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Reinforcement Learning-Based Trajectory Control for Mecanum Robot with Mass Eccentricity Considerations

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***Abstract*—This article presents a robust optimal tracking control approach for a Four Mecanum Wheeled Robot (FMWR) using an online actor-critic reinforcement learning (RL) algorithm to address the challenge of precise trajectory tracking problem in the presence of mass eccentricity and friction uncertainty. In order to handle these obstacles, a detailed dynamics model is derived using Lagrange's equation, and the Hamilton–Jacobi–Bellman (HJB) equation is solved by iteration algorithm with policy evaluation and improvement. The training laws of optimal control law and value function are proposed after minimizing the modified Hamiltonian function. Moreover, to handle the time-varying property of tracking error model, a transform is given with the addition of time derivative term. Simulation Studies demonstrate the approach's effectiveness, significantly improving trajectory tracking accuracy and robustness against disturbances. This research contributes to mobile robotics by enhancing control precision and reliability in dynamic environments.**

***Keywords*—Optimal Control, Reinforcement Learning, Trajectory Tracking, Friction Uncertainty**

I. INTRODUCTION

In the field of robotics, the control of mobile robots is becoming increasingly important for both industrial automation and research activities. High precision, real-time performance, and robustness to external disturbances are essential requirements for the design of control systems for these robots. Among various mobile robot designs, the Mecanum wheel stands out because it can move in any direction while supporting high loads [1]. This study addresses a specific problem: developing a robust tracking control system for a Four Mecanum Wheeled Robot (FMWR) that accounts for mass eccentricity using an online actor-critic reinforcement learning algorithm.

Extensive research has been conducted on both system modeling and tracking control algorithms to ensure that robots can accurately and swiftly follow desired trajectories. The dynamic model of a mobile robot equipped with four Mecanum wheels is detailed in [2], [3], while its kinematic model and the relationships between kinematics and platform dynamics are presented in [4], [5]. A comprehensive dynamics model that accounts for mass eccentricity and friction uncertainty is provided in [6]. Additionally, a mathematical model integrating both dynamic and kinematic aspects is analyzed and evaluated in [7].

Several studies have focused on control design for mobile robots. Enhancements in control performance for forward and inverse motion have been achieved using an optimal PID controller based on a differential evolution algorithm [7], as well as a PID controller designed for more accurate point tracking of the Mecanum robot [8]. Additionally, a nonlinear Backstepping control law, validated for its efficiency, has been applied based on the dynamic and kinematic model [9], [10]. A robot dynamics-based sliding mode controller for trajectory tracking is presented in [11], [52]–[54], [56].

Reinforcement learning (RL) offers distinct advantages by not requiring precise system models, as demonstrated in [12]. For instance, a reinforcement learning controller in [13] achieves optimal morality for wheeled mobile robots (WMR), with control PD parameters determined within the action space. However, continuous control remains a challenge due to the decomposition of the action space into multiple subspaces.

Studies such as [14], [57]–[60] have designed reinforcement learning algorithms for systemless MIMO routing uncertainty with prioritized control of output signals. The steering control algorithm for WMR, based on the proposed RL algorithm, was examined considering system time delay and slip effects [15]–[21], [63]–[70]. The RL algorithm for solving the optimal control solution for the Hamilton–Jacobi–Bellman (HJB) equation facilitates policy evaluation and improvement [25]–[28], [71]–[80]. Optimal regulation and tracking of nonlinear systems have been addressed using an actor-critic architecture incorporating both actor and critic neural networks (NNs) [20], [49]–[51].

However, these methods typically solve the optimal control problem offline, requiring multiple learning trials before the controller is trained, without ensuring system stability during the learning phase [22]–[24].

An intelligent actor-critic learning control method augmented by a fuzzy broad learning system with output recurrent feedback has been proposed for obstacle-avoiding trajectory tracking of heterogeneous Mecanum-wheeled omnidirectional mobile robots, accounting for unmodeled errors and varying parameters in response to the working environment [35]–[48]. Furthermore, the optimal tracking control for a three-omni-wheeled mobile robot under external disturbances, using an online actor-critic synchronous learning algorithm, was investigated in [34]. However, this study did not consider the impact of the robot's center of gravity deviation and changes in mass.

Despite these advancements, a significant gap persists in the literature concerning the integration of mass eccentricity into the control scheme for FMWR. Most existing studies either neglect mass eccentricity or address it inadequately, resulting in suboptimal control performance. This research bridges this gap by developing a robust tracking control system that incorporates mass eccentricity through an online actor-critic reinforcement learning algorithm. Section II presents the kinematic and dynamic modeling of the FMWR. Section III details the design of the controller using the online actor-critic algorithm. Section IV provides the results of simulation experiments, followed by a discussion in Section V. Finally, Section VI concludes the paper, summarizing the key findings and contributions.

