Design and Analysis of IO and FO Controllers to Investigate the Effects of Process Parameter Perturbations on Lag-Dominant Time Delay Systems

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Abstract*—This paper focuses on the design, analysis and implementation of Integer-order (IO) and Fractional-order (FO) controllers for systems characterized by lag-dominant time delays. The existing literature has been examined to analyze the methodology employed in tuning IO and FO controllers for first-order time delay system for perturbations in process parameters. It is observed that there is scope to investigate better controllers for lag-dominant time delay systems. The five different structures of controllers are chosen. The paper proposes IO and FO controllers tailored for a test group comprising 16 first-order systems with time delays. These IO and FO controllers are designed to fulfil design specifications: phase margin, peak overshoot, IAE, ITAE and ISE using Modified Bode's Ideal Loop Transfer Function with delay method. For comparison conventional IO tuning method, Gain-Phase Margin Tester(GPMT) and Fractional M_s Constrained Integral Gain Optimization Method (F-MIGO) is used. The simulation results and performance evaluation for both IO and FO controllers are obtained for a range of values of relative dead time of the system represented by τ. The τ value is obtained by varying conditions of delay (L) and time constant (T). Two scenarios are taken into account: the first involves varying L while keeping T constant, and the second involves keeping L constant while varying T. The main objective of the paper is to analyze IO and FO controllers based on time and frequency domain parameters, performance error indices, disturbance rejection, gain variations, Total Variation (TV) and control efforts for perturbations in process parameters. The simulation results indicate that FO controllers show superior tolerance to perturbations in L and T when compared to IO counterparts. This observation was noted during the analysis of the control system by varying values of L and T to obtain a consistent value of τ. Thus, the extensive simulation studies demonstrate that the FO controller tailored for lag-dominant time delay systems outperforms its IO counterpart in terms of robustness, closed-loop stability and error performance metrics.

Keywords*—First-Order Plus Time Delay System; Perturbations; Time Delay; Time Constant; Robustness; Stability; Error Metrics; Integer-Order Controller; Fractional-Order Controller.

I. INTRODUCTION

The lag-dominant time delay systems are prevalent across diverse domains such as process control, robotics, transportation, and communication networks [1]. These systems are characterized by a primary dynamic component involving a noticeable time delay, resulting in a substantial delay in the system's response relative to its input or stimulus. The delay can lead to instability, oscillations, or even system failure [2]. Therefore controlling lag-dominant time delay systems poses several intricate challenges that have been emphasized by numerous researchers [3]. These challenges encompass concerns about stability, robustness, performance degradation, phase cancellation, control signal smoothing, and stability margins. Therefore, there is a pressing need for a robust control design methodology that can ensure stability and consistent performance, even in the face of variations [4]. From a control perspective, effectively managing variations in delay time and time constant within lag-dominant time delay systems necessitates a meticulous approach to control system design and execution.

The literature offers several approaches commonly utilized to tackle these variations, including Adaptive Control, Self-Tuning Control, Estimation and Feedback Compensation. Adaptive controllers [5] modify their parameters to accommodate variations in the process over time, such as shifts in plant load. Consequently, an adaptive controller assesses the requirements for optimal control with updated process conditions and implements the necessary modifications. The self-tuning capability, commonly referred to as auto-tuning, found in numerous commercially available controllers is a form of adaptive control. Thus, the choice of a controller for a system dominated by lag and time delays depends on multiple factors, such as the system's specific characteristics, control objectives, and performance requirements [6] [7]. Nonetheless, Lead-Lag compensators, Smith Predictors, Model Predictive Control (MPC) and Proportional-Integral-Derivative (PID) controllers are frequently employed and can deliver stable control and satisfactory performance in many lag-dominated time delay systems. This study [8] introduces a novel method for configuring phase-lead/lag compensators to meet specified gain and phase margins in the context of all-pole stable plants with time delay. The Smith predictor control scheme [9] for time-delay systems discusses a new control scheme which allows zero-latency tracking of predictable targets by a time-delay system. A perceived limitation of the classical Smith predictor lies in its fixed time delay. The paper [10] presents theoretical and practical findings concerning the implementation of the Smith predictor in scenarios involving a fluctuating time delay. The model predictive control [11] paper presents a design predictive algorithm for control of time-delayed systems with measurable disturbance compensation. It's worth noting that the tuning of these controllers may require adjustments to accommodate the system's time delays. Proportional-Integral-Derivative (PID) controllers are frequently employed and can deliver stable control and satisfactory performance in many lag-dominated time delay systems [12]- [19]. Consequently, PID controllers with Fractional Calculus (FC) notations are widely adopted, known as Fractional-order control (FOC) [20]- [22]. FOC proves effective in addressing the challenges posed by such systems, and as a result, it has gained significant importance [23].

A thorough analysis of the literature on the integration and use of Fractional-order proportional integral derivative (FOPID) control in industry was provided in the studies [24]- [25]. Important problems with the industrialization of FOPID controllers were noted and explained. Studies [26]- [32] have indicated that FOPID controllers, as opposed to integer-order PID controllers, can improve a system's closed loop performance. This advantage is due to availability of five parameters for tuning FOPID controllers whereas three parameters for tuning IO controllers. Both the time and frequency domain tuning techniques for FOPID controllers which manage higher order processes are compared in the research [33] [34]. The applications of FO controllers to name a few include PV integrated power system, oscillatory system, tracking control, magnetic levitation, process control system, space technology, etc [35]- [45]. There have been numerous publications that review the application of FC in control systems. These papers predominantly concentrate on general tuning methodologies, particularly for the fractional-order proportional integral derivative controller. Nevertheless, not all tuning approaches are universally applicable, especially in the case of time delay systems. This underscores the necessity to compile a comprehensive body of literature on FOC applications, challenges, and advancements specifically tailored for time delay processes. The objective oF [46] is to present a state-of-the-art overview that can serve an insight into fractional-order tuning strategies designed for systems with time delays. The literature [47]- [49] reviews different tuning methods of FO controllers for time delay systems. The paper [50] proposes a FOPI controller in order to fulfill different robustness design specifications like overshoot, noise rejection constraints. The optimization technique used to tune the controller is based on a nonlinear function minimization subject to few given nonlinear constraints. This study [51]- [58] introduces an optimal fractional-order controller designed for a specific FO model by employing the direct synthesis method, identification and tuning FO controllers for time delay system using fractional pole, design technique based on gain and phase margin specifications. To conduct the study of tuning of FO controllers, this paper explores five different tuning rules, drawing from a review of rules published in existing literature. Initially, the paper delves into the IOPI and IOPID tuning rules [15]- [19], followed by an investigation of FOPI and FOPID tuning rules [59], [61], [62], [63], [64], [66]. The outcomes of this analysis showcase notable distinctions in the robustness of these controllers. An important takeaway from this research is the enhanced modeling capability and robust control performance facilitated by fractional-order control systems over integer-order control systems. This comparative evaluation provides valuable insights into the effectiveness of these controllers in addressing the specific needs of time delay system to perturbations in L and T. Furthermore, the paper includes a comparative assessment of tuning rules, taking into account various performance metrics such as the integral of absolute error (IAE), integral of time-weighted absolute error (ITAE), integral of square error (ISE), along with time and frequency domain parameters. The robustness based on a flattened phase characteristic is considered a major asset of robust controller system design in frequency-domain tuning efforts. It is achieved through Bode's Ideal Loop Transfer Function Method. The literature [65]- [73] suggests tuning methods based on Bode's Ideal Loop Transfer Function Method for design of FOPI and FOPID controllers. The primary advantage provided by this structure is iso-damping, meaning that overshoot remains consistent regardless of system gain. In systems demonstrating iso-damping, the overshoots in closed-loop step responses stay nearly unchanged across various controller gain settings. This guarantees robustness to variations in gain within the closed-loop system. This work is sincere effort to present FOPID controller design using Modified Bode's Ideal Loop Transfer Function Method with delay for first-order time delay lag-dominant systems. Also the proposed work is compares three tuning methods and five different structures of controllers.

The remaining part of this paper is organized as follows: section II introduces time delay systems with prime focus on first-order time delay system ; section III presents IO and FO control strategies for time delay system, section IV shows simulation for test batch of 16 first-order lag dominant time delay process and in section V, results and discussions are presented. Finally, conclusions are drawn in Sec. VI.

