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Tracking Control for Affine Time-Varying Nonlinear Systems with Bounds

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***Abstract*—In practice, there exist systems with high nonlinearity and time-varying functions. Time-varying nonlinear systems (TVNS) present inherent challenges due to their high nonlinearity and time-varying nature, especially when unknown input disturbance and model uncertainties occur. In this work, a class of single input single output (SISO) uncertain affine TVNS is considered for tracking controller design in the presence of unknown disturbance, in which both the disturbance and model uncertainties are assumed to be bounded. Based on these bounds, a tracking controller will be proposed for first-order uncertain TVNS with unknown input disturbance, and then it is extended for second-order uncertain affine TVNS with unknown input disturbance. Unlike other existing works, the proposed controller does not use fuzzy systems, neural networks or any adaptive mechanism to cope with uncertainties and disturbances. It only uses the bounds of disturbance and model uncertainties, the information of tracking error to compute the control signal, and Lyapunov stability theory is applied to analyze stability of the closed-loop system. In addition, the convergence rate of tracking error can be adjusted by tuning parameters. Some numerical simulations with a first-order system and a model of inverted pendulum are given to verify the developed controller. These systems are uncertain and disturbed by unknown external signals and the proposed controller does not know this information but the tracking error still converges to a small circle containing the origin. The proposed controller can be extended for higher-order systems or MIMO systems such as robotic manipulators.**

Keywords*—Nonlinear Systems; Time-Varying Systems; Input Disturbance; Boundedness; Robust Control; Model Uncertainties.

I. INTRODUCTION

Time-varying nonlinear systems (TVNS) have been seen in many real systems such as under-actuated surface vessels with time-varying external disturbances [1], bilateral teleoperation manipulators with time-varying delays due to communication channels [2], systems with time-varying actuator failures [3], quadrotors with time-varying disturbances [4]–[6], servo mechanisms [7], autonomous underwater vehicles [8], and worm drive system [9], consensus of multi-agent systems with time-varying delays [10], and cable-driven parallel robot [11].

Control of TVNS is very challenging, so it gained a lot of interests from the research community with different aspects of interests. First, adaptive state observers for SISO TVNS were investigated in [12]. In [13], an extension of Lyapunov-density condition for TVNS has been achieved. An averaging technique for TVNS independent of local Lipschitz continuity was studied in [14]. Next, a practical issue of prescribed-time control with time-varying gain [15] was considered. In addition, speed-gradient adaptive control with Lyapunov-Bregman functions was revisited and developed in [16]. Nonlinear system identification problem [17] was investigated with application of neural network and sliding mode technique. Moreover, Deep Koopman learning for TVNS was analyzed in [18]. Recently, stability analysis for a class of incommensurate real order uncertain TVNS [19] was considered. These systems were modeled using Caputo operator. Some novel results on stabilization of time-varying nonlinear Caputo derivative systems were introduced in [20].

There have been many control problems solved for TVNS. In [21], the model uncertainty was assumed to be bounded by a linear function. Nonlinear systems with time-delay was analyzed for prescribed time convergence in [22]. Time-varying parameters were studied for strict-feedback nonlinear systems in [23], in which the input function is lower and upper bounded by some positive constants and system functions are known. Unlike [23], the system functions in [24] were assumed unknown but they were both lower and upper bounded by positive constants and their signs were also certain. A model-free adaptive control based on linearization method was proposed in [27].

To deal with unknown input disturbance, a simple way is to apply some disturbance observers [28], disturbance estimators [29],[30], or extended state observer [31], and then the estimated disturbance is used to compensate in the input of the system [32]–[36]. However, to utilize these disturbance observers, the system model must be known and exact enough.

Otherwise, a nominal model is needed to estimate both the model uncertainties and unknown input disturbance.

Sliding mode control (SMC) [37]–[41] has been also a very effective tool to cope with model uncertainties and external disturbances [42]. Higher-order SMC with time-varying gain [43] was proposed for SISO nonlinear systems. The work [44] combined the SMC technique and backstepping control to deal with time-varying parameters and unwanted disturbances. In [45], a SMC strategy was studied to overcome the problem of unknown bounds of time-varying uncertainties. SMC with reduced-order observer was addressed in [46]. Systems with time-varying delays were considered in [47] using SMC. Switched hybrid systems with time-varying delays [48] was studied with application of SMC. A SMC controller for discrete-time linear systems with time-varying delays was taken in [49]. The research [50] applied the SMC method for higher-order TVNS.

Advanced SMC techniques have been also studied and applied in several existing works. Terminal SMC [51] was designed for fourth-order nonlinear systems. Fast terminal sliding mode control was also applied for second-order systems in [52]. Chattering-free SMC controllers were proposed in [53]–[55]. Time-varying non-singular terminal SMC was designed for second-order systems [56]. Time-varying sliding surface was previously studied in [57]. It was developed in [58] for second-order systems in which the compound disturbance is bounded by a second-order function of the state vector norm. The proposed control can achieve pre-specified finite-time convergence. In addition, time-varying sliding modes were proposed in [59] for second-order TVNS, in which the control gain was a known constant. Finally, super-twisting SMC observer was developed in [60].

Intelligent control [61]–[64] has been also applied for the TVNS with learning capacity of fuzzy systems [65]–[67], neural networks [68]–[70], approximate dynamic programming (ADP) [71]–[73] or reinforcement learning (RL) [74]–[77]. Both ADP and RL techniques often utilize neural networks to approximate optimal control signal, plant model and cost function. An ADP based control was also developed for TVNS in [78], but no input disturbance was addressed. ADP with policy iteration for discrete-time TVNS was taken in [79]. In [80], strict-feedback systems with input saturation issue was solved with neural networks. Moreover, [81] has dealt with both input saturation and time-varying delays using fuzzy systems. The article [82] developed a neural network based controller with disturbance observer for SISO non-affine nonlinear systems. An adaptive neural fault-tolerance control of a helicopter system was studied in [83]. In [84], neural networks were applied to approximate the model uncertain functions and disturbances. In addition, the control coefficient was assumed to be lower bounded and its sign was also given. Interestingly, iterative learning control was also applied in [85] with time-varying constraints for the output in presence of disturbances. Lastly, model predictive control based on learning was studied for systems with time-varying parameters in [86].

