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Enhancing Multi-Robot Systems Cooperation through Machine Learning-based Anomaly Detection in Target Pursuit

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Abstract—**Effectively pursuing dynamically moving targets in the domain of multi-robot systems (MRS) poses a significant challenge. This paper proposes an innovative leader-follower strategy within the MRS framework, enabling robots to dynamically adjust their roles based on target proximity. This approach fosters coordination, allowing robots to act cohesively when pursuing diverse targets, from other robots to mobile objects. The centralized architecture of the MRS facilitates wireless communication, enabling robots to share sensor-derived data providing proximity cues rather than precise location information. However, data anomalies arising from sensor errors, transmission glitches, or encoding issues pose challenges, compromising the reliability of target-related information. To mitigate this, the paper introduces an advanced methodology integrating the leader-follower strategy with Discriminant Analysis (DA)-based anomaly detection. This novel approach validates and filters data, enhancing data integrity and supporting decision-making processes. The integration of DA methods within the leader-follower strategy is detailed, emphasizing steps in anomaly detection implementation, showcasing robustness in selecting high-quality information for decision-making in dynamic environments. The research's real-world relevance addresses the problem of the impact of sensor anomalies on the performance and reliability of MRS in dynamic environments. By integrating machine learning-based anomaly detection, this methodology enhances MRS adaptability and robustness, particularly in scenarios requiring precise target tracking and coordination. Numerical experiments and simulations demonstrate the efficacy of the DA-based anomaly detection and collaborative hunting strategy in MRS. This method contributes to improved target tracking, enhanced system coordination, and streamlined pursuit of dynamic targets, affirming its practical applicability in surveillance, search and rescue operations, and industrial automation.**

Keywords—Multi-Robot Systems; Machine Learning; Anomaly Detection; Centralized Architecture; Collaborative Pursuit; Sensor-Derived Data.

I. INTRODUCTION

In a multi-robot system (MRS) [1], a group of robots collaboratively pursues a dynamically moving target, which may range from another robot to any movable object [2]. This collective effort is managed through a strategy known as the leader-follower approach [3][4]. Within this strategy, each robot assumes a specific role based on its proximity to the target [5].

The leader-follower strategy operates on a simple principle: one robot, often referred to as the leader, takes charge of guiding the group towards the target [6]. In contrast, the remaining robots, referred to as followers, adjust their positions according to their distance from the target. This approach streamlines coordination, heightening the prospects of a successful target capture. Essentially, the closer a robot is to the target, the more actively it participates in the pursuit. In this manner, the robots collaborate harmoniously, with each robot's role tailored to its relative position in relation to the moving target [7]. The architecture of this multi-robot system adopts a centralized structure [8]–[13]. This means that a central hub or server serves as the primary communication hub for the robots. The robots within the system communicate with this central server via wireless technology, sharing data collected by their onboard sensors [14]–[18]. The robots' sensors are specialized for detecting signals emanating from the target. However, it is essential to emphasize that these signals do not convey precise location information about the target. Instead, they serve as indicators of the target's presence, without specifying its exact coordinates. This configuration enables the robots to discern when the target is in close proximity but does not furnish them with precise positional data.

The central server plays a pivotal role by continually gathering data from the robots at regular intervals. It leverages this data to determine and assign specific roles to each robot within the group. Nevertheless, it is important to acknowledge that the information provided by the robots regarding the target's status is not consistently dependable. Periodically, irregularities or anomalies surface within this information. These anomalies may originate from various sources, including sensor inaccuracies [19], data transmission glitches, air quality [20], [21], and encoding or formatting issues. In other words, occasional inaccuracies in the data the robots provide about the target result from problems in their sensors, the way data is transmitted, or how data is packaged and sent to the server. These challenges pose complexities for the central server's task of assigning roles to the robots within the system. To address and mitigate these challenges, machine learning [22]–[27] methods can be employed, providing a proactive approach to anomaly detection and enhancing the system's ability to adapt to dynamic and unpredictable conditions [28]–[33].

As related works, the research [34] explores using IOTA and distributed ledger technologies to detect anomalies and byzantine agents in decentralized multi-robot systems. Unlike traditional blockchain methods facing challenges in real-world robotic environments, our approach leverages recent advancements in collaborative decision-making through IOTA smart contracts [35]. By adapting vision-based anomaly detection, we identify byzantine agents with minimal computational overhead, fostering trust among robots. The proposed methodology effectively detects anomalies and changes between robots operating in the same environment. In [36], they present a groundbreaking framework for identity authentication and consensus algorithms. The devised scheme features an identity-based authentication model employed by all communication nodes to establish connections and facilitate data exchange. Simultaneously, a hash pool-based joint consensus algorithm is introduced. In this algorithm, transmission data undergoes rigorous protection through the permutation of hash functions drawn from a hash pool, combined with the use of generated random numbers. This innovative approach markedly bolsters the security of multi-robot systems. Introducing the Proactive Anomaly Detection Network (PAAD) for enhancing robot navigation in unpredictable environments [37]. PAAD stands out by anticipating failure probabilities through the analysis of planned motions and current observations. This innovative system ensures robust detection, even in settings with sensor occlusion in the field.

The proposed method is tailored for unsophisticated robots, utilizing a single cost-effective sensor with occasional instability in measurements. Despite this, it remains cost-efficient and suitable for less sophisticated robotic platforms, addressing computational constraints through algorithms compatible with low-configuration cards, expanding its deployment potential to scenarios like ships and robots with lower-end configuration cards. However, the initial reliance on a centralized architecture may pose scalability challenges in extensive multi-robot systems (MRS). The occasional instability in measurements, influenced by the lone sensor, affects target tracking accuracy, and the efficacy of the machine learning model relies on the quality and diversity of training data. Our research addresses limitations in specialized sensors in MRS, enhancing the overall performance of Ultrasound, Chemical, Smoke, and Infrared (IR) sensors. It tackles inaccuracies, occasional false readings, and variations due to environmental conditions. Furthermore, it deals with data transmission glitches during wireless communication and encoding or formatting issues, ensuring reliable data interpretation at the central server. This comprehensive approach ensures the effectiveness and accuracy of the proposed methodology within the challenging context of multi-robot systems and anomaly detection. The methodology's practical applications extend to critical domains, addressing challenges in search and rescue missions, environmental monitoring, and surveillance in confined spaces. Tailored for scenarios where less sophisticated robots excel, it proves adaptable and effective. In search and rescue, a swarm efficiently locates survivors and assesses damage, while in environmental monitoring, it enables cost-effective data collection in remote or hazardous areas. Surveillance in confined spaces benefits from a swarm of robots with limited sensors, ensuring efficient reconnaissance and safety in complex environments. The method integrates the principles of a leader-follower strategy within a centralized architecture and employs machine learning techniques, specifically focusing on Discriminant Analysis (DA) methods. The primary goal is to optimize the efficiency of the central computing system by filtering out erroneous data from the robots, ensuring that only accurate information influences decision-making.