II. Time delay systems(TDS)

Time delays are experienced in modern applications, biological systems, mechanical systems and electrical fields. The literature survey shows many specific examples across various fields like liquid level system, Servo control and regulatory control of lag-dominant process [63], PMSM speed servo model [70], the dynamics of wind turbine model for energy generation system [74], fractional-order plant model for heating furnace [75] modeled as time delay transfer functions. From a frequency domain perspective, the process phase experiences an additional lag when there is delay thus making closed-loop control of these processes ultimately more difficult. To deal with time delay characteristics, numerous control strategies have been developed over time [46]. By applying FOC design techniques to the time delay field, the goal is to combine the superior performance of FC to the time delay control issue [76] [77]. In the research discussed in [59], the focus is on systems with time delays, where any action taken within the system affects the closed-loop behavior only after the process's dead time. These time delays can lead to unwanted effects such as oscillations, extended settling times, and diminished overall control precision. Designing a controller capable of addressing these performance issues and ensuring the system's desirable behavior becomes a challenging task. Analyzing and creating controllers for systems with dead time is, therefore, a complex endeavor. As a response to these challenges, the paper places significant emphasis on the development of fractional-order systems tailored for first-order lag-dominant systems. Fractional-order systems [63]- [64] are seen as a promising solution to tackle the intricacies of dead-time systems and improve their control performance. As a result, this paper places a strong emphasis on the development of fractional-order systems specifically tailored for first-order lag-dominant systems. The paper primarily concentrates on the First-Order Plus Delay Time (FOPDT) model [21], which is represented by (1),

$$P(s) = \frac{K}{1+Ts}e^{-Ls}, \tag{1}$$

where, K - Process gain, T - Time constant, L - Delay. A crucial property of the FOPDT models is the relative dead time of the system, which is defined as

$$\tau = \frac{L}{L+T} \tag{2}$$

The range of parameter τ is between 0 and 1. Systems in which $L \gg T$ are called delay dominant, $L \ll T$ are called lag dominant, $L \approx T$ are called balanced lag, delay system. τ is a significant factor which affects the controller performance and closed-loop response of control system. In this paper, the impact of τ on various tuning strategies is concentrated on by variation in delay L and time constant T. The paper focusses on closed loop response of control system for different values of τ.

III. Control strategies for Time Delay System with IO and FO Controllers

Control strategies are of most importance in governing the behavior of dynamic systems. When dealing with systems characterized by time delays, there are typically two predominant approaches employed; IO and FO control. The block diagram of the IO and FO control system is shown in Fig. 1, There are three

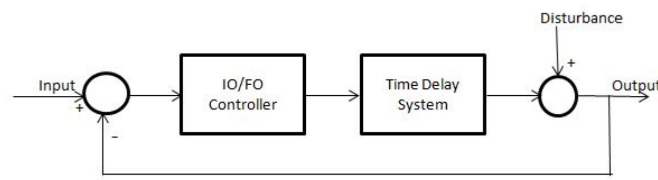

Fig. 1. Block diagram of IO and FO controllers with Time Delay System [2]

distinct control strategies like Rule-based methods, Analytical methods and Numerical methods. The literature demonstrates that various approaches for controller design are commonly employed, including Manual Tuning, the Ziegler-Nichols Method, Model-based Methods, and Optimization Techniques [78] [79] [80] [81]. Manual Tuning can often lead to satisfactory results for relatively simple systems and does not require extensive mathematical modeling or computation. It is time consuming and provides suboptimal tuning especially for complex systems. Therefore manual tuning may not always result in robust performance under varying operating conditions or disturbances. The Ziegler-Nichols Method provides a systematic approach for tuning PID controllers and requires minimal prior knowledge of the system dynamics. It can be effective for stabilizing a wide range of systems. But may result in oscillatory or unstable behavior if not carefully implemented. The method relies on the system being linear and stable, which may not always be the case. It may not be suitable for systems with nonlinearities or complex dynamics. The Model-based Methods utilizes mathematical models to predict system behavior and optimize controller parameters based on specific performance criteria. But the drawback that it requires accurate knowledge of the system dynamics, and therefore performance may degrade if the system deviates significantly from the assumed model. Optimization Techniques allows automated and systematic tuning of controller parameters. They can handle complex systems with nonlinearities and uncertainties and provides flexibility in defining and optimizing performance criteria. But such methods are computationally intensive. Each method has its trade-offs in terms of simplicity, accuracy, robustness, and computational complexity. The choice of method depends on factors such as the system's complexity, system dynamics and desired performance criteria. The structure of IO [16] and FO [27], [46] controllers chosen for implementation are given in (3), (4), (5), (6) below;

$$C_{IOPI}(s) = K_p + \frac{K_i}{s}, \tag{3}$$

$$C_{IOPID}(s) = K_p + \frac{K_i}{s} + K_d s, \tag{4}$$

The corresponding generalized FO controller is given by [7],

$$C_{FOPI}(s) = K_p + \frac{K_i}{s^{\lambda}}, \tag{5}$$

$$C_{FOPID}(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu}, \tag{6}$$

where K_p is the proportional gain, K_i is the integration gain, and K_d is the derivative gain and λ, μ are the fractional-order operators. Self-tuning [46] and auto-tuning [28], [37] methods are also used for tuning of fractional-order controllers. FOC have proved their efficacy over the conventional IO controllers by providing design flexibility with two more parameters namely λ and μ, guaranteeing a more robust closed-loop configuration enhancing the system control performance. In this paper, two IO controllers and three FO controllers are tuned with the purpose of dealing with time delay. The FO control strategies for time delay system discussed in this paper are Gain-Phase Margin Tester for FOPI controllers, Fractional M_s Constrained Integral Gain Optimization Method (FMIGO) and Modified Bode's Ideal Loop Transfer Function with delay for FOPID controllers.

A. FOPI Controller design using Gain-Phase Margin tester(GPMT)

In paper [59], the author proposes that any stable FOPDT system can be used to create a two-dimensional representation of complete set of achievable gain crossover frequencies(ω_{gc}) and phase margins(PM). Before designing the controller, all possible combinations of ω_{gc} and phase margin can be tested using this comprehensive set as a foundation. The stabilizing and desired FOPI can only be guaranteed if the combination is selected from achievable region. The system is FOPDT and is linear time invariant system. The paper proposes FOPI controller as,

$$C_1(s) = K_p + \frac{K_i}{s^r}, \tag{7}$$

where, K_p, K_i are the controller gains, and the real number $r \in (0, 2)$ is the fractional order. GPMT is a Gain-Phase Margin Tester [59] provides data for plotting the parameter plane boundaries of constant GM and PM and the equations for K_p and K_i are as follows;

$$K_p = \frac{-(B_1 S_1 + B_2 C_1)}{AKS_2 \omega^r}, \tag{8}$$

$$K_i = \frac{B - B_1 S_1 C_1 + B_2 C_1^2}{AKS_1} + \frac{B_1 S_1 C_2 + B_2 C_1 C_2}{AKS_2}. \tag{9}$$

The controller is obtained using (8) and (9).

B. PI^{λ} Controller design using Fractional M_s Constrained Integral Gain Optimization Method (F-MIGO)

The FMIGO method, which is based on M_s constrained integral gain optimization, is used to create tuning rules for the FOPDT class of dynamic systems and extended to handle the PI^{λ} case with the fractional order α. The final tuning rules only use the FOPDT model's relative dead time τ to determine the best fractional order α gains and PI^{λ} gains [21]. The frequency domain description of the PI^{λ} can be found in by

$$C(s) = K_p + \frac{K_i}{s}, \tag{10}$$

The rejection of load disturbances is this method's primary design objective. The effect of load disturbance at output will be minimal if the integral gain K_i is maximized, as demonstrated in [11]. The F-MIGO design algorithm allows to find controller gains at any α. And optimal gain K_p is calculated using following relation:

$$K_p = \frac{1}{k}\left(\frac{0.2978}{\tau + 0.00307}\right). \tag{11}$$

The optimal integral gain K_i is calculated using following relation:

$$K_i = \frac{K_p(\tau^2 - 3.4022\tau + 2.405)}{0.8578T}. \tag{12}$$

The controller is obtained using (11) and (12).

C. Modified Bode's Ideal Loop Transfer Function with delay

It is suggested that the Bode's ideal open-loop transfer function [60] is,

$$G(s) = (\omega_{gc}/s)^{\alpha}, \alpha \in R, \tag{13}$$

where ω_{gc} is a real and gain crossover frequency of $G(s)$. In FOPID design, the robustness against gain variation is frequently used as an additional specification. Under unity feedback, the ideal closed-loop model with infinite gain and constant phase margin(PM) is obtained by selecting $G(s)$ as the open-loop transfer function.

$$G(s) = \frac{\omega_c^{\alpha}}{s^{\alpha} + \omega_c^{\alpha}}, \tag{14}$$

Bode's ideal loop transfer function(BLTF), was Bode's elegant solution to this robust design problem [65]- [72]. The iso-damping property of Bode's ideal control loop frequency response around the gain crossover frequency is provided by its fractional integrator shape. A desired closed-loop model for TDS is selected in conjunction with the time-delay and the Bode's ideal transfer function and the model is,

$$H(s) = \frac{\omega_c^{\alpha}}{s^{\alpha} + \omega_c^{\alpha}} e^{-Ls}, \tag{15}$$

so that time delay in $H(s)$ is same as the actual delay. The fundamental idea behind the FOPID controller design is to close the parametric model $\tilde{G}_p$ to the actual plant $G_p(s)$ through data fitting by solving all of the unknown parameters. It is assumed that some prior knowledge of the process will be utilized in the design. Under the constraints of time-delay, bandwidth, such as ω_c, cannot be arbitrary large. The possible choice of ω_c should therefore satisfy

$$\omega_c^\alpha \leq (\frac{\epsilon}{1-\epsilon})/ \parallel (1-e^{-Ls})/s^\alpha \parallel_\infty \tag{16}$$