In summary, to regulate the TVNS systems, some main control techniques have been applied such as SMC control with adaptive mechanism, intelligent control, disturbance observer based control, and combination of these techniques (for example, SMC with neural network, SMC with fuzzy systems, SMC with disturbance observers) to deal with model uncertainties and unknown external disturbancesm. However, the pure SMC control has not dealt with the model uncertainties and unknown disturbance concurrently for the considered systems to the best of my knowledge. This motivate us to study an additional controller design method without using fuzzy systems, neural networks and disturbance observers, which only utilizes a signum function.

Initial objects will be second-order TVNS with unknown disturbance. Control of second-order nonlinear systems drew a lot of interests from the researchers and scientists since most of engineering systems have this form such as a continuous stirred tank reactor [87], manipulators [88], spacecraft [89] and multi-agent systems [90]–[92]. These systems in general are MIMO [15],[93] or SISO [94] as a special case. In this work, tracking controller design for a class of SISO second-order uncertain affine TVNS systems with unknown disturbance is addressed.

The main contribution of this work to propose novel tracking controller for a class of SISO second-order uncertain affine TVNS systems with unknown disturbance which only uses the information of the bounds, the tracking error, and a signum function to compute the control signal in place of using fuzzy systems or neural networks to approximate uncertainties or any adaptive mechanism.

The rest of this work is organized as follows. Next section will present main contributions consisting of novel tracking controllers based on boundedness for first-order and second-order TVNS systems. Then, some numerical simulations are implemented to verify the proposed controller in section III. Final section will give conclusions and future works.

II. Main Results

A. First-order uncertain affine TVNS with unknown disturbance

Consider a first-order uncertain SISO affine TVNS system with unknown input disturbance as follow:

$$\dot{x} = f(x,t) + g(x,t)(u + d(t)), \tag{1}$$

where $x \in R$ (R: set of real numbers) is the system state and also the system output, $u \in R$ is the system input, the functions $f, g : R^2 \to R$ are unknown but bounded, and $d(t)$ is the unknown time-varying input disturbance.

The target is to design a tracking controller so that the tracking error $e = x - x_d \to 0$, where $x_d(t)$ is a desired reference.

Assumption 2.1: [80]

$$|\dot{x}_d(t)| \le \delta_1, \quad |\ddot{x}_d(t)| \le \delta_2, \quad \forall t \tag{2}$$

where $\delta_1, \delta_2 > 0$, $|*|$ denotes the absolute value of $*$.

The condition (2) will be used to prove the convergence of tracking error. In addition, the velocity and acceleration of physical systems are always bounded such as robotic manipulators. So, their corresponding desired trajectories must be bounded either.

Assumption 2.2: The function $f(x,t)$ satisfies the following condition (3), [94]:

$$|f(x,t)| \le \bar{f}, \quad \forall x, t, \tag{3}$$

where $\bar{f}$ is a known positive constant.

Assumption 2.3: The input function is lower and upper bounded as (4), [24],[84]

$$\underline{g} \le g(x,t) \le \bar{g}, \quad \forall x, t \tag{4}$$

where $\underline{g}$ and $\bar{g}$ are known positive constants.

Assumption 2.4: The unknown disturbance is bounded as (5), [59],[88]

$$|d(t)| \le \epsilon, \epsilon > 0, \tag{5}$$

where ϵ is known.

Unlike other works such as [88] in which both first and second derivatives of the disturbance are also assumed to be bounded, the proposed method only used the bound of the disturbance. This condition (5) is understandable since the system can be uncontrollable or unstable or even damaged if the disturbance is not bounded (goes to infinity).

Theorem 1: Under the following controller (6)

$$u = \frac{-\alpha|e| - \bar{f} - \delta_1 - \bar{g}\epsilon}{\underline{g}} sign(e), \tag{6}$$

where $\alpha \ge 0$ is a tuning parameter, $sign$ is a *signum* function, and Assumptions 2.1, 2.2, 2.3 and 2.4 hold, the tracking error e of the system (1) converges asymptotically to zero.

Proof 1: Define a Lyapunov function

$$V = \frac{1}{2}e^2. \tag{7}$$

So, $V \ge 0$ $\forall e \ne 0$, $V = 0$ if and only if $e = 0$, and its first time derivative is

$$\begin{aligned} \dot{V} &= e\dot{e} \\ &= e(\dot{x} - \dot{x}_d) \\ &= e(f + g(u + d) - \dot{x}_d). \end{aligned} \tag{8}$$

Substitute the controller (6) into (8) to get

$$\dot{V} = e\Big(f - \dot{x}_d + gd + g\frac{-\alpha|e| - \bar{f} - \delta_1 - \bar{g}\epsilon}{\underline{g}} sign(e)\Big) \tag{9}$$

When $e \ge 0$, $sign(e) = 1$, $|e| = e$, $ef \le e\bar{f}$ due to Assumption 2.2 ($f \le \bar{f}$), $-e\dot{x}_d \le e\delta_1$ according to Assumption 2.1 ($-\dot{x}_d \le \delta_1$), and $egu \le e\underline{g}u$ because $u = \frac{-\alpha e - \bar{f} - \delta_1 - \bar{g}\epsilon}{\underline{g}} < 0$ and $g \ge \underline{g}$ due to Assumption 2.3, $egd \le e\bar{g}\epsilon$ due to Assumptions (2.3, 2.4 $gd \le \bar{g}\epsilon$), and then (9) becomes

$$\dot{V} = e\Big(f - \dot{x}_d + gd + g\frac{-\alpha e - \bar{f} - \delta_1 - \bar{g}\epsilon}{\underline{g}}\Big). \tag{10}$$