The integration of DA methods in anomaly detection is groundbreaking, reshaping the landscape of multi-robot systems, promising efficiency and adaptability in dynamic environments. Addressing data anomalies in MRS, stemming from sensor inaccuracies, transmission glitches, and environmental factors, our research focuses on sensor dependability, communication robustness, and environmental adaptability. Machine learning methods enhance anomaly detection, contributing to system reliability in real-world applications. Metrics and evaluation results confirm DA's robustness, tailored for applications of this nature. Data acquired from the robots undergoes thorough verification using a trained model, ensuring its reliability for subsequent processing and decision-making. The combined performance of DA algorithms for anomaly detection and the hunting strategy within the Multi-Robot System (MRS) is showcased through numerical experiments and simulations.

The main contributions of this paper are: (1) Innovative Integration of Leader-Follower Strategy and Machine Learning, where the method integrates a leader-follower strategy within a centralized architecture and employs Discriminant Analysis (DA) methods, optimizing the central computing system's efficiency by filtering erroneous data. (2) Groundbreaking Anomaly Detection with DA Methods by the integration of DA methods for anomaly detection reshapes the landscape of multi-robot systems, promising efficiency and adaptability in dynamic environments. (3) Adaptable Methodology for Unsophisticated Robots where it is designed for less sophisticated robots, utilizing a cost-effective single sensor and addressing computational constraints, making it suitable for a variety of robotic platforms.

The paper is organized into sections, delving into the background, detailing the proposed method, presenting and interpreting results, and encapsulating the conclusion with outlines for further research.

II. Method

Within the swarm, numerous robots undertake the roles of trackers or hunters with the shared objective of locating and capturing a predefined target. Effective communication and information exchange among these robots are paramount to establishing a robust formation. This collaborative effort aims to expedite the target-finding process and prevent its evasion. The target, in this context, is characterized by its unpredictable movement patterns and behaviors, adding an element of complexity to the pursuit. To illustrate the elusive nature of the target, we conceptualize it as akin to another robot, emitting signals that deliberately withhold any information regarding the target's precise location. This deliberate lack of locational cues challenges the tracking robots to rely on adaptive strategies and dynamic coordination within the swarm to swiftly and efficiently apprehend the elusive target, navigating the complexities of its random velocity and behavior. In adherence to the hunting strategy, robots within the swarm are tasked with measuring the signal generated by the target. The value obtained from this measurement plays a pivotal role in determining the roles assigned to individual robots and, consequently, shaping the overall formation of the swarm. It is imperative to underscore that the accuracy and reliability of these measured values are paramount for the success of the pursuit. The correctness of these measurements directly influences the strategic assignment of roles among the robots and the subsequent coordination within the entire swarm.

A. Pursuit Strategy in Multi-Robot Systems

This hunting strategy employs an improvised multi-robot cooperation approach to pursue a dynamic target, proving to be an effective method for addressing such multi-robot tasks [38]. Inspired by the hunting behavior of wolves, this method has been adapted with certain modifications to suit the robotic context. The robots involved in this strategy assume distinct roles, including the leader wolf, antagonist wolves, and follower wolves. The N robots utilize a hunting strategy based on detecting the odor concentration of the prey, expressed through the fitness function $\theta(Rr,T)$, where T represents the target vector position and $r = (1 \dots . N)$. The fitness function $\theta(Rr,T)$ increases as the robot gets closer to the target. Notably, the function value $\theta(Rr,T)$ remains independent of the number of dimensions in the search space (n). Mathematically, $\theta: IRn \rightarrow IR$.

The leader wolf, denoted as R^l, is expected to possess extensive knowledge about the prey's position and be the closest to the target. When the leader wolf initiates howling, all follower wolves join forces to pursue the target. Roles among wolves are dynamically exchanged based on the fitness function: if $\theta(Rr,T) > \theta(R1,T)$, then Robot Rr becomes the new leader, and the former leader assumes the role of an antagonist.

The robots assigned the role of the antagonist wolf follow the method outlined below to track the target independently of the leader. Within the swarm, the number of antagonists varies between $[(N-1)/(\lambda+1), (N-1)/\lambda]$, where $\lambda = [1, N/2]$, representing the antagonism proportion factor. The search location is determined based on the position of the leader, who is consistently the closest to the prey. This particular wolf focuses its search within a specific angle.

The measurement of this angle, situated between the x-axis and the straight line connecting the antagonist and the leader, is calculated as follows:

$$\Theta = 2\arctan \frac{R^l_y - R^a_{ky}}{\sqrt{(R^l_x - R^a_{kx}+1)^2+(R^l_y - R^a_{ky})^2} + R^l_x - R^a_{kx} + 1} \quad (1)$$

The coordinates of the antagonist wolf R_k with $1 \leq k \leq$ A_wolves are denoted as R^a_{ky} and R^a_{kx}, and the angle θ is measured in radians. The value of this angle is converted according to equation (2) to align it with the algorithmic method.

$$\Theta' = \begin{cases} \Theta, & \Theta \in [0, \pi] \\ \Theta + 2\pi, & \Theta \notin [0, \pi] \end{cases} \quad (2)$$

Employing the aforementioned angle measurement, the antagonist wolf assesses the prey's odor in specific positions. These temporary positions $(R^{\tilde{a}}_{kx}, R^{\tilde{a}}_{ky})$ are calculated using equation (3).

$$\begin{pmatrix} \widetilde{R^a_{kx}} \\ \widetilde{R^a_{ky}} \end{pmatrix}_{\varphi} = \begin{pmatrix} R^a_{kx} \\ R^a_{ky} \end{pmatrix} + \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} \Theta \begin{pmatrix} \cos(\frac{2\pi\varphi}{\psi}) \\ \sin(\frac{2\pi\varphi}{\psi}) \end{pmatrix} \quad (3)$$

The vector β represents the seeking vector, where the factor ψ determines the extent of global directions that the antagonist R_k a will explore in search of the prey. Additionally, φ is the factor governing the advancing direction, where:

$$\varphi = \left[\frac{\psi \times \Theta'}{2\pi} - l_1, \frac{\psi \times \Theta'}{2\pi} + l_2\right] \quad (4)$$

The two integers l_1 and l_2 define the boundaries of the seeking area, with their values falling within the interval $[1, \psi]$. The antagonist R_k updates its position to the temporary position that registers the highest value of the prey's odor concentration, on the condition that $\max(\theta(Rk\tilde{\varphi}a,T)) > \theta(Rka,T)$. The antagonist secures the leader position if $\theta(Rka,T) > \theta(Rl,T)$.

In this swarm, there are N-A_wolves-1 followers. This hunting strategy defines two types of behavior for each follower. The follower wolf switches between the two behaviors according to the distance between this wolf and the leader $Dconv$. The distance of convergence Dconv is described as:

$$Dconv = \left(\frac{1}{\max(xs) - \min(xs)} + \frac{1}{\max(ys) - \min(ys)}\right)^{-1} \times \sigma \quad (5)$$

Where $\sigma = [0, 1]$ is the convergence factor. (xs, ys) are the boundaries of the search space.