In this paper, we present a novel approach to developing a robust tracking control scheme for a Fast Mobile Wheeled Robot (FMWR), utilizing an online actor-critic algorithm to specifically tackle the challenge posed by mass eccentricity. Our primary aim is to ensure precise adherence to a predefined trajectory within the shortest possible timeframe and with minimal deviation. This research significantly advances the field of mobile robot control by seamlessly integrating mass eccentricity into the control scheme, effectively addressing a notable gap in the existing literature.

Our experimental results underscore the efficacy of the proposed control system, demonstrating its ability to guide the robot along the desired trajectory with remarkable accuracy and efficiency, thus bolstering the overall performance and reliability of FMWR in practical applications. Section II elucidates the kinematic and dynamic modeling of the FMWR, laying the foundation for our subsequent controller design discussed in Section III, which leverages the online actor-critic algorithm. Section IV presents the outcomes of simulation experiments, providing valuable insights discussed further in Section V. Finally, Section VI concludes the paper, summarizing the key findings and contributions made throughout our research endeavor.

II. The Four Mecanum Wheel Mobile Robot

A. Kinematic of FMWR

The mecanum wheel is designed with passive rollers placed around the main wheel, ensuring the robot can move in all directions. The FMWR model uses four active wheels guided independently by four special motors. Robot moving, the rollers convert part of the translation force into horizontal sliding force, helping the robot move flexibly in many different directions. Let's consider the wheel frame in the robot coordinate [5] being shown in Fig. 1.

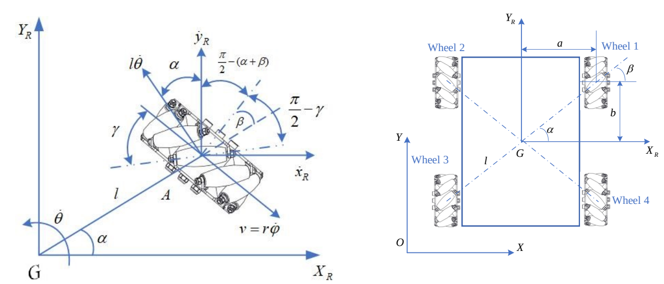


Fig. 1. The four mecanum wheel robot in the world frame

In the case of no slipping occurs along the contact of roller's axis, the same velocity can also be computed from the wheel's rotation speed as:

$$[\sin(\alpha+\beta+\gamma) - \cos(\alpha+\beta+\gamma) - l\cos(\beta+\gamma)]R(\theta)\dot{q} = r\dot{\varphi}\cos\gamma \tag{1}$$

where α is angle of GA with horizontal axis, β is the angle between the vector GA and the main wheel axis, and $\gamma = 45^0$ is the deflection angle of the passive roller. The geometric center G to the wheel center A is l, and r is the main wheel's radius. The orientation of the inertia frame with respect to the robot frame can be expressed as matrix

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

where θ is the angle between axes X_R and X. In the next section, We will construct the kinematic and dynamic equations for FMWR.

In the considered FMWR, assuming that ach mecanum wheel is controlled respectively by an independent motor as arranged in Fig. 1 with wheel has equal radius and distances. The parameters α, β and γ for each wheel are presented in Table I. and substituted into equation (1)

TABLE I. The Parameters of Each Mecanum Wheel

Wheels	α_i	β_i	γ_i
1	$\tan^{-1}(b/a)$	$-\tan^{-1}(b/a)$	$(\pi/2+\pi/4)$
2	$\pi-\tan^{-1}(b/a)$	$\tan^{-1}(b/a)$	$-(\pi/2+\pi/4)$
3	$\pi+\tan^{-1}(b/a)$	$-\tan^{-1}(b/a)$	$(\pi/2+\pi/4)$
4	$2\pi+\tan^{-1}(b/a)$	$\tan^{-1}(b/a)$	$-(\pi/2+\pi/4)$

After computation, the Jacobian matrix is identified as follows:

$$J=\begin{bmatrix}\sqrt{2}/2 & \sqrt{2}/2 & l\sin(\pi/4-\alpha)\\ \sqrt{2}/2 & -\sqrt{2}/2 & l\sin(\pi/4-\alpha)\\ -\sqrt{2}/2 & -\sqrt{2}/2 & l\sin(\pi/4-\alpha)\\ -\sqrt{2}/2 & \sqrt{2}/2 & l\sin(\pi/4-\alpha)\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix} \tag{3}$$

The forward kinematic equation of the FMWR can be obtained as follows:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}=-(\sqrt{2}/2)rJ^{+}\begin{bmatrix}\dot{\varphi}_1\\ \dot{\varphi}_2\\ \dot{\varphi}_3\\ \dot{\varphi}_4\end{bmatrix} \tag{4}$$

where $J^{+}=\left(J^{T}J\right)^{-1}J^{T}$ is the pseudoinverse of J. The velocities of wheels and angular velocities are determined by

$$\begin{bmatrix}\dot{x}_R\\ \dot{y}_R\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} \tag{5}$$

B. Dynamic of FMWR

Consider the FMWR to be show in Fig. 2, where G, G' are the geometric and mass center, respectively [5].