So,

$$\begin{aligned} \dot{V} &\le e\Big(\bar{f} + \delta_1 + \bar{g}\epsilon + \underline{g}\frac{-\alpha e - \bar{f} - \delta_1 - \bar{g}\epsilon}{\underline{g}}\Big) \\ &\le -\alpha e^2 \\ &\le 0. \end{aligned} \tag{11}$$

If $e < 0$, $sign(e) = -1$, $|e| = -e$, $ef \le -e\bar{f}$ due to Assumption 2.2 ($f \ge -\bar{f}$), $-e\dot{x}_d \le -e\delta_1$ according to Assumption 2.1 ($-\dot{x}_d \ge -\delta_1$), and $egu \le e\underline{g}u$ because $u = \frac{-\alpha e + \bar{f} + \delta_1 + \bar{g}\epsilon}{\underline{g}} > 0$ and $g \ge \underline{g}$ due to Assumption 2.3, $egd \le -e\bar{g}\epsilon$ due to Assumptions (2.3, 2.4 $gd \ge -\bar{g}\epsilon$), and then (9) becomes

$$\dot{V} = e\Big(f - \dot{x}_d + gd + g\frac{-\alpha e + \bar{f} + \delta_1 + \bar{g}\epsilon}{\underline{g}}\Big). \tag{12}$$

So,

$$\begin{aligned} \dot{V} &\le e\Big(-\bar{f} - \delta_1 - \bar{g}\epsilon + \underline{g}\frac{-\alpha e + \bar{f} + \delta_1 + \bar{g}\epsilon}{\underline{g}}\Big) \\ &\le -\alpha e^2 \\ &\le 0. \end{aligned} \tag{13}$$

Thus, $\dot{V} \le -\alpha e^2 \le 0$ for $\forall e, \alpha \ne 0$, this implies $V \to 0$ or $e(t) \to 0$ asymptotically as $t \to \infty$. The convergence rate of the tracking error will be faster if the tuning parameter α is larger.

A structure diagram of the proposed control system is displayed in Fig. 1 in which the plant is modeled as the system (1) and the tracking controller is designed as (6).

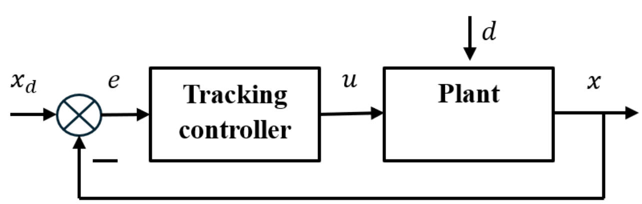

Fig. 1. A Structure Diagram of the Control System

Next, this result will be extended for second-order uncertain affine TVNS with unknown input disturbance.

B. Second-order uncertain TVNS with unknown disturbance

Consider a second-order uncertain SISO affine TVNS system with unknown input disturbance as follows

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x_1, x_2, t) + g(x_1, x_2, t)(u + d(t)), \end{cases} \tag{14}$$

where $x_1, x_2 \in R$ are state variables, x_1 is the system output, both functions $f(x_1, x_2, t) : R^3 \to R$ and $g(x_1, x_2, t) : R^3 \to R$ are uncertain but bounded, and $d(t) \in R$ is unknown input disturbance.

A similar system was also studied in [52],[58] with different assumptions. However, the work [52] applied the fast terminal SMC technique while the reseach [58] used a time-varying sliding surface.

Assumption 2.5: The function $f(x_1, x_2, t)$ satisfies the following condition (15), [94]:

$$|f(x_1, x_2, t)| \leq \bar{f}, \quad \forall \underline{x}, t, \quad \underline{x} = [x_1 \quad x_2]^T, \tag{15}$$

where $\bar{f}$ is a known positive constant.

Assumption 2.6: The input function is lower and upper bounded as (16), [24],[84]

$$\underline{g} \leq g(x_1, x_2, t) \leq \bar{g}, \quad \forall \underline{x}, t \tag{16}$$

where $\underline{g}$ and $\bar{g}$ are known positive constants.

These conditions (3) and (15) for the function f, and (4) and (16) for the function g are helpful in the proof of convergence for tracking error. For continuous-time systems such as robotic manipulators and inverted pendulum, the function f, g are continuous. Moreover, the working space $x, \dot{x}$ (for example joint angle and its derivative) is also limited due to physical constraints. Hence, the system functions f, g are bounded.

Remark 2.1: In [58], the unknown functions were presented as $f = f_0 + \Delta_f$ and $g = g_0 + \Delta_g$ where f_0, g_0 were known functions and Δ_f, Δ_g were uncertain. Then, a compounded disturbance was defined as $d_c(t) = \Delta_f(\underline{x}, t) + \Delta_g(\underline{x}, t)u(t) + g(\underline{x})d(t)$ and it is assumed to be bounded by a positive second-order function of the state vector norm $\delta = a_0 + a_1||\underline{x}|| + a_2||\underline{x}||^2$ with positive constants a_0, a_1, a_2. The controller [58] based on a time-varying sliding surface needs to know the functions f_0, g_0 and the bound $\delta(\underline{x})$ to compute the control signal. However, the proposed controller in this work requires only the bounds for unknown functions and disturbance. Hence, for practical systems with totally model uncertainties and unknown disturbance, these bounds can be selected as large as possible for $(\bar{f}, \bar{g}, \epsilon)$ and as small as possible for $\underline{g}$.

Assumption 2.7:

$$|\dot{x}_1| \leq \nu, \quad \forall x_1 \text{ and } \nu > 0. \tag{17}$$

Remark 2.2: In practical systems, such as mobile robots, manipulators, the velocity is bounded due to the physical limitation of the actuator. So, the assumption (17) is a reasonable condition for the state x_2.