When the distance between the leader and the follower exceeds Dconv, the follower wolf adopts summoning behavior. In this mode, the follower advances towards the leader according to equation (6):

$$\begin{pmatrix} R^f_{jx}(i+1) \\ R^f_{jy}(i+1) \end{pmatrix} = \begin{pmatrix} R^f_{jx}(i) \\ R^f_{jy}(i) \end{pmatrix} + \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \Theta \begin{pmatrix} R^l_x - R^f_{jx}(i) \\ R^l_y - R^f_{jy}(i) \end{pmatrix} \Theta \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} \Theta \begin{pmatrix} |R^l_x - R^f_{jx}(i)|^{-1} \\ |R^l_y - R^f_{jy}(i)|^{-1} \end{pmatrix} \quad (6)$$

The follower wolf exhibits predatory behavior when the distance between itself and the leader is less than Dconv. In this behavior, the follower gradually encircles and captures the prey. The position update for the follower is expressed by the following expression:

$$\begin{pmatrix} R_{jx}^{f}\ (i+1) \\ R_{jy}^{f}\ (i+1) \end{pmatrix} = \begin{pmatrix} R_{jx}^{f}\ (i) \\ R_{jy}^{f}\ (i) \end{pmatrix} + \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} \odot \begin{pmatrix} R_{x}^{l} - R_{jx}^{f}\ (i) \\ R_{y}^{l} - R_{jy}^{f}\ (i) \end{pmatrix} \odot \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} \tag{7}$$

B. Linear Discriminant Analysis (LDA) and it's Variants

The goal of the LDA technique is to project the original data matrix onto a lower dimensional space [39]–[42]. To achieve this goal, three steps had to be carried out. The first step is to calculate the separability between classes (the distance between the means of the different classes), which is called the between classes variance S_B. The second step is to calculate the distance between the mean and the samples of each class, which is called the within-class variance S_w [43]–[46]. After having calculated the between-class variance (S_B) and the within-class variance (S_W), the transformation matrix (Π) of the lower dimensional space can be calculated as in the equation (8), called Fisher criterion. This formula can be reformulated as in the equation (9).

$$\underset{W}{\operatorname{argmax}} \frac{\Pi^T S_B \Pi}{\Pi^T S_W \Pi} \tag{8}$$

$$S_B \Pi = \lambda S_w \tag{9}$$

The challenge of linear separability in Linear Discriminant Analysis (LDA) refers to the method's limitation in modeling linear relationships between variables and classes [47], [48]. LDA assumes that data is linearly separable in the projection space, meaning that classes can be optimally discriminated using a linear combination of input variables [49]–[53]. However, in many real cases, the relationships between variables and classes can be nonlinear. To overcome this issue, extensions of LDA have been proposed [54], [40]–[42], [55]–[65]. One of these extensions is Quadratic Discriminant Analysis (QDA), which allows for modeling quadratic relationships between variables and classes. Additionally, Kernel Discriminant Analysis (KDA) is another extension that handles nonlinear relationships by using kernel functions to project data into a higher-dimensional space where linear separation can be achieved, as shown in Fig. 1. This enables KDA to capture nonlinear relationships between variables and classes.

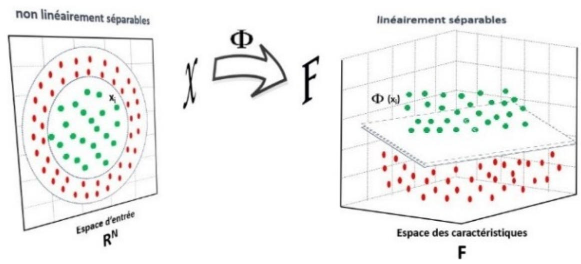

Fig. 1. Nonlinear transformation

C. Anomaly Detection in Target Pursuit

The proposed method is specifically designed for unsophisticated robots, lacking an abundance of sensors or high-quality sensor capabilities. Additionally, it caters to robots operating in unstable conditions. The primary challenge stems from the fact that the measured values from these robots are not consistently reliable or accurate. This sensitivity becomes a critical concern for the hunting strategy, as inaccuracies in these values can lead to various issues such as disruptions in the formation of the robots and potential prolongation or failure of the pursuit.

In this method, a centralized architecture is employed to organize the robots. The computing center, serving as the central hub for coordination, can take the form of a computer within the same local network (Fig. 2), a server accessible to all robots connected to the internet, or even a designated robot within the swarm as shown in Fig. 3. The computing center plays a crucial role in aggregating information about the measured values from all other robots. Subsequently, based on this collective data, the computing center undertakes the responsibility of assigning specific roles to each robot within the swarm. This centralized approach ensures a cohesive and synchronized operation, where the computing center acts as the brain of the system, orchestrating the collaboration of robots by leveraging the information received from each unit.

In a misguided collective movement away from the target. This not only compromises the pursuit objective but may also introduce a new layer of complexity and potential conflict among the robots. Therefore, the computing center's ability to discern and rectify anomalies becomes crucial for the successful execution of the collaborative pursuit strategy. By accurately identifying and addressing anomalies, the computing center ensures that the assigned roles and movements within the robot swarm align with the actual circumstances, enhancing the overall effectiveness and reliability of the multi-robot system in dynamic environments.

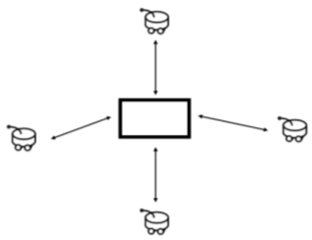

Fig. 2. A centralized architecture involves the use of a server or a central computer center

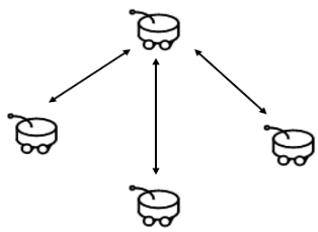

Fig. 3. Centralized architecture centered around a robot as the computing hub

In its role of orchestrating the team, the computing center assumes a dual responsibility by not only coordinating the actions of the robots but also diligently detecting and managing anomaly information sent by these robots (Fig. 4). Anomalies can stem from various sources, including inaccuracies in sensor measurements, coding issues, transmission errors, and more. The impact of an erroneous measure is substantial, capable of disrupting the entire formation of the robots. Consider a scenario where a robot within the swarm detects the weakest signal, yet due to an error, the value received by the computing center is erroneously amplified. Subsequently, other robots might align their positions with this misidentified leader, resulting. In the context of this problem, an anomaly is defined by a significant variation in the values transmitted by the robots. This variance is attributed to the inherent instability of the signal generated by the target, influencing all robots within the system. The essence of this anomaly is manifested in a scenario where the value transmitted at time t deviates considerably from the value transmitted at the preceding instant, $t-1$. Table I illustrates specific instances of these anomalies, providing a tangible representation of the observed variations.