Next data fitting to be carried out with the constraint in (16), to have $G_p(s) \approx \tilde{G}_p(s)$. In this way, the open-loop transfer function satisfies

$$G_p(s)G_c(s) \approx \tilde{G}_p(s)G_c(s) = \frac{\omega_c^\alpha e^{-Ls}}{s^\alpha}(1-\Delta(s)). \tag{17}$$

Recalling that $\parallel \Delta(s) \parallel_\infty < \epsilon$ and ϵ is a small positive constant, we have

$$G_p(s)G_c(s) \approx= \frac{\omega_c^\alpha e^{-Ls}}{s^\alpha}(1-\Delta(s)) \approx \frac{\omega_c^\alpha e^{-Ls}}{s^\alpha} \tag{18}$$

The gain and PM of $G_p(s)G_c(s)$ are expressed by, using the approximation in (18)

$$A_m \approx \parallel \left(\frac{(2-\alpha)\pi}{2L\omega_c}\right)^\alpha \parallel, \tag{19}$$

$$\gamma = (\pi - \frac{\alpha}{2}\pi - L\omega_c)rad = (180 - 90\alpha - 57.3L\omega_c)^\circ, \tag{20}$$

respectively. To assure the closed-loop stability, requirement is $A_m > 1$ and $\gamma > 0$, to have

$$\omega_c < \frac{\pi}{L} - \frac{\alpha\pi}{2L}, \tag{21}$$

when $0 < \alpha < 2$. And (19) and (20) give the estimation of margins of stability.

Using the above design methods the parameters of proposed FOPID design are given as following.

$$k_i = \frac{\omega_c^\alpha}{1 + \omega_c^\alpha \lim_{s\to 0} \frac{1-e^{-Ls}}{s^\alpha} G_p(j0)}, \tag{22}$$

$$k_d(\mu) = -\frac{(dp+cq)\omega_x^\alpha k_p + k_i q}{(pb+qa)\omega_x^t}, \tag{23}$$

$$k_p(\mu) = \frac{bk_i(p^2+q^2) - (pb+qa)\omega_c^\alpha}{(da-cb)(p^2+q^2)\omega_x^\alpha}. \tag{24}$$

Up-till now, 2 parameters λ and k_i are found. k_d and k_p are determined with the variable μ.

In (24), k_d and k_p therefore is uniquely determined if μ is iterated. The controller is obtained using (22), (23), (24).

IV. Simulation Case Study of Lag-Dominant Time Delay System

In this paper, a case study of a lag dominant FOPDT model is chosen and the design methods mentioned above are tested on the 16 processes. A test batch of 16 simulated FOPDT lag dominant processes for K=1 is given in (25) as,

$$P(s) = \frac{1}{1+Ts}e^{-Ls} \tag{25}$$

The 16 processes distilled for 2 different scenarios are represented in (26) below,

$$\begin{aligned} P(s) &= \frac{1}{1+Ts}e^{-\Delta Ls} \\ P(s) &= \frac{1}{1+\Delta Ts}e^{-Ls} \end{aligned} \tag{26}$$

This work presents the implementation of τ shifts targeting lag dominant time delay significant process. The range of τ is selected between 0.001 to 0.444 to maintain lag dominance. Hence 16 different FOPDT processes served as the basis for the simulations are shown in this paper. For process perturbations, the changes in value of L and T are simulated for two scenarios as follows,

1) $L = 0.01, 0.03, 0.05, 0.1, 0.5, 0.8$ with $T = 1$
2) $T = 0.3, 0.5, 1.5, 2, 4, 6, 8, 10, 20, 50$ with $L = 0.1$

The values of gain cross-over frequency (ω_{gc}) are determined by setting the gain cross-over frequency at a specific point in the Bode diagram of the process, where the magnitude reaches a predefined value. To ensure an adequate level of stability in the controller and minimize additional phase lag before the closed-loop system becomes unstable, a constant phase margin (PM) of 60° is maintained for all processes. This consistent PM value is applied to all processes in the test bench to facilitate result comparability. By employing these controller tuning methods, various frequency domain parameters like PM, gain margin (GM), and ω_{gc}, as well as time domain parameters including settling time T_s, rise time T_r, and overshoot M_p are determined, and the responses are illustrated. Additionally, an analysis of control efforts is also conducted. The controllers designed and simulated for a test batch of 16 lag dominant time delay process are given below in (27), (28), (29), (30) , (31).

1. IOPI controller [16]

$$C_1(s) = 2.8236 + \frac{4.6464}{s} \tag{27}$$

2. IOPID controller [62]

$$C_2(s) = 0.3582 + \frac{0.7783}{s} + 0.032s \tag{28}$$

3. FOPI controller(GPMT) [59]

$$C_3(s) = 3.3367 + \frac{4.64}{s^{1.21}} \tag{29}$$

4. FMIGO controller [21]

$$C_4(s) = 3.26 + \frac{7.99}{s^{0.7}} \quad (30)$$

5. FOPID controller(Modified BLTF with Delay)

$$C_5(s) = 3.1498 + \frac{4.93}{s^{1.01}} + 0.149s^{0.68} \quad (31)$$

The trends of the performance indices [21] IAE, ITAE and ISE are observed for the entire range of τ and these performance criteria are given in (32), (33), (34);

$$IAE = \int_0^\infty \mid e(t) \mid d(t) \quad (32)$$

$$ITAE = \int_0^\infty t \mid e(t) \mid d(t) \quad (33)$$

$$ISE = \int_0^\infty e^2(t)d(t) \quad (34)$$

The other metric for performance is total variation(TV) given by (35),

$$TV = \sum_{k+1}^{\infty} |u_{k+1} - u_k|. \quad (35)$$

where u_k and u_{k+1} are the controller output signals. TV is used as an indication of the smoothness of the control action for input changes for two conditions. It is observed that the performance of the closed loop system and the controller's robustness are the primary effects of the τ variation. For each variation of τ, closed-loop responses are inspected. The step responses nad load disturbance responses are illustrated to show the set-point tracking and disturbance rejection performance. The loop gain variations are presented to compare robustness of these controllers. Finally, the frequency and time-domain specifications of the closed-loop control system and control efforts of the IO and FO controllers are examined for variations in value of τ. All the simulations are carried out using MATLAB R2017b and Simulink. The FMIGO uses Curve Fitting Toolbox of MATLAB [21]. Thus, the three tuning strategies GPMT, FMIGO and Modified BLTF with delay are implemented and examined. The primary design aim of FMIGO method is the load disturbance rejection with a constraint on the maximum or peak sensitivity using PI^λ controller. The GPMT method uses gain crossover frequency and phase margin satisfying flat phase constraint also to achieve a FOPI controller. But achieving the desired controller requires understanding the entirety of the stabilizing region within the system's domain. The presented Modified BLTF with delay method analysis robustness and stability in terms of gain and phase margins using (19), (20), (21). Therefore, it offers an analytical result into the impact of time delay on the performance of a control system. However, it relies on prior knowledge of the process, which is assumed and utilized in the design process. This means that if the desired model is unsuitable, a stabilizing controller may not be identified.

V. Results and Discussions

The simulation on the test batch of 16 first-order plus time delay processes is presented. A step input is applied to closed loop system with lag-dominant time delay plant and five different structures of controllers namely, integer-order (IOPI & IOPID), fractional-order (FOPI & FOPID) and FMIGO controllers. A variation in delay L and time constant T of process parameters is simulated to validate stability and robustness against perturbations. When system parameters and uncertainties are taken into account and altered, the subsequent changes in closed-loop transient performance is observed. It is observed that there are negligible changes for τ value less than 0.1 but a large change is observed when τ value is greater than 0.1. The closed loop performance of all the controllers is examined. Focusing on lag-dominant systems; the results are evaluated in terms of changes in delay and time constant of closed-loop system for two scenarios stated in section 4 (26). The observations in Table I show that the FOPID has lesser overshoots across all controllers for first condition.

TABLE I. Peak Overshoot M_p for T=1 and ΔL

Controllers/ΔL	L0.01	L0.03	L0.05	L0.1	L0.5	L0.8
C1 IOPI	5.851	5.851	6.989	9.341	37.706	124.94
C2 IOPID	-	-	-	-	14.368	25.5949
C3 FOPI	8.152	8.152	8.152	9.341	3.96	119.21
C4 FMIGO	6.989	8.152	10.556	24.375	-	-
C5 FOPID	3.464	4.737	4.737	5.851	41.95	234.375

For the value of $L = 0.08$ and $T = 1$, a significant overshoot is observed for all controllers as the system tends to getting closer to balanced lag system i.e. $L \approx T$. For second condition, the Table VIII shows peak overshoot values for $L = 0.1$ and ΔT. The FO controllers show lesser overshoots for τ values greater than 0.0476 as compared to IO controllers. The rise time parameter for first and second conditions are shown in Table II and Table IX respectively.