Theorem 2: Under the following controller (18)

$$u = \frac{-\alpha|s| - \bar{f} - \delta - \bar{g}\epsilon}{\underline{g}} sign(s), \tag{18}$$

where $s = \dot{e} + \beta e$ is a sliding surface, $e = x_1 - x_d$, $\alpha \geq 0$, $\beta > 0$ are tuning parameters, $\delta = \delta_2 + \beta(\nu + \delta_1)$, and Assumptions 2.1, 2.5, 2.6 and 2.4 hold, the tracking error e of the system (14) converges asymptotically to zero.

Proof 2: Define a Lyapunov function

$$V = \frac{1}{2}s^2. \tag{19}$$

Do the same things as in the previous proof 1 with s is in the place of e. So,

$$\begin{aligned} \dot{V} &= s\dot{s} \\ &= s(\ddot{e} + \beta\dot{e}) \\ &= s[\ddot{x}_1 - \ddot{x}_d + \beta(\dot{x}_1 - \dot{x}_d)] \\ &= s[f + g(u + d) - \ddot{x}_d + \beta(\dot{x}_1 - \dot{x}_d)]. \end{aligned} \tag{20}$$

When $s \geq 0$,

$$\begin{aligned} \dot{V} &= s\Big(f - \ddot{x}_d + gd + \beta(\dot{x}_1 - \dot{x}_d) + g\frac{-\alpha s - \bar{f} - \delta - \bar{g}\epsilon}{\underline{g}}\Big) \\ &\leq s\Big(\bar{f} + \delta_2 + \beta(\nu + \delta_1) + \bar{g}\epsilon + \underline{g}\frac{-\alpha s - \bar{f} - \delta - \bar{g}\epsilon}{\underline{g}}\Big) \\ &\leq -\alpha s^2, \end{aligned} \tag{21}$$

since $u < 0$ and $s \geq 0$.

If $s < 0$,

$$\begin{aligned} \dot{V} &= s\Big(f - \ddot{x}_d + gd + \beta(\dot{x}_1 - \dot{x}_d) + g\frac{-\alpha s + \bar{f} + \delta + \bar{g}\epsilon}{\underline{g}}\Big) \\ &\leq s\Big(-\bar{f} - \delta - \bar{g}\epsilon + \underline{g}\frac{-\alpha s + \bar{f} + \delta + \bar{g}\epsilon}{\underline{g}}\Big) \\ &\leq -\alpha s^2, \end{aligned} \tag{22}$$

since $u > 0$ and $s < 0$.

Thus, $\dot{V} \leq -\alpha s^2 \leq 0$ for $\forall s, \alpha \neq 0$, this makes $V \to 0$ and consequently $s \to 0$, and $\dot{V} = 0$ if and only $s = 0$. Hence, this implies $e(t) \to 0$ asymptotically as $t \to \infty$ since $\beta > 0$. If β increases then $e(t) \to 0$ more quickly but the control signal is also larger because s is bigger. In summary, the bigger parameter α will force the sliding surface to converges to zero more quickly and the larger value β will cause the tracking error to go to the origin faster, however the control signal will becomes larger.

Next section will provide some numerical simulations with two examples to demonstrate how the proposed tracking controller works.

III. Numerical Simulations

A. Example 1

Consider the system (1) with the system functions

$$f(x,t) = 1 + sin^2(x) + \frac{1}{1+cos^2(t)}, \tag{23}$$

$$g(x,t) = \frac{2}{1+sin^2(t)}, \tag{24}$$

and the input disturbance

$$d = cos(t). \tag{25}$$

Clearly, these system functions and disturbance are bounded by $\bar{f} = 3$, $\underline{g} = 1$, $\bar{g} = 2$, and $\epsilon = 1$, respectively. The desired reference is chosen as $x_d = sin(t)$. So, $\delta_1 = 1$. The proposed controller is

$$u = (-\alpha|e| - 5)sign(e), \tag{26}$$

where $\alpha = 20$. Sampling time is selected as $T_s = 0.001$ seconds. Initial output $x(0) = \pi/3$ is chosen for comparison purpose.

Numerical simulation results are shown in following figures. For easy view, all curves are plotted on logarithm scale for the time axis.

In Fig. 2, the system output $x(t)$ and the reference signal $x_d(t)$ are plotted. The output converges to the desired output after 0.05 seconds. The tracking error is displayed in Fig. 3. It converges to a small ball including the origin and oscillates with high frequency due to the chattering phenomenon.