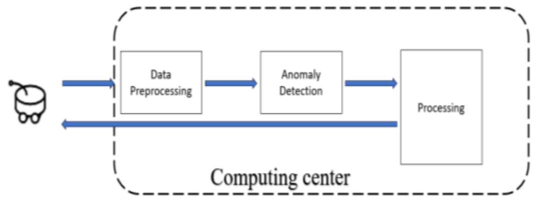

Fig. 4. Critical components in a computing center

TABLE I. IDENTIFICATION OF ANOMALIES

Case	1	2	3	4	5	6	7
Robot 1	20	80	87	120	125	70	13
Robot 2	22	79	88	126	127	64	9
Robot 3	31	86	85	122	123	62	11
Robot 4	2	63	61	80	150	59	0

In Case 5, an anomaly is clearly identified. Robot 4, which was initially the farthest from the target at time $t-1$, unexpectedly sends the highest value among all robots at time t. This abrupt and substantial increase in the transmitted value contradicts the anticipated behavior, marking it as an anomaly.

In the process of anomaly detection within a multi-robot system, the initial step involves comprehensive data preparation by collecting pertinent information from sensors on each robot. This dataset encompasses features crucial to understanding robot behavior, such as sensor readings, movement patterns, and communication signals. Subsequently, the data is labeled to distinguish between normal and anomalous instances, acknowledging potential anomalies arising from sensor errors, unexpected movements, or communication disruptions. Following dataset splitting for training and testing, feature scaling is applied to ensure optimal performance. The different classifiers, then trained on the feature matrix (X_train) for binary classification of normal and anomalous behavior. Predictions are made on the testing set, and model performance is evaluated using metrics like accuracy, precision, recall, and F1-score, with parameter adjustments made as necessary. The trained classifier model is integrated into the central computing center of the multi-robot system, where it continuously monitors incoming data, classifying instances as normal or anomalous. In response to detected anomalies, a proactive mechanism is devised, potentially involving role reassignment, behavior recalibration, or system-wide alerts.

III. RESULTS AND DISCUSSION

In the experimental phase, the pursuit of a target involves four robots that establish communication through a centralized unit. This unit plays a pivotal role in data retrieval from the robots and responds with assigned tasks. Within this central unit, a supervised machine learning model is implemented to detect data anomalies. To identify the most suitable machine learning algorithm for this task, data has been systematically collected from a hunting pursuit involving the four robots. This data has been meticulously labeled to discern whether each measurement signifies an anomaly or not.

A. Competitors

This part provides an overview of several competing classification methods commonly used for anomaly detection, each with its own set of parameters that can be tuned to achieve optimal performance as shown in Table II.

TABLE II. ALGORITHM DESCRIPTION

Algorithm	Designation	Parameters
RF	Random Forest [40]	Number of trees= 100 Tree depth =None Minimum samples split =5
SVM	Support Vector Machine [49]	Kernel type (rbf) Kernel width =0.001 C=1
k-NN	k-Nearest Neighbors [48]	Number of neighbors (k=3)
DT	Decision Tree [41]	Tree depth= None Minimum samples split=5
LR	Logistic Regression [42]	C=0.1 (regularization)
QDA	Quadratic Discriminant Analysis [47]	Tol=1e-4 Priors=None
KDA	Kernel Discriminant Analysis [43]	Kernel type (rbf) Kernel width =0.001
Nys-KDA	KDA with Nystrom approximation [44]	Number of landmarks points (ratio=0.5)
Bag-LDA	Bagging applied to LDA [45]	Number of LDA Classifiers =2
LDA	Linear Discriminant analysis [46]	Solver type (svd)

Algorithm's default parameters are chosen as reasonable starting points, but they may not be suitable for all situations. Therefore, an in-depth understanding of each parameter and its implications on model behavior is crucial to ensure that our models make accurate and reliable predictions. In the process of parameter tuning, a cross-validation techniques, such as k-fold cross-validation, allow us to assess a model's performance.

B. Evaluation

The ensuing Table III showcase the outcomes of various machine learning algorithms deployed for supervised anomaly detection. Performance evaluation is conducted using metrics such as F1-Score and Accuracy. Each metric offers distinctive insights into the efficacy of the algorithms in identifying anomalies within the communication network. This rigorous evaluation process ensures the selection of an optimal machine learning algorithm, thereby enhancing the system's ability to detect and respond to anomalies effectively in the pursuit of the target.

TABLE III. F1-SCORE RESULTS

Algorithm	F1 Score
Random Forest	0.835443
SVM	0.9235802
k-NN	0.860465
Decision Tree	0.820000
Logistic Regression	0.870588
QDA	0.9866666
KDA	0.9219999
Nystrom-KDA	0.9487179
Bagging-LDA	0.8841509
LDA	0.8769574

Anomaly detection is a pivotal task in various applications, and the F1-Score proves to be a valuable metric that balances precision and recall (see Fig. 5), providing a comprehensive evaluation of how well an algorithm performs in identifying anomalies. Let's delve into the F1-Scores of these algorithms to gain insights into their anomaly detection capabilities. QDA, Nystrom-KDA, KDA and SVM stand out as the top performers in anomaly detection, with F1-Scores of 0.987, 0.949, 0.924, and 0.923, respectively. They effectively strike a balance between precision and recall, making them well-suited for applications where accurate anomaly detection is essential. Bagging LDA, Linear Discriminant Analysis, k-NN, and Logistic Regression also deliver solid performances, with F1-Scores of 0.885, 0.877, 0.860, and 0.871, respectively. While Random Forest and Decision Tree perform well with F1-Scores of 0.835 and 0.820, they slightly lag behind the top performers.

Accuracy measures the proportion of correctly classified instances, both normal and anomalous (Table IV and Fig. 6). In this context, QDA excels with the highest accuracy of 0.960, signifying its proficiency in correctly classifying anomalies. Nys-KDA follows closely with an accuracy of 0.922, demonstrating robust performance. SVM and KDA also deliver impressive accuracy rates of 0.8823 and 0.8827, respectively, showing their effectiveness in distinguishing between normal and anomalous cases. Decision Tree achieves a respectable accuracy of 0.843, outperforming Logistic Regression (0.784), Random Forest (0.800) and Bagging-LDA (0.781). Linear Discriminant Analysis and K-NN while delivering accuracies of 0.772 and 0.765, respectively, slightly lag behind the top performers. k-NN, with an accuracy of 0.765, demonstrates the lowest accuracy among the algorithms, indicating room for improvement in correctly classifying instances.