TABLE II. Rise time T_r for T=1 and ΔL

Controllers/ΔL	L0.01	L0.03	L0.05	L0.1	L0.5	L0.8
C1 IOPI	0.536	0.507	0.463	0.372	0.379	0.337
C2 IOPID	-	-	-	-	1.545	-
C3 FOPI	0.574	0.541	0.503	0.387	0.540	0.325
C4 FMIGO	0.303	0.275	0.244	0.188	1.757	-
C5 FOPID	0.561	0.511	0.477	0.371	0.340	0.369

It is seen that even if FOPID requires sufficient time for the response to reach its final value but FOPID does not show large variations as compared to other controllers. For validation, the step responses with disturbance rejection and gain variation for ΔL and $T = 1$ as shown in Fig. 18 to 25 and for ΔT and $L = 1$ are shown in Fig. 26 to 35. The IO controllers show large load disturbance for $L = 0.1$ and $T = 1$ in Fig. 18, 20, 22, 24 and are more variant in Fig. 19, 21, 23, 25 for step response with gain variations. The figures clearly show that the FO controllers outperform IO controllers. The Table III and X shows the settling time for all controllers with both first and second condition respectively. The settling time is almost same for all controllers in first condition but for second condition of perturbations, the IO controller varies from 5.852seconds to 6.47 seconds while FO controller varies from 2.253seconds to 2.619seconds. Thus FO controllers exhibiting faster stable performance as compared to IO controllers.

TABLE III. SETTLING TIME T_s FOR T=1 AND ΔL

Controllers	L0.01	L0.03	L0.05	L0.1	L0.5	L0.8
C1 IOPI	2.287	2.236	2.177	2.024	8.942	oscillations
C2 IOPID	5.960	5.905	5.852	5.727	5.155	oscillations
C3 FOPI	2.674	2.629	2.581	2.449	2.353	oscillations
C4 FMIGO	1.678	1.633	1.595	1.546	9.971	oscillations
C5 FOPID	2.370	2.312	2.253	2.091	8.335	oscillations

For frequency domain analysis, the value of phase margin is constant for all controllers for range of L from 0.01 to 0.8. The bode plots for ΔL and $T = 1$ are shown in Fig. 2 to Fig. 7. A fall in value of PM is observed when T is varied between 0.3 to 50. This occurs as L is much less than T. The bode plots for ΔT and $L = 1$ are also shown in Fig. 8 to Fig. 17. Thus, for perturbed condition, it is observed that FOPID achieves a maximum PM of 110.537 and minimum of 14.253 which is more as compared to other controllers as shown in Table IV and Table XI. This frequency domain property of phase margin is a benefit of using BLTF [60], [65], [67]. The performance error metric for IAE, ITAE and ISE are shown in Table V, VI, VII respectively for the first condition i.e. $T = 1$ & ΔL and Table XII, XIII, XIV respectively for the second condition i.e. $L = 1$ & ΔT.

TABLE IV. PHASE MARGIN PM FOR T=1 AND ΔL

Controllers/ΔL	L0.01	L0.03	L0.05	L0.1	L0.5	L0.8
C1 IOPI	79.802	79.802	79.802	79.802	79.802	79.802
C2 IOPID	74.611	74.610	74.610	74.610	74.610	74.611
C3 FOPI	86.828	86.828	86.828	86.828	86.828	86.828
C4 FMIGO	73.811	73.811	73.811	73.811	73.811	73.811
C5 FOPID	87.25	87.257	87.257	87.257	87.257	87.25

TABLE V. IAE FOR T=1 AND ΔL

Controllers/ΔL	L0.01	L0.03	L0.05	L0.1	L0.5	L0.8
C1 IOPI	0.337	0.346	0.355	0.346	276.200	1.156E+06
C2 IOPID	1.438	1.447	1.458	1.489	1.899	-
C3 FOPI	0.436	0.442	0.449	0.496	17.590	2.565E+12
C4 FMIGO	0.303	0.317	0.335	0.533	177E+09	2.566
C5 FOPID	0.31	0.316	0.322	0.35	420.9	2.02E-07

For values of L between 0.01 to 0.1 the FO controllers exhibit lesser errors for IAE, ITAE, ISE. For values of L above 0.1 all controllers shows larger error metric. For ΔT all error metrics are consistent but FO controllers shows less error value as compared to IO controllers.

TABLE VI. ITAE FOR T=1 AND ΔL

Controllers/ΔL	L0.01	L0.03	L0.05	L0.1	L0.5	L0.8
C1 IOPI	0.4970	0.5060	0.5160	0.5570	6.45E+03	1.80E+08
C2 IOPID	3.2250	3.2510	3.2800	3.4040	5.07E+00	8.84E+00
C3 FOPI	0.9060	0.9110	0.9170	1.3680	2.48E+02	3.22E+07
C4 FMIGO	0.8250	0.8430	0.8680	3.2020	5.12E+12	7.44E+13
C5 FOPID	0.4726	0.4780	0.4840	0.5500	1.00E+04	5.69E+08

TABLE VII. ISE FOR T=1 AND ΔL

Controllers/ΔL	L0.01	L0.03	L0.05	L0.1	L0.5	L0.8
C1 IOPI	0.139	0.177	0.189	0.228	6774.000	1.29E+13
C2 IOPID	0.835	0.847	0.860	0.892	1.210	1.574
C3 FOPI	0.163	0.176	0.187	0.223	13.500	3.914E+11
C4 FMIGO	0.114	0.127	0.143	0.198	1.77E+22	4.209E+24
C5 FOPID	0.139	0.150	0.162	0.197	17400.000	1.36E+14

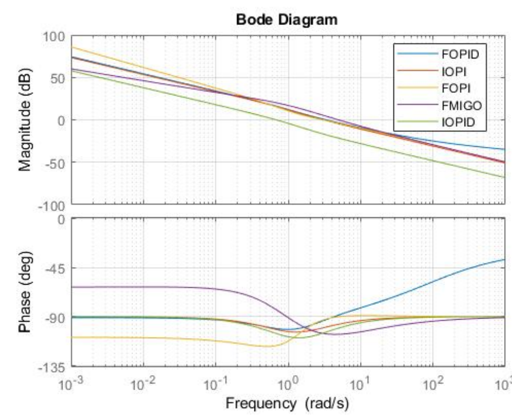

Fig. 2. Bode plot for $L = 0.01$ and $T = 1$

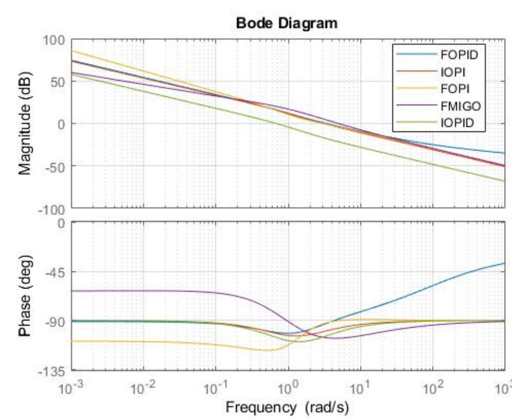

Fig. 3. Bode plot for $L = 0.03$ and $T = 1$

TABLE VIII. PEAK OVERSHOOT M_p FOR L=0.1 AND ΔT

Controllers/ΔT	T0.3	T0.5	T1.5	T2	T4	T6	T8	T10	T20	T50
C1 IOPI	22.84	3.646	15.698	21.341	30.921	40.141	46.324	50.758	170.832	0.714
C2 IOPID	0.501	0.504	10.556	15.698	27.564	30.921	77.679	91.346	-	-
C3 FOPI	27.564	2.43	17.059	24.375	43.143	108.319	143.288	248.274	2.872	-1.992
C4 FMIGO	77.679	44.203	21.341	21.341	24.375	25.949	27.564	29.221	32.667	34.459
C5 FOPID	29.221	1.531	13.068	18.452	30.921	38.194	44.203	48.507	166.595	0.704

TABLE IX. RISE TIME T_r FOR L=0.1 AND ΔT

Controllers/ΔT	T0.3	T0.5	T1.5	T2	T4	T6	T8	T10	T20	T50
C1 IOPI	0.092	0.166	0.518	0.624	0.966	0.012	1.394	1.595	1.099	4.282
C2 IOPID	2.867	0.0026	2.423	2.562	3.163	3.765	3.333	3.534	-	-
C3 FOPI	0.082	0.146	0.544	0.654	0.983	0.842	0.831	0.449	3.36	-
C4 FMIGO	0.644	0.986	0.281	0.364	0.627	0.841	1.022	1.178	1.834	3.272
C5 FOPID	0.068	0.127	0.508	0.616	0.944	0.011	1.713	1.534	1.068	4.198