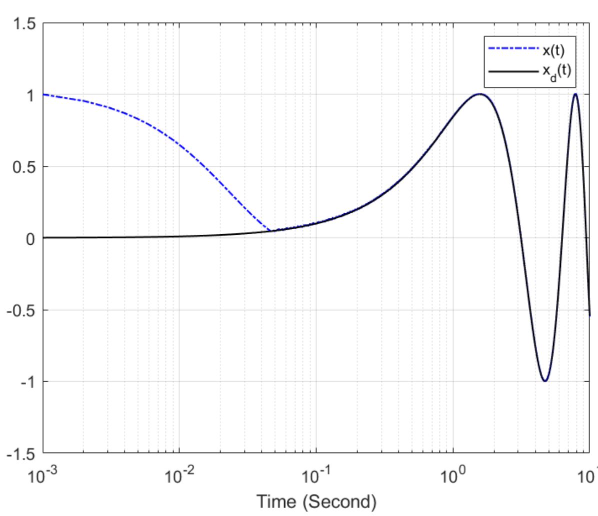

Fig. 2. System Output and Desired Reference

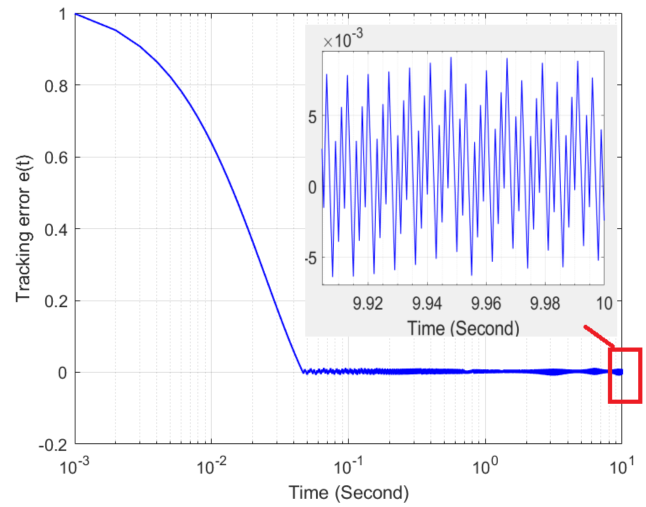

Fig. 3. Tracking Error

In Fig. 4, a plot of the Lyapunov function is shown. It is smaller than 10^{-4} but oscillatory. The control signal is plotted in Fig. 5. It oscillates with magnitude of ± 5 as expected when the tracking error converges to zero. This chattering phenomenon can be removed if the sign function is replaced with a saturation function or tanh function.

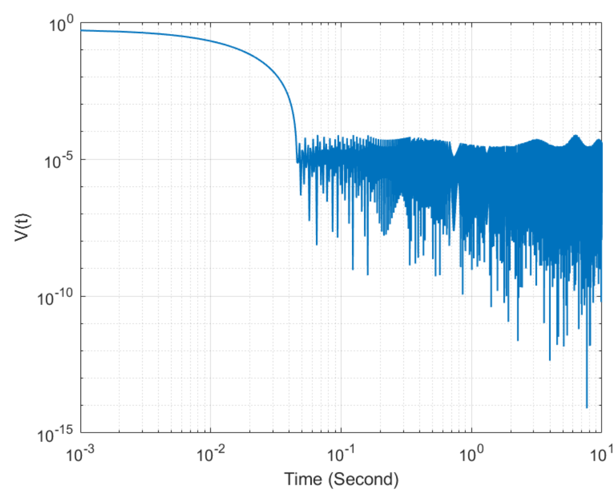

Fig. 4. Lyapunov Function

In this case, a modified tanh function $u = tanh(qe) = \frac{v^{qe} - v^{-qe}}{v^{qe} + v^{-qe}}$ is applied in place of $sign(e)$ where $v \approx 2.7183$ is the base of the natural logarithm and $q > 0$ is big enough. Then, the control signal is obtained as in Fig. 6. There is no chattering phenomenon as compared with Fig. 5 and it is much more smooth.

The corresponding Lyapunov function is shown in Fig. 7. It also converges to the domain containing the origin with much

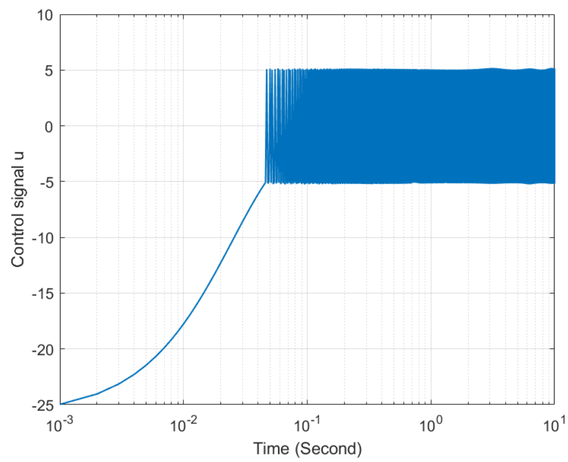

Fig. 5. Control Signal

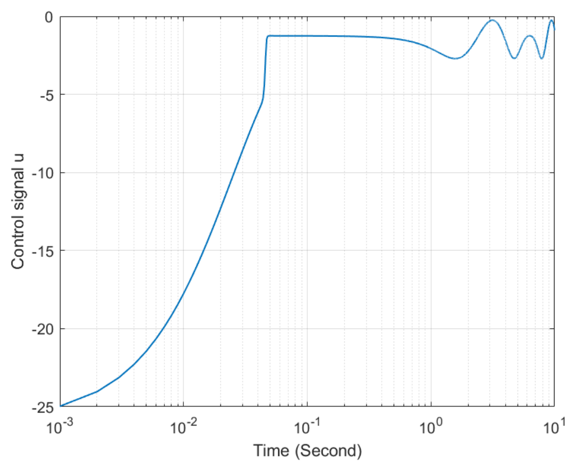

Fig. 6. Control Signal With the Function $tanh(100e)$ in Place of $sign(e)$

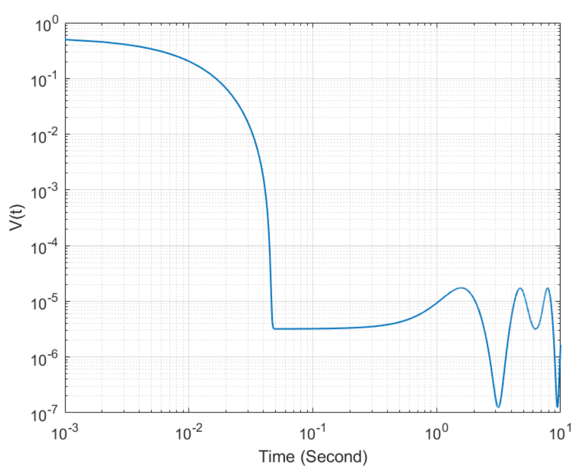

Fig. 7. The Lyapunov Funtion $V(e)$

less oscillation.

To compare the effectiveness of the function $tanh(100e)$ with the signum function $sign(e)$, three performance indicators are computed as

$$\begin{aligned} MAE =& \frac{1}{N}\sum_{i=0}^{N}|e(i)|, \text{ Mean Absolute Error} \\ ME =& \frac{1}{N}\sum_{i=0}^{N}e(i), \text{ Mean Error} \\ RMSE =& \frac{1}{N}\sqrt{\sum_{i=0}^{N}e^2(i)}, \text{ Root Mean Squared Error,} \end{aligned} \tag{27}$$

where $N = 1000$. Calculated indicators are shown in Table I. It can be observed that the proposed controller with the modified function $tanh(100e)$ provided closely the same performance as the one using the signum function $sign(e)$ but there is no chattering phenomenon as in Fig. 6.