TABLE IV. ACCURACY RESULTS

Algorithm	Accuracy
Random Forest	0.8000000
SVM	0.8823450
k-NN	0.7647058
Decision Tree	0.8431372
Logistic Regression	0.7843137
QDA	0.9603921
KDA	0.8827529
Nys-KDA	0.9215686
Bagging-LDA	0.7807973
LDA	0.7716525

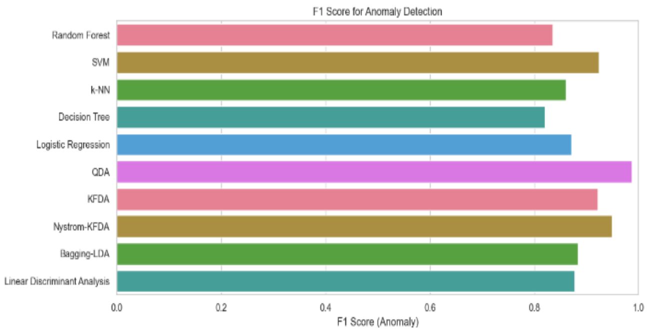

Fig. 5. F1 score of ML Algorithms

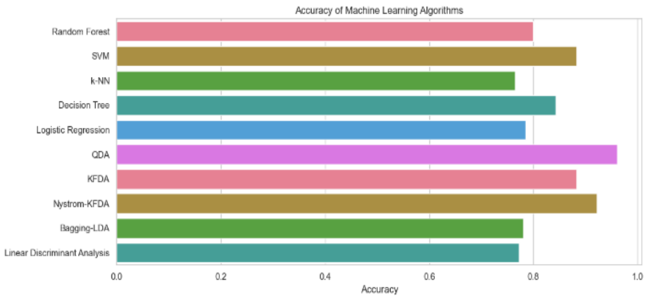

Fig. 6. Accuracy of ML Algorithms

C. *Oversampling Data*

Expanding the dataset using techniques such as data duplication or oversampling can significantly enhance model performance, especially in cases of imbalanced or small original datasets. This approach often results in a more balanced representation of different classes, aiding the model in effectively learning underlying patterns. The advantages include improved classification accuracy, a better trade-off between precision and recall, and a reduction in overfitting. This is why I noticed a remarkable performance boost when I increased the number of samples in my dataset.

However, this method can lead to an increase in dataset size, posing challenges in terms of memory, computational resources, and scalability. It emphasizes the importance of considering techniques suitable for handling large and real-time datasets.

D. *LDA and its Variants*

Linear Discriminant Analysis (LDA) provides a strong baseline for anomaly detection. Bagging-LDA builds upon this foundation, demonstrating that an ensemble approach can enhance performance. By aggregating multiple LDA models trained on different subsets of the data, Bagging-LDA mitigates the risk of overfitting and can capture more nuanced patterns within the dataset. This transition from LDA to Bagging-LDA reflects the value of leveraging ensemble techniques to boost anomaly detection capabilities. Kernel Discriminant Analysis (KDA) marks a significant shift. KDA introduces non-linearity through the kernel trick. This change enables KDA to capture complex, non-linear relationships in the data, making it more adept at handling intricate anomalies. The improvement signifies the importance of considering non-linear relationships in anomaly detection tasks, especially when dealing with complex, multi-dimensional data. KDA is further enhanced with Nystrom sampling. The Nystrom method provides a computationally efficient way to approximate kernel matrices, enabling the processing of large datasets. The boost in performance from KDA to Nystrom-KDA showcases the significance of scalability and efficiency in anomaly detection. This transition reflects the need to handle real-world, extensive datasets, making anomaly detection more applicable in practical scenarios.

In essence, LDA provides a solid starting point for anomaly detection, more advanced techniques such as ensemble learning, non-linear transformations, and efficient computations can significantly enhance anomaly detection capabilities. This observation aligns with the evolving needs of data analysis, where complex, high-dimensional data is increasingly prevalent. To tackle such challenges effectively, leveraging variant LDA methods becomes essential.

E. Hunting Process Simulation

In a simulated hunting scenario conducted on MATLAB (Fig. 7 and Fig. 8), the same set of robots and computing unit from previous experiments are employed. The objective of this simulation involves four robots tasked with capturing a target within a confined environment. The robots transmit information to the central computing unit, where the received data undergoes preprocessing. Subsequently, the data is fitted into the trained model for anomaly detection, a process that determines the legitimacy of the information for use in the hunting endeavor. This integrated system ensures that only validated and accurate data contributes to the decision-making process during the simulated hunting operation, emphasizing the significance of effective anomaly detection in optimizing the overall performance and success of the hunting simulation.

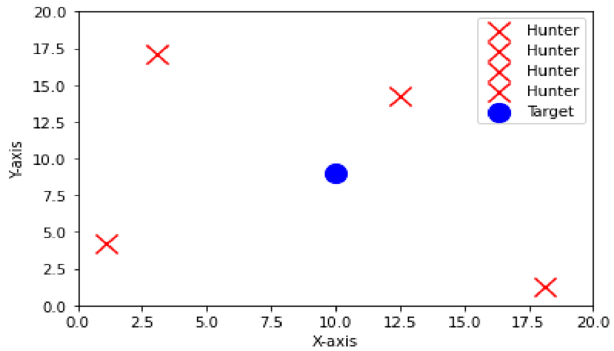

Fig. 7. The initial position for the robots and target

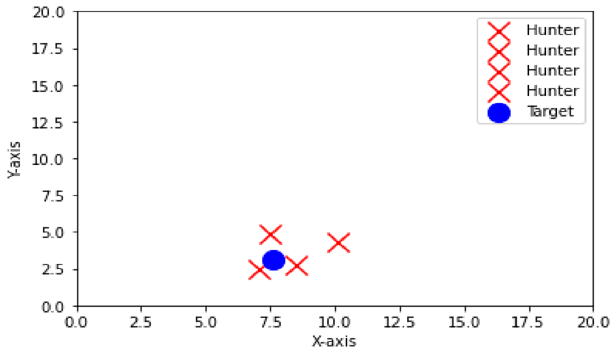

Fig. 8. The robots hunt the target successfully

IV. CONCLUSION

This research marks a new contribution for multi-robot systems (MRS) by introducing the leader-follower approach, redefining how robots collaborate during pursuit scenarios. This strategy optimizes pursuit efficiency by assigning roles based on proximity to the target, streamlining collaboration within MRS. The benefits derived from role assignment based on proximity are profound, as it ensures that robots closer to the target assume more active roles in the pursuit, leading to a more streamlined and optimized collaborative effort. This optimization, in turn, contributes to an overall improvement in the performance of multi-robot systems, making them more effective and adaptable in dynamically evolving pursuit scenarios. The leader-follower strategy, therefore, stands as a cornerstone in facilitating effective collaboration, heralding a new era in the capabilities of robot teams engaged in pursuit missions. This advancement propels robot teams into a new era of capabilities during pursuit missions. In the centralized MRS architecture, challenges with data reliability and anomaly detection take the spotlight. Sensor inaccuracies and data transmission issues threaten information integrity. To overcome these challenges, our research introduces an innovative solution—merging the leader-follower strategy with machine learning techniques, specifically Linear Discriminant Analysis (LDA) variants, for anomaly detection. This integration effectively addresses sensor-related complexities, ensuring shared data integrity among robots. This approach marks more than incremental progress, signaling a paradigm shift in the conception and implementation of Multi-Robot Systems (MRS). It extends beyond anomaly detection, fundamentally reshaping adaptability and efficiency in dynamic environments. With applications spanning search and rescue, environmental monitoring, and confined space surveillance, its impact reverberates across industries, prioritizing operational efficiency and decisive decision-making. In shaping the future of robotics, our method elevates MRS beyond mere tools, transforming them into dynamic collaborators capable of seamless adaptation and evolution. This paper presents novel contributions by integrating a leader-follower strategy and machine learning within a centralized architecture to enhance computing efficiency. It pioneers anomaly detection using DA methods, reshaping multi-robot systems for improved efficiency in dynamic environments. Additionally, the paper introduces an adaptable methodology designed for less sophisticated robots, utilizing a cost-effective single sensor and addressing computational constraints across diverse robotic platforms.