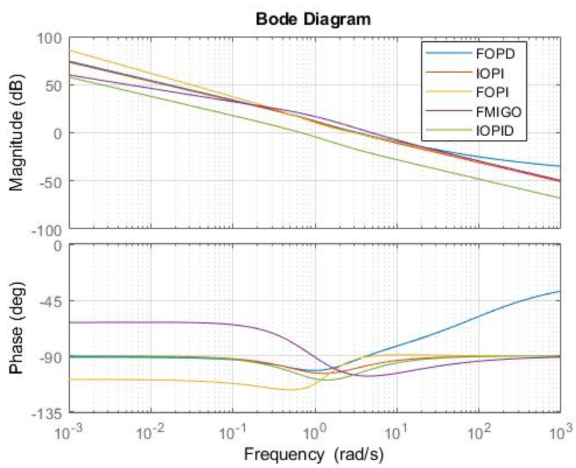

Fig. 4. Bode plot for $L = 0.05$ and $T = 1$

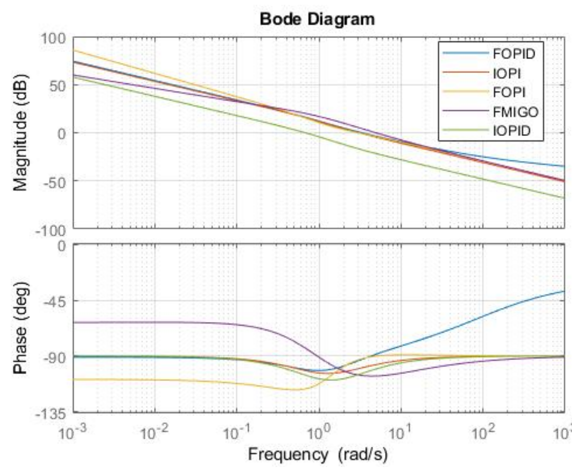

Fig. 6. Bode plot for $L = 0.5$ and $T = 1$

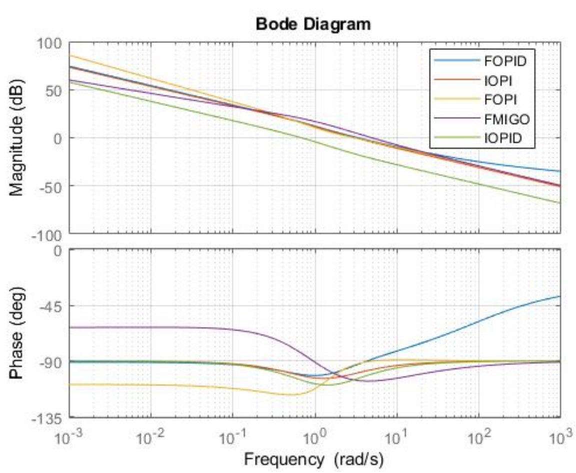

Fig. 5. Bode plot for $L = 0.1$ and $T = 1$

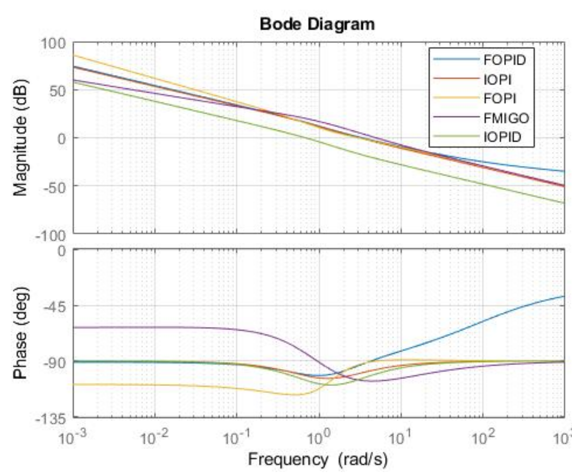

Fig. 7. Bode plot for $L = 0.8$ and $T = 1$

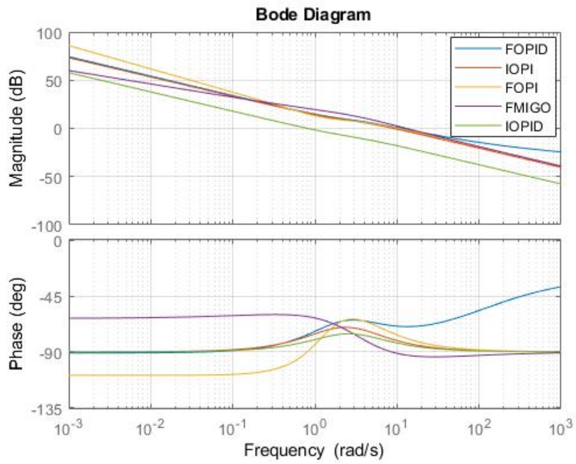

Fig. 8. Bode plot for $L = 0.1$ and $T = 0.3$

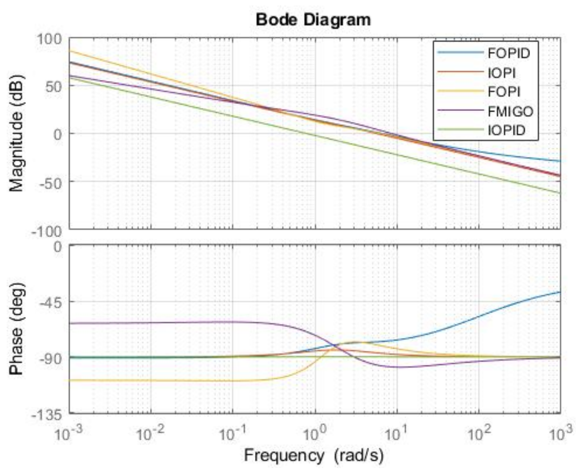

Fig. 9. Bode plot for $L = 0.1$ and $T = 0.5$

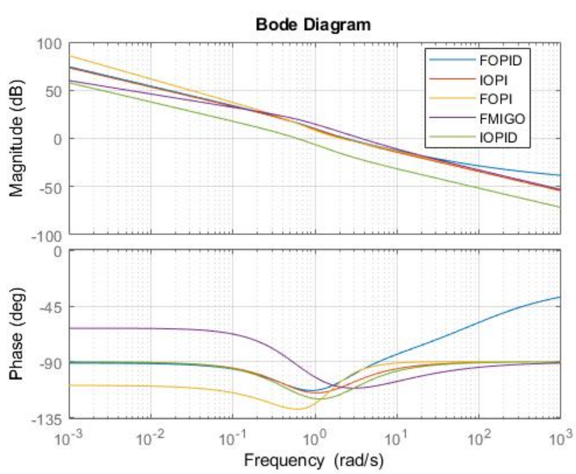

Fig. 10. Bode plot for $L = 0.1$ and $T = 1.5$

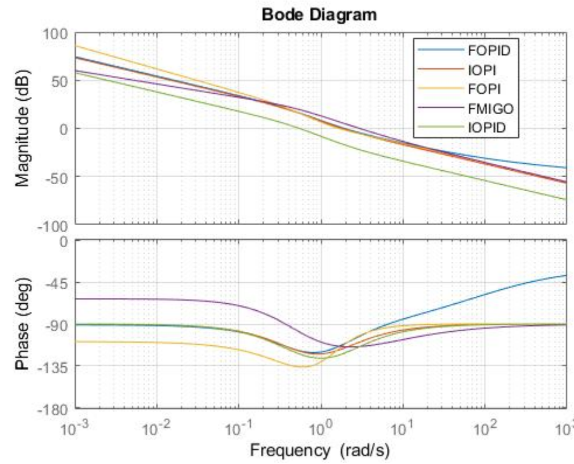

Fig. 11. Bode plot for $L = 0.1$ and $T = 2$

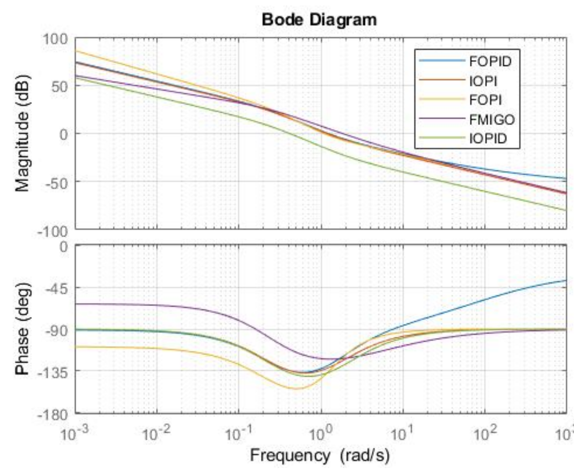

Fig. 12. Bode plot for $L = 1$ and $T = 4$

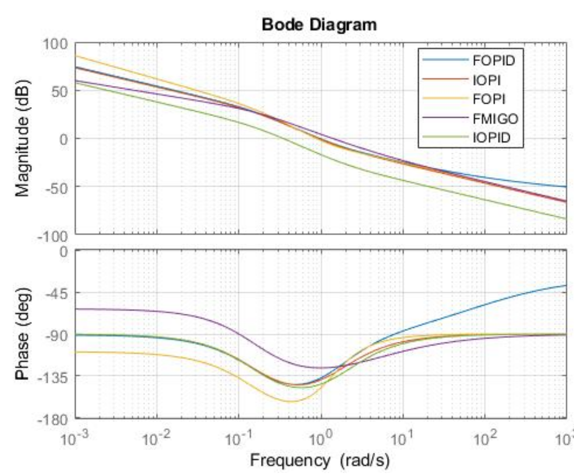

Fig. 13. Bode plot for $L = 0.1$ and $T = 6$

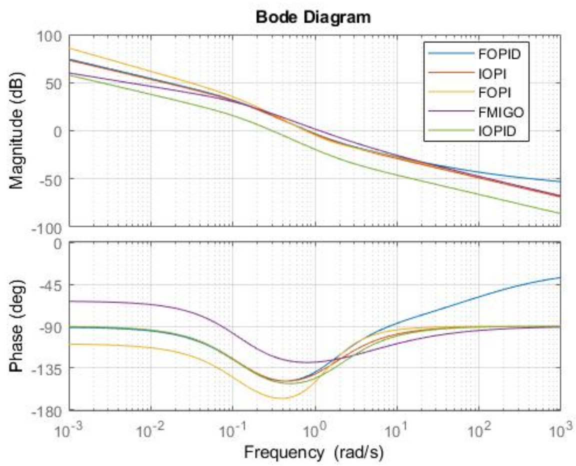

Fig. 14. Bode plot for $L = 0.1$ and $T = 8$

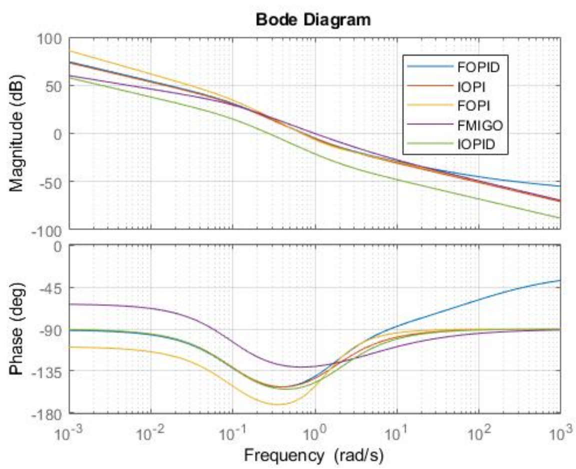

Fig. 15. Bode plot for $L = 0.1$ and $T = 10$

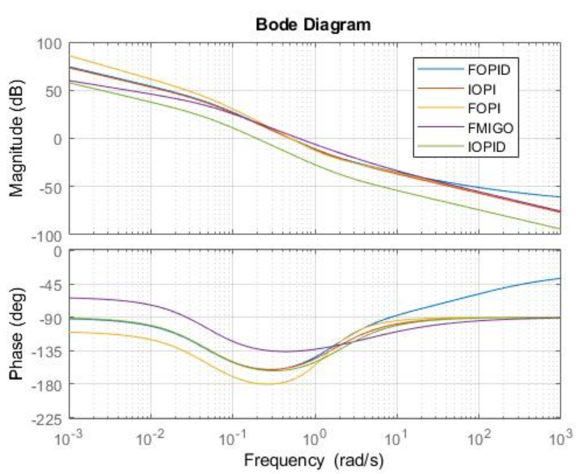

Fig. 16. Bode plot for $L = 0.1$ and $T = 20$

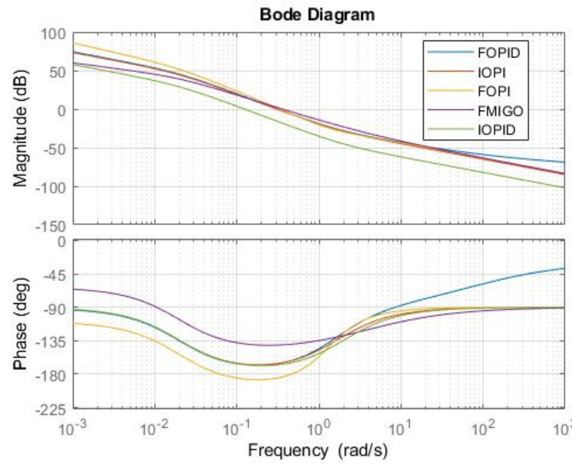

Fig. 17. Bode plot for $L = 0.1$ and $T = 50$

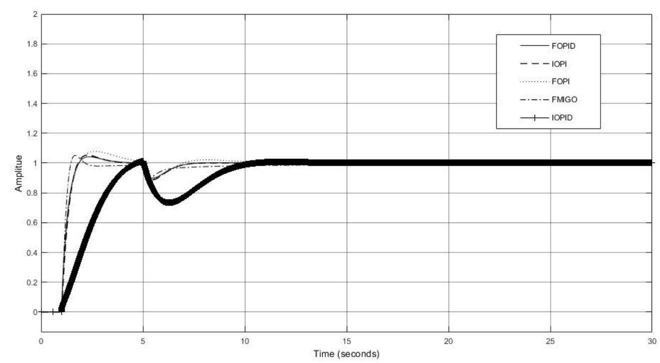

Fig. 18. Step response and load disturbance plot for $L = 0.01$ and $T = 1$

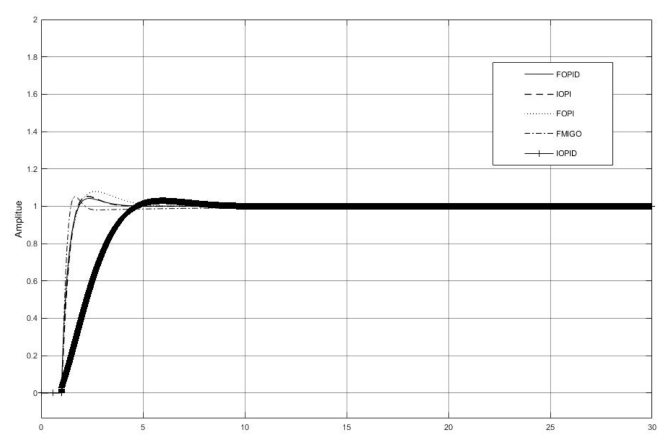

Fig. 19. Step response with gain variation for $L = 0.01$ and $T = 1$

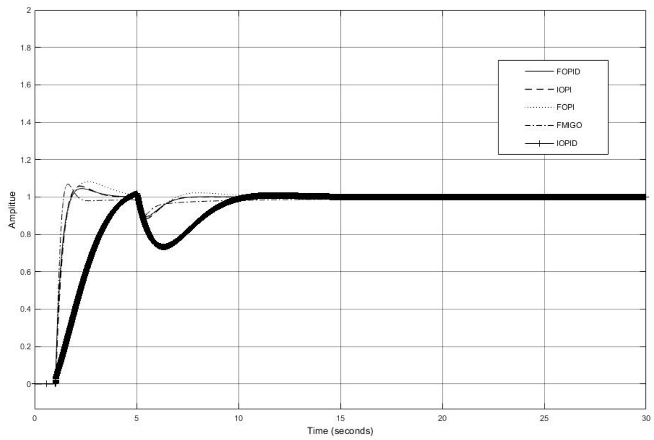

Fig. 20. Step response and load disturbance plot for $L = 0.03$ and $T = 1$

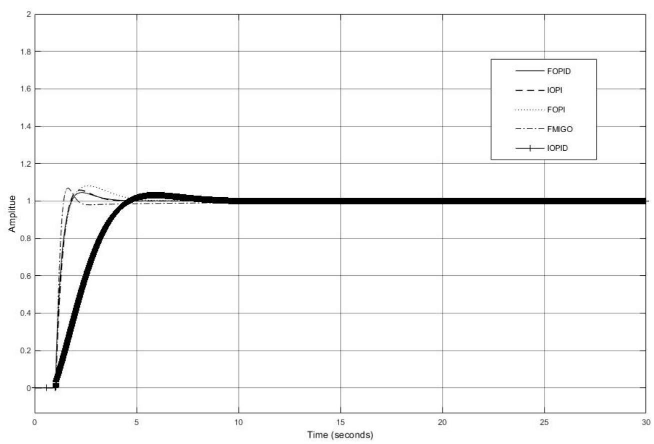

Fig. 21. Step response with gain variation for $L = 0.03$ and $T = 1$

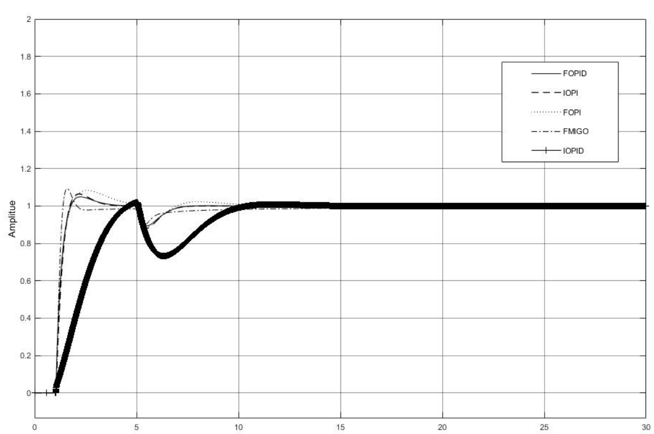

Fig. 22. Step response and load disturbance plot for $L = 0.05$ and $T = 1$

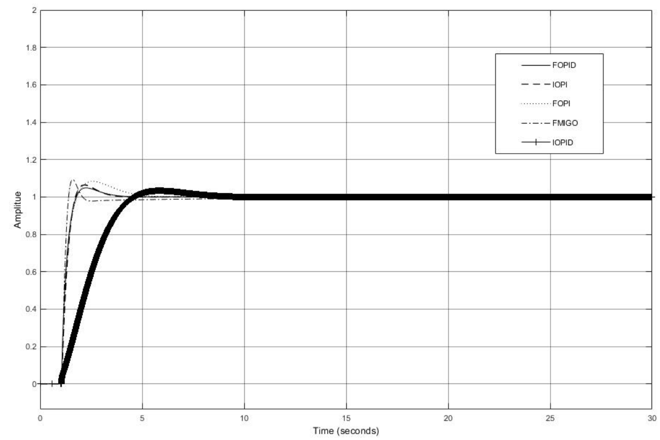