TABLE I. Comparison of Three Performance Indicators

Function	MAE	ME	RMSE
$tanh(100e)$	0.6189	0.1889	0.0069
$sign(e)$	0.6184	0.1877	0.0069

In summary, the proposed controller with the modified function $tanh(100e)$ gave the similar performance indicators but its control signal is much more smooth with no chattering phenomenon. So, the function $tanh(qe)$ can be applied in place of $sign(e)$ to remove the oscillation with high frequency of the control signal. Next subsection will verify the proposed controller with a benchmark system.

B. Example 2

To illustrate the proposed controller for second-order systems, a cart-pole inverted pendulum (IP) will be used, which is very challenging in control since its unstable nature, underactuation, and non-linearity. For simulation, the model is described as follows [52],[95].

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(\underline{x}) + g(\underline{x})(u + d(t)), \end{aligned} \tag{28}$$

where, $\underline{x} = [x_1\ x_2]^T$, $x_1 = \theta$ is the tilt angle of the pendulum, $x_2 = \dot{\theta}$.

$$\begin{aligned} f(\underline{x}) =& \frac{asin(x_1) - mlx_2^2 cosx_1 sinx_1/(m_c + m)}{l[4/3 - mcos^2x_1/(m_c + m)]}, \\ g(\underline{x}) =& \frac{cosx_1/(m_c + m)}{l[4/3 - mcos^2x_1/(m_c + m)]}, \end{aligned}$$

$a = 9.8\ m/s^2$ is gravity acceleration, m_c is the mass of the cart, m is the mass of pendulum, the length of the pendulum is $2l$, $P = masin\theta$ is the gravitational force, and u is the force applied to the cart as in Fig. 8. In addition, the input disturbance is selected as $d(t) = 0.3sin(0.5t)$ which is time-varying.

The IP's parameters are given as follows: $m_c = 1\ kg$, $m = 0.1\ kg$ và $l = 0.5\ m$. The controller's parameters are chosen as: $\alpha = 200$, $\beta = 100$, and the bounds are obtained from the model (28) as: $\underline{g} = 0.375$, $\bar{f}+\delta_2+\beta(\nu+\delta_1)+\bar{g}\epsilon = 10$. The sampling time $T_s = 0.001$, the reference is $x_d(t) = 0.1sin(t)$, and the initial state vector is $\underline{x}_0 = [0.3 \quad 0]^T$. Simulation results

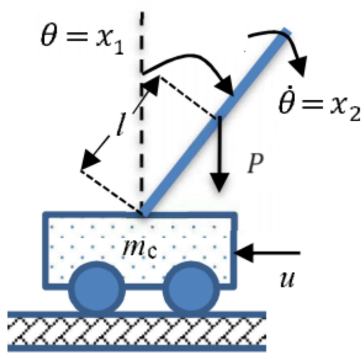

Fig. 8. Diagram of the IP

are displayed in following Fig. 9 and 10 with logarithm scale on the time axis. The tracking error converges to zero after 0.08 seconds as in Fig. 9. The control signal $u(t)$ is plotted in Fig. 10 with chattering phenomenon, but this bang-bang issue is removed with $tanh(50s)$ in place of $sign(s)$ as in Fig. 11. The corresponding tilt angle is obtained as in Fig. 12 with similar performance as the case using $sign(s)$.

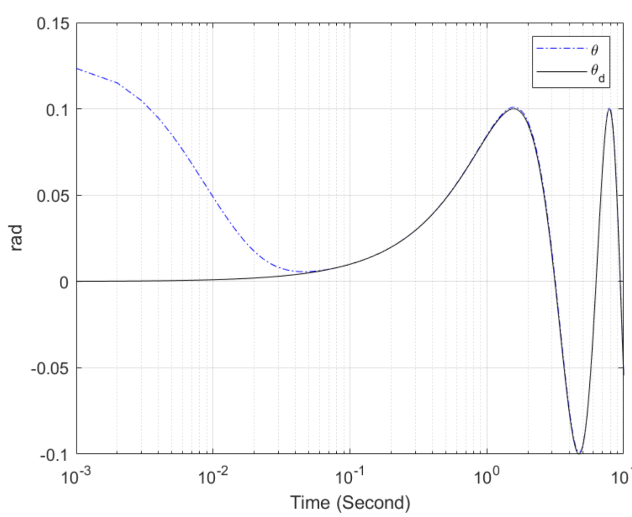

Fig. 9. Tilt angle θ and its reference

The proposed controller (Proposal) is also compared with an adaptive fuzzy controller (AFC) in which its parameters are chosen as $\alpha = 0.01$, $\beta = 10$, the modified function $tanh(50s)$ is in place of the signum function $sign(s)$.

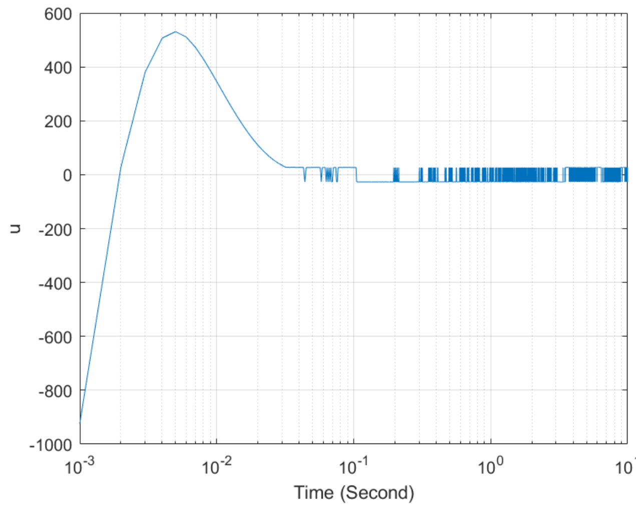

Fig. 10. Control signal u for the IP.