In the research's forward-looking perspective, it paves the way for exploring the Collaborative Kernel Discriminant Analysis (CKDA) approach, emphasizing its potential to address large-scale problems and real-time data streams in multi-robot systems (MRS). By incorporating ensemble learning and compression techniques, CKDA emerges as a transformative tool capable of handling vast datasets in dynamic environments. The significance of probing its scalability and adaptability is paramount, as CKDA holds the potential to revolutionize anomaly detection not only in pursuit scenarios but also in broader applications like cybersecurity and industrial automation. This exploration signifies a strategic step toward more robust, efficient, and real-time anomaly detection methodologies, establishing the groundwork for heightened performance and adaptability across diverse domains.

REFERENCES

[1] Y. Msala, M. Hamlich, and A. Mouchtachi, “A new Robust Heterogeneous Multi-Robot Approach Based on Cloud for Task Allocation,” in *2019 5th International Conference on Optimization and Applications (ICOA)*, pp. 1–4, Apr. 2019, doi: 10.1109/ICOA.2019.8727618.

[2] O. Hamed, M. Hamlich, and E. Mohamed, “Hunting strategy for multi-robot based on wolf swarm algorithm and artificial potential field,” *Indones. J. Electr. Eng. Comput. Sci*., vol. 25, pp. 159–171, Jan. 2022, doi: 10.11591/ijeecs.v25.i1.pp159-171.

[3] X. Cao and L. Guo, “A leader–follower formation control approach for target hunting by multiple autonomous underwater vehicle in three-dimensional underwater environments,” *Int. J. Adv. Robot. Syst*., vol. 16, no. 4, p. 1729881419870664, Jul. 2019, doi: 10.1177/1729881419870664.

[4] R. Chand, K. Raghuwaiya, J. Vanualailai, and J. Raj, “Leader-Follower Based Control of Fixed-Wing Multi-Robot System (MRS) via Split-Rejoin Maneuvers in 3D,” in *Proceedings of Third International Conference on Advances in Computer Engineering and Communication Systems*, pp. 195–209, 2023, doi: 10.1007/978-981-19-9228-5_18.

[5] X. Cao and C. Sun, “A potential field-based PSO approach to multi-robot cooperation for target search and hunting,” *Autom*., vol. 65, no. 12, pp. 878–887, Dec. 2017, doi: 10.1515/auto-2017-0080.

[6] Q. Pan *et al*., “Adaptive Cooperative Gene Regulatory Network Optimized by Elastic Deformation Algorithm for Multirobot Hunting,” *IEEE Syst. J*., vol. 17, no. 3, pp. 4843–4854, Sep. 2023, doi: 10.1109/JSYST.2023.3285736.

[7] O. Hamed and M. Hamlich, “Hybrid Formation Control for Multi-Robot Hunters Based on Multi-Agent Deep Deterministic Policy Gradient,” *MENDEL*, vol. 27, no. 2, Dec. 2021, doi: 10.13164/mendel.2021.2.023.

[8] X. An, C. Wu, Y. Lin, M. Lin, T. Yoshinaga, and Y. Ji, “Multi-Robot Systems and Cooperative Object Transport: Communications, Platforms, and Challenges,” *IEEE Open J. Comput. Soc*., vol. 4, pp. 23–36, 2023, doi: 10.1109/OJCS.2023.3238324.

[9] H. Kabir, M.-L. Tham, and Y. C. Chang, “Internet of robotic things for mobile robots: concepts, technologies, challenges, applications, and future directions,” *Digit. Commun. Netw*., vol. 9, no. 6, pp. 1265-1290, 2023, doi: 10.1016/j.dcan.2023.05.006.

[10] F. -J. Mañas-Álvarez, M. Guinaldo, R. Dormido, and S. Dormido, "Robotic Park: Multi-Agent Platform for Teaching Control and Robotics," in *IEEE Access*, vol. 11, pp. 34899-34911, 2023, doi: 10.1109/ACCESS.2023.3264508.

[11] F. Corradini, S. Pettinari, B. Re, L. Rossi, and F. Tiezzi, “A BPMN-driven framework for Multi-Robot System development,” *Robot. Auton. Syst.*, vol. 160, p. 104322, Feb. 2023, doi: 10.1016/j.robot.2022.104322.

[12] R. Muthusamy *et al*., “Strictly Decentralized Approaches for Multi-Robot Grasp Coordination,” in *2023 IEEE 19th International Conference on Automation Science and Engineering (CASE)*, pp. 1–8, Aug. 2023, doi: 10.1109/CASE56687.2023.10260355.

[13] S. Leder and A. Menges, “Architectural design in collective robotic construction,” *Autom. Constr*., vol. 156, p. 105082, Dec. 2023, doi: 10.1016/j.autcon.2023.105082.

[14] Z. Lv, C. Cheng, and H. Lv, “Multi-Robot Distributed Communication in Heterogeneous Robotic Systems on 5G Networking,” *IEEE Wirel. Commun.*, vol. 30, no. 2, pp. 98–104, Apr. 2023, doi: 10.1109/MWC.001.2200315.

[15] P. -Y. Lajoie and G. Beltrame, "Swarm-SLAM: Sparse Decentralized Collaborative Simultaneous Localization and Mapping Framework for Multi-Robot Systems," in *IEEE Robotics and Automation Letters*, vol. 9, no. 1, pp. 475-482, Jan. 2024, doi: 10.1109/LRA.2023.3333742.

[16] J. P. Queralta, F. Keramat, S. Salimi, L. Fu, X. Yu, and T. Westerlund, “Blockchain and Emerging Distributed Ledger Technologies for Decentralized Multi-robot Systems,” *Curr. Robot. Rep*., vol. 4, no. 3, pp. 43–54, Sep. 2023, doi: 10.1007/s43154-023-00101-3.

[17] L. Bramblett and N. Bezzo, “Epistemic planning for multi-robot systems in communication-restricted environments,” *Front. Robot. AI*, vol. 10, 2023.

[18] Z. Xun *et al*., "CREPES: Cooperative RElative Pose Estimation System," *2023 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pp. 5274-5281, 2023, doi: 10.1109/IROS55552.2023.10342523.

[19] M. Dehghani, R. A. McKenzie, R. A. Irani, and M. Ahmadi, “Robot-mounted sensing and local calibration for high-accuracy manufacturing,” *Robot. Comput.-Integr. Manuf*., vol. 79, p. 102429, Feb. 2023, doi: 10.1016/j.rcim.2022.102429.

[20] P. Wei *et al*., “Impact Analysis of Temperature and Humidity Conditions on Electrochemical Sensor Response in Ambient Air Quality Monitoring,” *Sensors*, vol. 18, no. 2, Feb. 2018, doi: 10.3390/s18020059.