Fig. 23. Step response with gain variation for $L = 0.05$ and $T = 1$

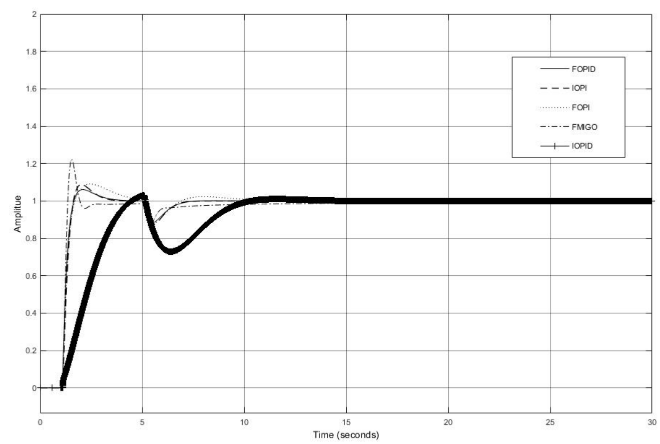

Fig. 24. Step response and load disturbance plot for $L = 0.1$ and $T = 1$

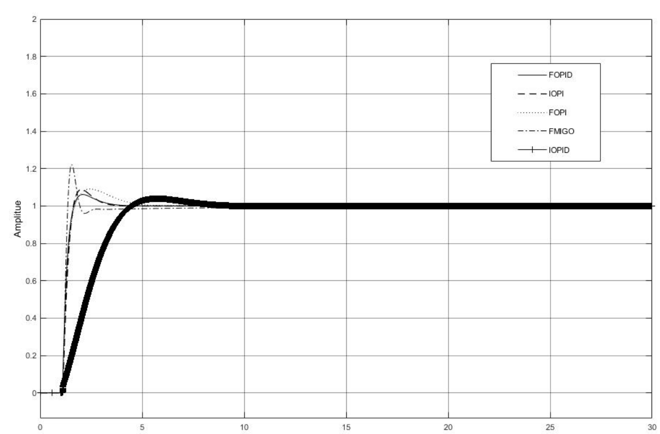

Fig. 25. Step response with gain variation for $L = 0.1$ and $T = 1$

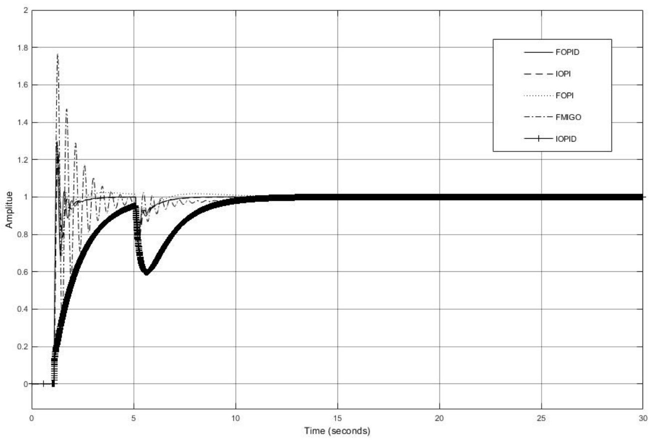

Fig. 26. Step response and load disturbance plot for $L = 0.1$ and $T = 0.3$

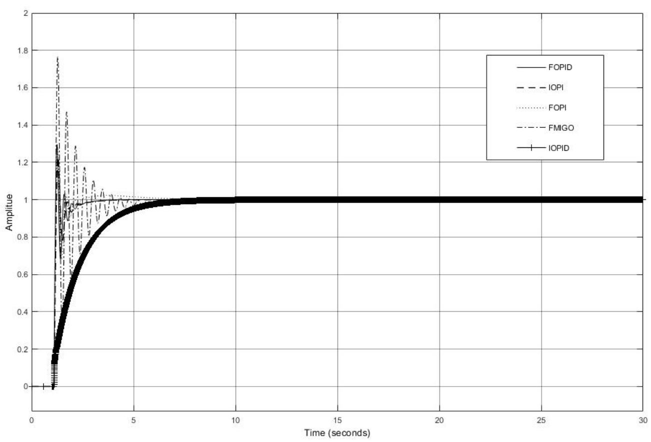

Fig. 27. Step response with gain variation for $L = 0.1$ and $T = 0.3$

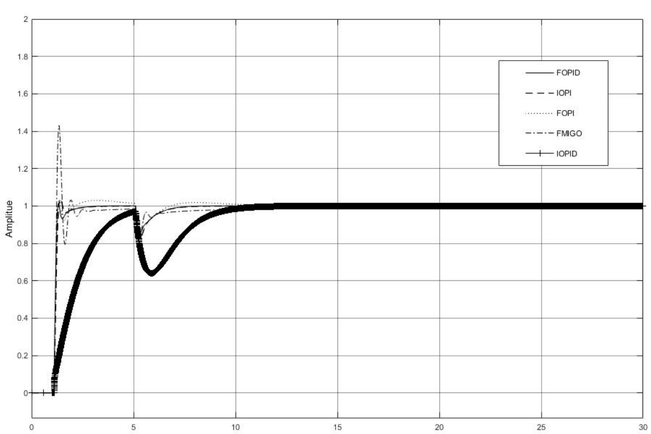

Fig. 28. Step response and load disturbance plot for $L = 0.1$ and $T = 0.5$

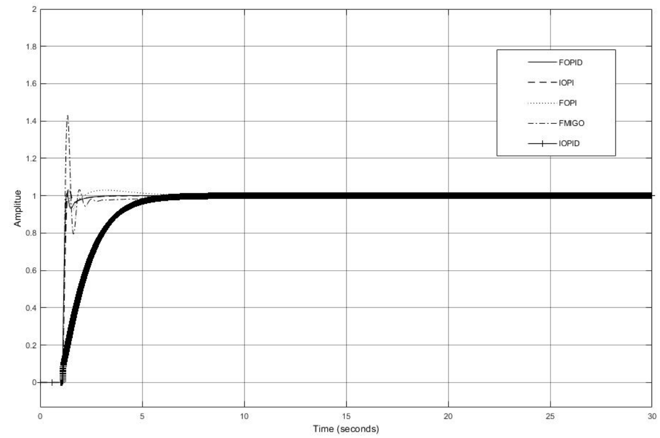

Fig. 29. Step response with gain variation for $L = 0.1$ and $T = 0.5$

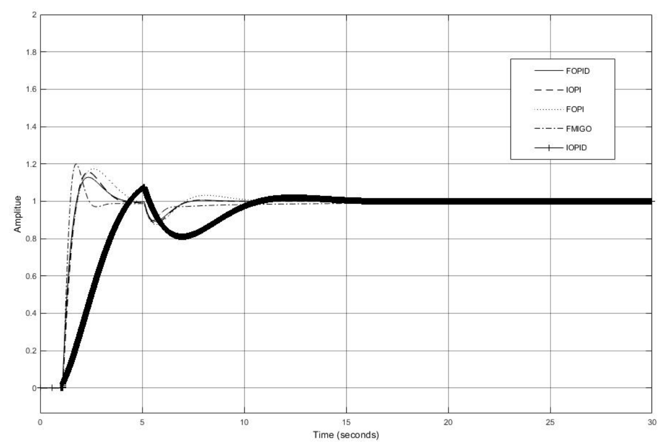

Fig. 30. Step response and load disturbance plot for $L = 0.1$ and $T = 1.5$

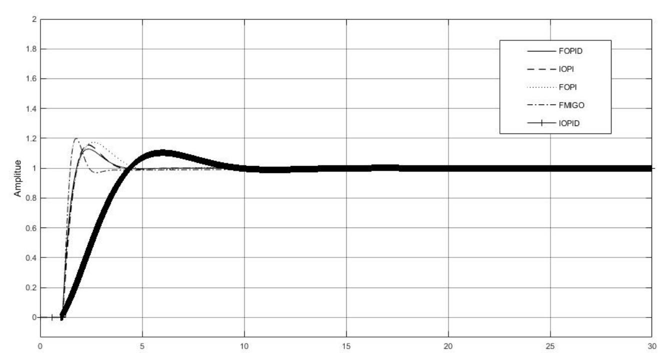

Fig. 31. Step response with gain variation for $L = 0.1$ and $T = 1.5$

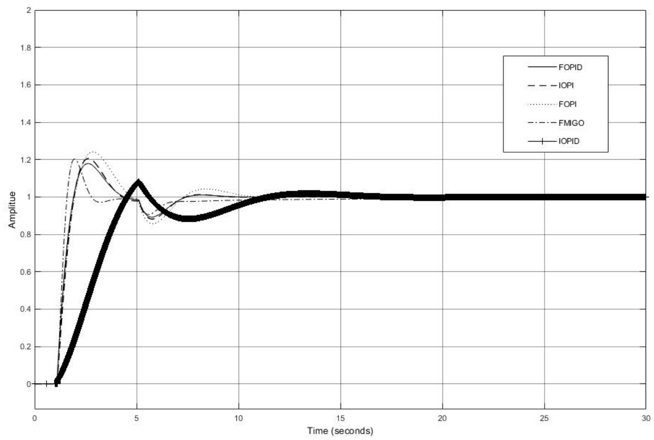

Fig. 32. Step response and load disturbance plot for $L = 0.1$ and $T = 2$

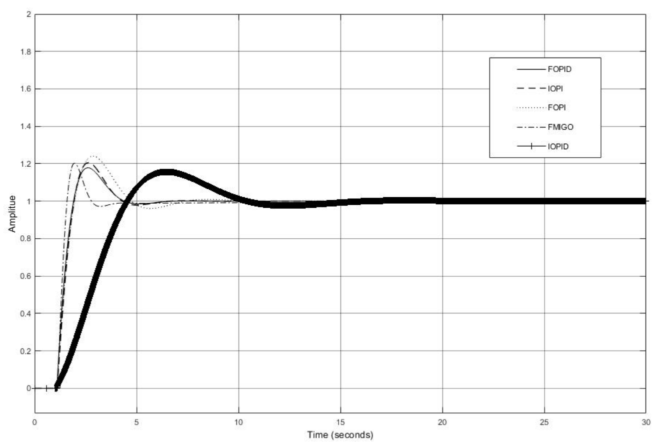

Fig. 33. Step response with gain variation for $L = 0.1$ and $T = 2$

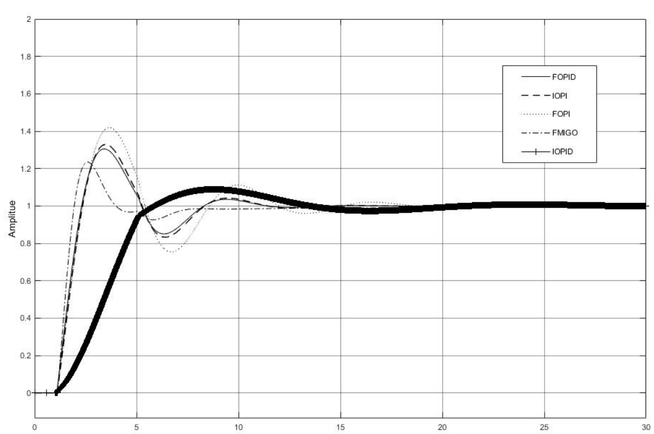

Fig. 34. Step response and load disturbance plot for $L = 0.1$ and $T = 4$

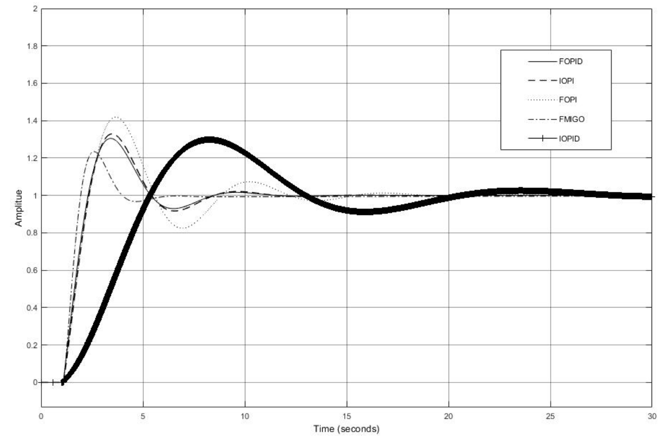

Fig. 35. Step response with gain variation for $L = 0.1$ and $T = 4$

VI. CONCLUSION

The closed-loop responses of first-order plus time delay lag dominant processes for variations in delay L and time constant T are presented in this paper. The tuning methods of five controllers are designed, implemented and compared for examining closed-loop control system. The simulations are carried out on 16 FOPDT processes. These processes are observed for different