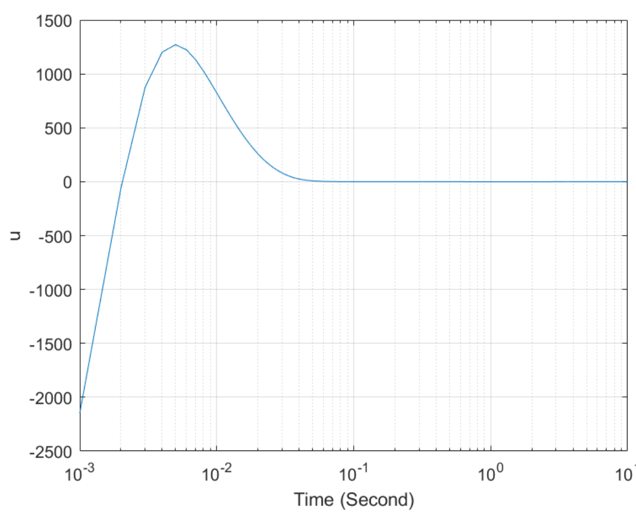

Fig. 11. Control signal when $tanh(50s)$ is applied

The AFC [95] is designed as follows.

$$u = \frac{1}{\hat{g}(\underline{x}, \underline{w}_g)}[-\hat{f}(\underline{x}, \underline{w}_f) + \ddot{x}_d + \underline{k}^T \underline{e}] \tag{29}$$

where $\underline{e} = [e \quad \dot{e}]^T$, $\underline{k} = [15 \quad 8]^T$, $\hat{f}$ and $\hat{g}$ are estimations by fuzzy systems of f and g, respectively, in which their parameters are updated as

$$\begin{aligned} \dot{\underline{w}}_f &= -\gamma_1 \underline{e}^T P \underline{b} \zeta(\underline{x}) \\ \dot{\underline{w}}_g &= -\gamma_2 \underline{e}^T P \underline{b} \eta(\underline{x}) u, \end{aligned} \tag{30}$$

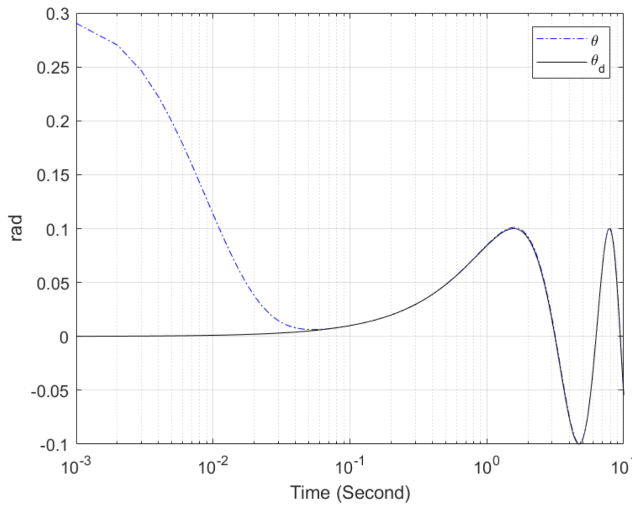

Fig. 12. The tilt angle θ when $tanh(50s)$ is applied

where $P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$, $\gamma_1 = 100$, $\gamma_2 = 1$, $\underline{b} = [0 \quad 1]^T$,

$$\begin{aligned} \mu_{1n} &= gaussmf(x_1, [0.1779; -\pi/6]), \\ \mu_{1z} &= gaussmf(x_1, [0.1779; 0]), \\ \mu_{1p} &= gaussmf(x_1, [0.1779; \pi/6]) \end{aligned} \tag{31}$$

$$\begin{aligned} \mu_{2n} &= gaussmf(x_2, [0.06795; -0.2]), \\ \mu_{2z} &= gaussmf(x_2, [0.06795; 0]), \\ \mu_{2p} &= gaussmf(x_2, [0.06795; 0.2]), \end{aligned} \tag{32}$$

$$\begin{aligned} \zeta(\underline{x}) =& [\mu_{1n}\mu_{2n} \quad \mu_{1n}\mu_{2z} \quad \mu_{1n}\mu_{2p} \quad \mu_{1z}\mu_{2n} \quad \mu_{1z}\mu_{2z} \\ & \mu_{1z}\mu_{2p} \quad \mu_{1p}\mu_{2n} \quad \mu_{1p}\mu_{2p} \quad \mu_{1p}\mu_{2p}]^T, \end{aligned} \tag{33}$$

$$\eta(\underline{x}) = \begin{bmatrix} \mu_{1n} & \mu_{1z} & \mu_{1p} \end{bmatrix}^T, \tag{34}$$

and $gaussmf(x, [\sigma; c]) = e^{-\frac{(x-c)^2}{2\sigma^2}}$ is a Gaussian membership function.

The setting for comparison is given with initial output $\theta_0 = 0.3(rad)$, sampling time $T_s = 0.01(second)$, simulation time $T_f = 20(second)$.

Simulation results are shown in the following table and figures. The output θ and its reference are displayed in Fig. 13 in which the proposed controller provided a faster response than the AFC controller while the tracking error are similarly shown in Fig. 14. In addition, performance indicators for the proposed controller are better than the AFC as shown in Table II in which MIN and MAX are minimum and maximum values of the tracking error, respectively. The control signals are compared in Fig. 15, in which the proposal generated a larger magnitude during the starting time but smaller after that.