[21] S. El Bied, M. V. de-la-Fuente-Aragon, L. Ros-McDonnell, and D. Ros-McDonnell, “Urban Ecosystem Sustainability Impacts of Air Quality,” in *IoT and Data Science in Engineering Management*, pp. 225–232, 2023, doi: 10.1007/978-3-031-27915-7_41.

[22] R. Al-amri, R. K. Murugesan, M. Man, A. F. Abdulateef, M. A. Al-Sharafi, and A. A. Alkahtani, “A Review of Machine Learning and Deep Learning Techniques for Anomaly Detection in IoT Data,” *Appl. Sci*., vol. 11, no. 12, Jan. 2021, doi: 10.3390/app11125320.

[23] S. Mokhtari, A. Abbaspour, K. K. Yen, and A. Sargolzaei, “A Machine Learning Approach for Anomaly Detection in Industrial Control Systems Based on Measurement Data,” *Electronics*, vol. 10, no. 4, Jan. 2021, doi: 10.3390/electronics10040407.

[24] M. Hamlich, A. E. Khantach, and N. Belbounaguia, “Machine learning methods against false data injection in smart grid,” *Int. J. Reason.-Based Intell*. Syst., vol. 12, no. 1, pp. 51–59, Jan. 2020, doi: 10.1504/IJRIS.2020.104991.

[25] A. B. Nassif, M. A. Talib, Q. Nasir, and F. M. Dakalbab, “Machine Learning for Anomaly Detection: A Systematic Review,” *IEEE Access*, vol. 9, pp. 78658–78700, 2021, doi: 10.1109/ACCESS.2021.3083060.

[26] S. Wang, J. F. Balarezo, S. Kandeepan, A. Al-Hourani, K. G. Chavez, and B. Rubinstein, “Machine Learning in Network Anomaly Detection: A Survey,” *IEEE Access*, vol. 9, pp. 152379–152396, 2021, doi: 10.1109/ACCESS.2021.3126834.

[27] M. Chaabi, M. Hamlich, and M. Garouani, “Product defect detection based on convolutional autoencoder and one-class classification,” *Int. J. Artif. Intell.*, vol. 12, no. 2, p. 912, Jun. 2023, doi: 10.11591/ijai.v12.i2.pp912-920.

[28] Y. Xu, L. Zhang, B. Du, and L. Zhang, “Hyperspectral Anomaly Detection Based on Machine Learning: An Overview,” *IEEE J. Sel. Top. Appl. Earth Obs. Remote Sens*., vol. 15, pp. 3351–3364, 2022, doi: 10.1109/JSTARS.2022.3167830.

[29] S.-V. Oprea, A. Bâra, F. C. Puican, and I. C. Radu, “Anomaly Detection with Machine Learning Algorithms and Big Data in Electricity Consumption,” *Sustainability*, vol. 13, no. 19, Jan. 2021, doi: 10.3390/su131910963.

[30] M. Mittal, R. P. de Prado, Y. Kawai, S. Nakajima, and J. E. Muñoz-Expósito, “Machine Learning Techniques for Energy Efficiency and Anomaly Detection in Hybrid Wireless Sensor Networks,” *Energies*, vol. 14, no. 11, Jan. 2021, doi: 10.3390/en14113125.

[31] A. Guezzaz, Y. Asimi, M. Azrour, and A. Asimi, “Mathematical validation of proposed machine learning classifier for heterogeneous traffic and anomaly detection,” *Big Data Min. Anal*., vol. 4, no. 1, pp. 18–24, Mar. 2021, doi: 10.26599/BDMA.2020.9020019.

[32] M. Ibrahim, A. Alsheikh, F. M. Awaysheh, and M. D. Alshehri, “Machine Learning Schemes for Anomaly Detection in Solar Power Plants,” *Energies*, vol. 15, no. 3, Jan. 2022, doi: 10.3390/en15031082.

[33] E. Šabić, D. Keeley, B. Henderson, and S. Nannemann, “Healthcare and anomaly detection: using machine learning to predict anomalies in heart rate data,” *AI Soc*., vol. 36, no. 1, pp. 149–158, Mar. 2021, doi: 10.1007/s00146-020-00985-1.

[34] S. Salimpour, F. Keramat, J. P. Queralta, and T. Westerlund, “Decentralized Vision-Based Byzantine Agent Detection in Multi-robot Systems with IOTA Smart Contracts,” in *Foundations and Practice of Security*, pp. 322–337, 2023, doi: 10.1007/978-3-031-30122-3_20.

[35] A. Khatib, M. Hamlich, and D. Hamad, “Machine Learning based Intrusion Detection for Cyber-Security in IoT Networks,” *E3S Web Conf.*, vol. 297, p. 01057, 2021, doi: 10.1051/e3sconf/202129701057.

[36] W. Liang, Z. Ning, S. Xie, Y. Hu, S. Lu, and D. Zhang, “Secure fusion approach for the Internet of Things in smart autonomous multi-robot

systems," *Inf. Sci.*, vol. 579, pp. 468–482, Nov. 2021, doi: 10.1016/j.ins.2021.08.035.

[37] T. Ji, A. N. Sivakumar, G. Chowdhary, and K. Driggs-Campbell, "Proactive Anomaly Detection for Robot Navigation With Multi-Sensor Fusion," *IEEE Robot. Autom. Lett.*, vol. 7, no. 2, pp. 4975–4982, Apr. 2022, doi: 10.1109/LRA.2022.3153989.

[38] O. Hamed and M. Hamlich, "Improvised multi-robot cooperation strategy for hunting a dynamic target," in *2020 International Symposium on Advanced Electrical and Communication Technologies (ISAECT)*, pp. 1–4, Nov. 2020, doi: 10.1109/ISAECT50560.2020.9523684.

[39] F. Zhu, J. Gao, J. Yang, and N. Ye, "Neighborhood linear discriminant analysis," *Pattern Recognit.*, vol. 123, p. 108422, Mar. 2022, doi: 10.1016/j.patcog.2021.108422.

[40] V. Amiri and K. Nakagawa, "Using a linear discriminant analysis (LDA)-based nomenclature system and self-organizing maps (SOM) for spatiotemporal assessment of groundwater quality in a coastal aquifer," *J. Hydrol.*, vol. 603, p. 127082, Dec. 2021, doi: 10.1016/j.jhydrol.2021.127082.

[41] Y. Li, B. Liu, Y. Yu, H. Li, J. Sun, and J. Cui, "3E-LDA: Three Enhancements to Linear Discriminant Analysis," *ACM Trans. Knowl. Discov. Data*, vol. 15, no. 4, p. 57:1-57:20, Mar. 2021, doi: 10.1145/3442347.

[42] S. D. Fabiyi, P. Murray, J. Zabalza, and J. Ren, "Folded LDA: Extending the Linear Discriminant Analysis Algorithm for Feature Extraction and Data Reduction in Hyperspectral Remote Sensing," *IEEE J. Sel. Top. Appl. Earth Obs. Remote Sens.*, vol. 14, pp. 12312–12331, 2021, doi: 10.1109/JSTARS.2021.3129818.