values of τ. The time and frequency domain parameters of 16 processes with five controllers are shown in paper. The simulation results show that a small perturbation in value of L and T significantly affect the stability and performance of the control system, thus limiting robustness. The relative dead time τ can take a same value for sets of L and T. The control system design uses this τ value for tuning of controller. It was examined during simulation that the closed loop behaviour changes even if the τ value is same. The simulation shows that FO controllers provide larger PM, lesser overshoots and faster settling time. It is observed that fractional-order controllers outperform integer-order controllers with same τ value also. Further observations shows that FO controllers exhibit better performance in terms of robustness and performance indices for perturbations as compared to IO controllers. The present work is limited to lag-dominant FOPDT systems. The further work is to investigate the implementation of FO controllers for higher-order time-delay systems using reference [33] for application in real control system.

TABLE X. SETTLING TIME T_s FOR L=0.1 AND ΔT

Controllers/ΔT	T0.3	T0.5	T1.5	T2	T4	T6	T8	T10	T20	T50
C1 IOPI	1.292	1.409	2.365	2.643	3.419	4.131	4.679	5.161	7.056	10.83
C2 IOPID	30	30	6	6.47	8.264	9.759	11.052	12.204	-	-
C3 FOPI	1.275	3.204	2.621	2.863	3.674	4.296	4.815	5.271	7.016	29.116
C4 FMIGO	1.283	1.344	1.767	1.965	2.603	3.115	3.562	3.963	5.612	9.154
C5 FOPID	1.242	1.313	2.365	2.619	3.419	4.04	4.566	5.028	6.856	10.491

TABLE XI. PHASE MARGIN PM FOR L=0.1 AND ΔT

Controllers/ΔT	T0.3	T0.5	T1.5	T2	T4	T6	T8	T10	T20	T50
C1 IOPI	99.99	93.307	70.267	63.328	47.647	39.739	34.785	31.315	22.432	14.298
C2 IOPID	98.502	90.052	64.692	57.809	42.914	35.628	31.116	27.972	19.983	12.716
C3 FOPI	103.29	99.316	73.992	63.412	39.291	27.824	20.963	16.31	4.988	-4.597