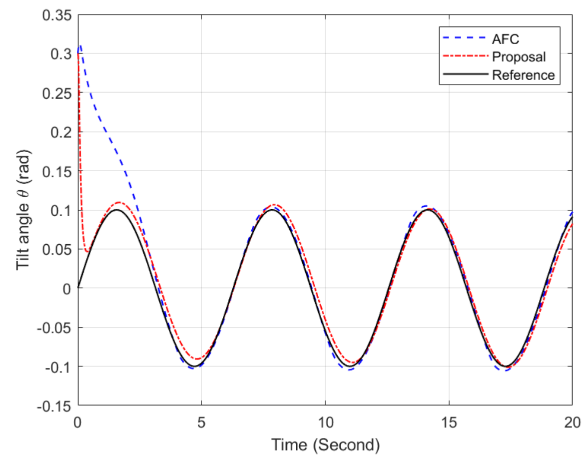

Fig. 13. Comparison of the tilt angle θ

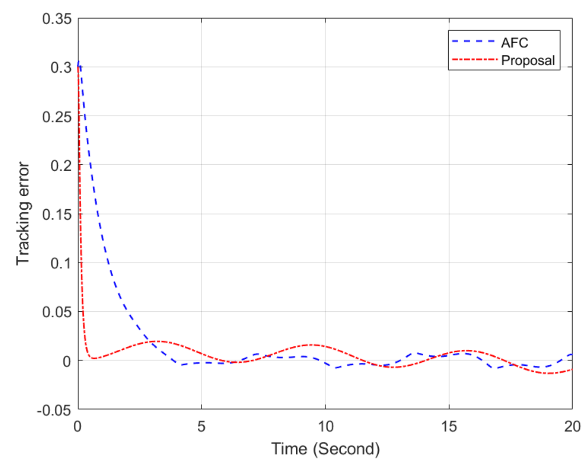

Fig. 14. Comparison of the tracking error $\theta - x_d(t)$

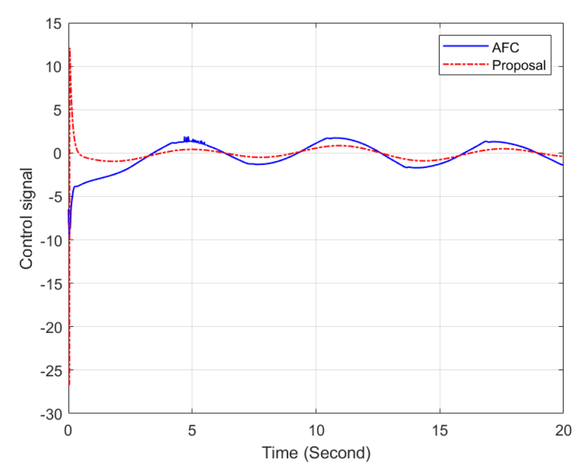

Fig. 15. Comparison of the control signal u

TABLE II. COMPARISON WITHOUT MEASUREMENT NOISE

Method	MAE	ME	RMSE	MAX	MIN
$Proposal$	0.0652	0.0091	0.0016	0.3000	-0.1011
AFC	0.0818	0.0193	0.0022	0.3126	-0.1058

To evaluate how the measurement noise can affect the proposed controller, a sensor noise $d_n(t)$ is added to the tilt angle x_1 for simulations as follow

$$d_n(t) = 0.0105sin(3t + \pi/2), \quad (35)$$

where the magnitude of the noise is selected as 3.5% of the maximum value of x_1 (that is 0.3) [17]. Most of the existing works cited here had not evaluated the impact of the measurement noise except the work [17]. The simulation results in comparison with the AFC controller are given in Table III in which the performance indicators of the proposed method are better than the AFC method but they are similar to those of the proposed method without measurement noise in Table II.

TABLE III. COMPARISON WITH MEASUREMENT NOISE

Method	MAE	ME	RMSE	MAX	MIN
$Proposal$	0.0656	0.0092	0.0017	0.3000	-0.1046
AFC	0.0831	0.0194	0.0022	0.3131	-0.0983

It can be observed that there are 12 parameters $(\underline{w}_f, \underline{w}_g)$ of the AFC updated online, which involves two integral operators. Moreover, the designer had to select suitably the basis function vector $\eta(\underline{x}), \zeta(\underline{x})$ beforehand. So, the proposed controller is simpler than the AFC in design and computational load. By trials, this AFC controller can not guarantee stability when the initial output θ_0 is far away from the equilibrium point $\theta_e = 0$ ($|\theta_0| > 0.3$) but the proposed controller still works.

From these simulation examples, it can be observed that the tracking error converges to the origin faster if the tuning parameters α, β are bigger but the control signal will be larger either, and vice versa. In addition, the proposed controller can cope with the measurement noise as the case with IP example. Finally, the modified function $tanh(qs)$, with $q > 1$ big enough will remove the chattering phenomenon effectively while keeping similar performance. This opens up practical applications.

IV. CONCLUSION AND FUTURE WORKS

In this work, a boundedness based tracking controller was proposed for a class of first-order uncertain SISO affine TVNS systems with unknown input disturbance. Then, the proposed controller was extended for second-order uncertain SISO affine TVNS systems with unknown input disturbance. This controller can be applied in case that the functions of the system and input disturbance are unknown but their bounds can be obtained. Desired convergence rate of the tracking error can be achieved more quickly by tuning the controller parameters α, β. If these factors are bigger, the tracking error will converge to the origin faster but the control signal will be larger, and vice versa.

Numerical simulations for a first-order system and a IP model were carried out to verify the designed controllers. The bounds of uncertainties and disturbance were determined from the prior information of the system. Simulation results show that the tracking error converges to the origin with chattering effects of the control signal as expected. To avoid the chattering effects, the modified function $tanh(qe)$ was proposed to replace the signum function $sign(e)$. With this replacement, the performance indicators were maintained and the control signal was much more smooth without oscillation. The proposed method can also deal with the sensor's noise for the IP robot since it provided better performance than the AFC controller and kept similar performance as the case without measurement noise. Future works will focus on uncertain MIMO TVNS and higher-order uncertain TVNS systems with unknown disturbance.

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