[43] Y. Zhou, S. Yan, Y. Ren, and S. Liu, "Rolling bearing fault diagnosis using transient-extracting transform and linear discriminant analysis," *Measurement*, vol. 178, p. 109298, Jun. 2021, doi: 10.1016/j.measurement.2021.109298.

[44] W. Lin, Q. Gao, M. Du, W. Chen, and T. Tong, "Multiclass diagnosis of stages of Alzheimer's disease using linear discriminant analysis scoring for multimodal data," *Comput. Biol. Med.*, vol. 134, p. 104478, Jul. 2021, doi: 10.1016/j.compbiomed.2021.104478.

[45] L. Xu, J. Raitoharju, A. Iosifidis, and M. Gabbouj, "Saliency-Based Multilabel Linear Discriminant Analysis," in *IEEE Transactions on Cybernetics*, vol. 52, no. 10, pp. 10200-10213, Oct. 2022, doi: 10.1109/TCYB.2021.3069338.

[46] G. E. Castro Guzman and A. Fujita, "Convolution-based linear discriminant analysis for functional data classification," *Inf. Sci.*, vol. 581, pp. 469–478, Dec. 2021, doi: 10.1016/j.ins.2021.09.057.

[47] D. Albani, W. Hönig, D. Nardi, N. Ayanian, and V. Trianni, "Hierarchical Task Assignment and Path Finding with Limited Communication for Robot Swarms," *Appl. Sci.*, vol. 11, no. 7, Jan. 2021, doi: 10.3390/app11073115.

[48] F. Nie, H. Chen, S. Xiang, C. Zhang, S. Yan, and X. Li, "On the Equivalence of Linear Discriminant Analysis and Least Squares Regression," *IEEE Trans. Neural Netw. Learn. Syst.*, pp. 1–11, 2022, doi: 10.1109/TNNLS.2022.3208944.

[49] D. Kim and T.-Y. Heo, "Anomaly Detection with Feature Extraction Based on Machine Learning Using Hydraulic System IoT Sensor Data," *Sensors*, vol. 22, no. 7, Jan. 2022, doi: 10.3390/s22072479.

[50] M. O. Adebiyi, M. O. Arowolo, M. D. Mshelia, and O. O. Olugbara, "A Linear Discriminant Analysis and Classification Model for Breast Cancer Diagnosis," *Appl. Sci.*, vol. 12, no. 22, Jan. 2022, doi: 10.3390/app122211455.

[51] Q. Ma, C. Sun, B. Cui, and X. Jin, "A novel model for anomaly detection in network traffic based on kernel support vector machine," *Comput. Secur.*, vol. 104, p. 102215, May 2021, doi: 10.1016/j.cose.2021.102215.

[52] L. Kang, L. Zhang, X. Huang, W. Hu, and X. Yang, "Hardware Fingerprint Authentication in Optical Networks Assisted by Anomaly Detection," *IEEE Photonics Technol. Lett.*, vol. 34, no. 19, pp. 1030–1033, Oct. 2022, doi: 10.1109/LPT.2022.3199106.

[53] L. Begic Fazlic *et al.*, "A Novel Hybrid Methodology for Anomaly Detection in Time Series," *International Journal of Computational Intelligence Systems*, vol. 15, no. 1, p. 50, 2022.

[54] A. Khatib, F. Dufrenois, M. Hamlich, and D. Hamad, "Collaborative Kernel Discriminant Analysis for Large Scale Multi Class Problems," in *Smart Applications and Data Analysis*, pp. 34–50, 2022, doi: 10.1007/978-3-031-20490-6_4.

[55] K. Chumachenko, J. Raitoharju, A. Iosifidis, and M. Gabbouj, "Speed-up and multi-view extensions to subclass discriminant analysis," *Pattern Recognit.*, vol. 111, p. 107660, Mar. 2021, doi: 10.1016/j.patcog.2020.107660.

[56] R. Graf, M. Zeldovich, and S. Friedrich, "Comparing linear discriminant analysis and supervised learning algorithms for binary classification—A method comparison study," *Biom. J.*, vol. 66, no. 1, p. 202200098, 2024, doi: 10.1002/bimj.202200098.

[57] S. Vijh, H. M. Pandey, and P. Gaurav, "Brain tumor segmentation using extended Weiner and Laplacian lion optimization algorithm with fuzzy weighted k-mean embedding linear discriminant analysis," *Neural Comput. Appl.*, vol. 35, no. 10, pp. 7315–7338, Apr. 2023, doi: 10.1007/s00521-021-06709-w.

[58] C.-N. Li, Y.-F. Qi, D. Zhao, T. Guo, and L. Bai, "FF-norm two-dimensional linear discriminant analysis and its application on face recognition," *Int. J. Intell. Syst.*, vol. 37, no. 11, pp. 8327–8347, 2022, doi: 10.1002/int.22941.

[59] C. Ozdemir, R. C. Hoover, K. Caudle, and K. Braman, "High-Order Multilinear Discriminant Analysis Via Order-N Tensor Eigendecomposition," *arXiv preprint arXiv:2205.09191*, 2022.

[60] P. Buzzini, J. Curran, and C. Polston, "Comparison between visual assessments and different variants of linear discriminant analysis to the classification of Raman patterns of inkjet printer inks," *Forensic Chem.*, vol. 24, p. 100336, Jun. 2021, doi: 10.1016/j.forc.2021.100336.

[61] B. Bartan and M. Pilanci, "Neural Fisher Discriminant Analysis: Optimal Neural Network Embeddings in Polynomial Time," in *Proceedings of the 39th International Conference on Machine Learning, PMLR*, pp. 1647–1663, 2022.

[62] T. Yan, D. Wang, T. Xia, J. Liu, Z. Peng, and L. Xi, "Investigation on optimal discriminant directions of linear discriminant analysis for locating informative frequency bands for machine health monitoring," *Mech. Syst. Signal Process.*, vol. 180, p. 109424, Nov. 2022, doi: 10.1016/j.ymssp.2022.109424.

[63] T. Liu, Z. Yang, A. Marino, G. Gao, and J. Yang, "Joint Polarimetric Subspace Detector Based on Modified Linear Discriminant Analysis," *IEEE Trans. Geosci. Remote Sens.*, vol. 60, pp. 1–19, 2022, doi: 10.1109/TGRS.2022.3148979.

[64] W. Chang, F. Nie, Z. Wang, R. Wang, and X. Li, "Self-weighted learning framework for adaptive locality discriminant analysis," *Pattern Recognit.*, vol. 129, p. 108778, Sep. 2022, doi: 10.1016/j.patcog.2022.108778.

[65] M.-L. Zhang, J.-H. Wu, and W.-X. Bao, "Disambiguation Enabled Linear Discriminant Analysis for Partial Label Dimensionality Reduction," *ACM Trans. Knowl. Discov. Data*, vol. 16, no. 4, pp. 1-18, Jan. 2022, doi: 10.1145/3494565.