C4 FMIGO	87.351	81.886	69.014	65.68	58.149	54.187	51.6	49.727	44.634	39.442
C5 FOPID	110.537	102.421	76.5	68.631	50.928	42.082	36.589	32.762	23.044	14.253

TABLE XII. IAE FOR L=0.1 AND ΔT

Controllers/ΔT	T0.3	T0.5	T1.5	T2	T4	T6	T8	T10	T20	T50
C1 IOPI	0.266	0.224	0.577	0.766	1.525	2.283	3.036	3.776	6.976	11.6
C2 IOPID	1.283	1.285	1.884	2.388	4.297	6.058	7.446	8.774	11.92	16.41
C3 FOPI	0.334	0.3273	0.683	0.949	2.156	3.727	5.542	7.48	14.49	22.02
C4 FMIGO	0.752	0.539	0.478	0.664	0.947	1.212	1.465	1.706	2.779	5.278
C5 FOPID	0.261	0.207	0.517	0.693	1.394	2.104	2.816	3.524	6.631	11.54

TABLE XIII. ITAE FOR L=0.1 AND ΔT

Controllers/ΔT	T0.3	T0.5	T1.5	T2	T4	T6	T8	T10	T20	T50
C1 IOPI	0.372	0.276	0.995	1.549	4.893	10.03	3.036	24.89	70.31	148.3
C2 IOPID	2.994	2.756	5.241	8.814	27.8	52.03	7.446	95.62	145.8	244.2
C3 FOPI	0.712	1.106	1.465	2.708	9.981	26.44	5.542	83.62	210.8	373.3
C4 FMIGO	1.753	3.301	1.054	3.295	3.769	4.45	1.465	6.384	13.19	40.73
C5 FOPID	0.372	0.309	0.885	1.404	4.311	8.842	2.816	22.32	64.99	149.1

TABLE XIV. ISE FOR L=0.1 AND ΔT

Controllers/ΔT	T0.3	T0.5	T1.5	T2	T4	T6	T8	T10	T20	T50
C1 IOPI	0.149	0.162	0.3	0.374	0.67	0.967	1.265	1.562	3.031	6.346
C2 IOPID	0.619	0.697	1.087	1.282	2.063	2.84	3.587	4.319	6.882	11.32
C3 FOPI	0.154	0.156	0.304	0.394	0.829	1.401	2.144	3.09	9.152	21.09
C4 FMIGO	0.325	0.18	0.235	0.275	0.421	0.5537	0.675	0.789	1.286	2.443
C5 FOPID	0.141	0.14	0.262	0.328	0.599	0.876	1.155	1.436	2.845	6.227

TABLE XV. TOTAL VARIATION IN PERFORMANCE PARAMETERS FOR τ = 0.0476

Controllers	M_p	T_r	T_s	PM	ISE	Norm
C1 IOPI	14.351	0.161	0.466	16.474	0.185	0.258
C2 IOPID	-	-	0.618	16.801	0.422	0.002
C3 FOPI	16.598	0.151	0.282	23.416	0.207	0.156
C4 FMIGO	10.784	0.12	0.37	8.131	0.132	0.868
C5 FOPID	13.715	0.139	0.366	18.626	0.166	0.118

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