectively. Moreover, in this research work, at the age of eight years old, the IAU performs better, it reaches 16.3 for the hip joint and 11.57 for the knee joint.

Additionally, results illustrate that the robustness in the uncertainty case of the FOPID is studied varying 5% and 10% the shank and thigh masses. There is proven that the IAE is adjusted with 6%. Referred to the given tables, this proposed FOPID-FLC is still better than the FOPID in terms of tracking and robustness although the mass variations reach the $\pm 50\%$. These findings are due to the flexibility of the FOPID and the adaptability of the FLC to enhance the used torque in the control process.

In comparison also to a fractional order combined with sliding mode controller (SMC) that uses the grey wolf optimization technique for a lower limb rehabilitation exoskeleton [94], the ISE performance criteria, for the nominal case, reaches $2\ 10^{-3}$ and $7\ 10^{-3}$ for joint 1 and joint 2 respectively. In our proposed research, the ISE is reduced to $0.4\ 10^{-3}$ for joint 1 and $1.4\ 10^{-3}$

for joint 2. When uncertainties are considered, the ISE criteria of the present work are of $1\ 10^{-3}$ and $5\ 10^{-3}$ for joint 1 and joint 2, respectively, rather than $3\ 10^{-3}$ and $7\ 10^{-3}$ in the precited work.

As another example in the same study [94], in the nominal and uncertainty cases, the IAE performance criteria reaches $1\ 10^{-3}$ for both joints. Moreover, in this proposed work, it is comprised between $1.3\ 10^{-3}$ and $5.7\ 10^{-3}$. Thus, we can conclude that the FL associated with FO control holds significant improvements in terms of stability and smoothness compared to FO with SMC. However, the latter is used to emphasize the accurate tracking of the desired joint angles or positions.

Further, it's important to note that the choice of such controllers depends on the specific control problem, the dynamic model and the application conditions. In summary, both FOPID and FLC controllers are effective control systems for handling complex processes. Moreover, by combining fuzzy logic and fractional calculus, the control system provides more efficiency. It produces a robust and adaptive control for the lower limb exoskeleton.

The fuzzy fractional order PID controller with dynamic switching enhances patient adaptability, improves control performance, optimizes robustness to uncertainty, offers flexibility in control design, and adapts nonlinear and time-varying systems. These benefits make this controller an attractive approach for achieving optimal control in various applications.

FOPID-FLC controller has the added benefit of automatic parameterize but it requires more challenges in designing and implementation. Such as the input variables could include joint angles, muscle activation levels, patient feedback, and unknown parameters. These control approach limitations should be considered in the next work.

V. CONCLUSION

Rehabilitation robots are considered great equipment for the treatment of children with cerebral palsy. In this paper, an enhanced control approach is proposed at the aim of providing targeted therapy that deals with the specific needs of each child. The proposed FOPID-FLC ensures high accuracy, through the use of both the fuzzy logic approach and fractional order PID controller. The latter is established via oustaloup approximation and optimized via the GA process to improve the trajectory tracking performance. The former is employed to obtain a highly accurate estimation of the torque. Findings show that our proposal does not only ensure good performance but it also outperforms the traditional FOPID controller. In particular, at the age of four years old, for joint 2, the ITSE and the MSE are enhanced by 90%. In addition, at the age of three years old, for joint 1, the IAE enhancement is up to 60%.

In the future, we are working on the optimization of our controller through the consideration of joint angles, muscle activation levels, and unknown patient and environmental characteristics. The implementation of the optimal solutions of FOPID controllers using the Gradient-Based Optimizer (GBO) process will be also considered. Further studies and clinical trials are recommended to validate the controller's efficacy in real-world rehabilitation settings and assess its impact on patient recovery.

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