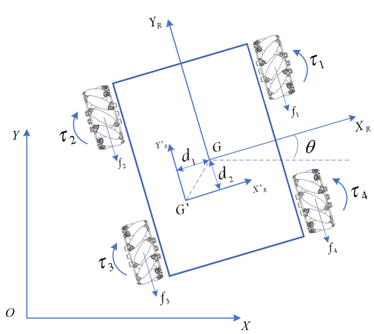


Fig. 2. Schematic of the FMWR

The total kinetic energy E of the mobile robot including those of the platform and four Mecanum wheels can be computed as below:

$$E=\frac{1}{2}\left[m_b v_{G'}^{T}v_{G'}+I_b\dot{\theta}^2+\sum_{i=1}^{4}m_{wi}(r\dot{\varphi}_i)^2+\sum_{i=1}^{4}I_i\dot{\varphi}_i^2\right] \tag{6}$$

where m_b is the mass of the platform, and m_{wi} is the mass of the ith wheel; I_b is the moment of inertia of the platform, and I_i is the moment inertia of the ith wheel about its main axis.

The system moves on the ground, the gravitational potential energy of the system $T=0$, The sum of the kinetic energies of the body and wheels is: $L=E+T$

$$L=\frac{1}{2}m_b v_{G'}^{T}v_{G'}+\frac{1}{2}I_b\dot{\theta}^2+\frac{1}{2}\sum_{i=1}^{4}m_{wi}(r\dot{\varphi}_i)^2+\frac{1}{2}\sum_{i=1}^{4}I_i\dot{\varphi}_i^2 \tag{7}$$

Using the Lagrange's equations [30]:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}-\frac{\partial L}{\partial q_i}\right)=Q_i \tag{8}$$

where $Q_i,(i=1,2,3)$ is torque generalized, $q_i=[x,y,\theta]^T$ is generalized coordinate vector, f_i is the contact friction force of the wheel with the floor, can be derived as follows [29]:
The dynamic equations for FMWR:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+B\xi=B\tau \tag{9}$$

where

$$\tau=\begin{bmatrix}\tau_1 & \tau_2 & \tau_3 & \tau_4\end{bmatrix}^T, f=\begin{bmatrix}f_1 & f_2 & f_3 & f_4\end{bmatrix}^T,$$

$$\xi=r.f.\text{diag}\begin{bmatrix}\text{sgn}(\dot{\varphi}_1) & \text{sgn}(\dot{\varphi}_2) & \text{sgn}(\dot{\varphi}_3) & \text{sgn}(\dot{\varphi}_4)\end{bmatrix}$$

$$M=[m_{ij}]_{3\times 3}, m_11=m_b+4\left(m_w+\tfrac{I}{r^2}\right);$$

$$m_{22}=m_b+4\left(m_w+\tfrac{I}{r^2}\right); m_{12}=m_{21}=0;$$

$$m_{13}=m_{31}=m_b\left(d_1\sin\theta+d_2\cos\theta\right);$$

$$m_{23}=m_{32}=m_b\left(-d_1\cos\theta+d_2\sin\theta\right)$$

$$m_{33}=m_b\left(d_1^2+d_2^2\right)+I_b+8\left(m_w+\tfrac{I}{r^2}\right)l^2\sin^2(\pi/4-\alpha)$$

$$C=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ m_b\dot{\theta}\left(d_1\cos\theta-d_2\sin\theta\right) & m_b\dot{\theta}\left(d_1\sin\theta+d_2\cos\theta\right) & 0\end{bmatrix}^T$$

$$B=\frac{1}{r}\begin{bmatrix}-(\cos\theta-\sin\theta) & -(\sin\theta+\cos\theta) & -\sqrt{2}l\sin(\pi/4-\alpha)\\ -(\cos\theta+\sin\theta) & -(\sin\theta-\cos\theta) & -\sqrt{2}l\sin(\pi/4-\alpha)\\ \cos\theta-\sin\theta & \sin\theta+\cos\theta & -\sqrt{2}l\sin(\pi/4-\alpha)\\ \cos\theta+\sin\theta & \sin\theta-\cos\theta & -\sqrt{2}l\sin(\pi/4-\alpha)\end{bmatrix}^T \tag{10}$$

According to equation (5), it obtains the kinematic equation of FMWR as follows:

$$v_q=R(\theta)\dot{q} \tag{11}$$

Taking the time derivative of equation (11), it implies that:

$$\dot{v}_q=R(\theta)\ddot{q}+\dot{\theta}\dot{R}(\theta)\dot{q} \tag{12}$$

Equation (9) implies the following equality:

$$\ddot{q}=M^{-1}B^T\tau-M^{-1}B^T\xi-M^{-1}C\dot{q} \tag{13}$$

Substituting (13) into equation (12), it obtains that:

$$\dot{v}_q=R(\theta)M^{-1}1B^T\tau-R(\theta)M^{-1}1B^T\xi-R(\theta)R(\theta)^{-1}1$$
$$M^{-1}1Cv_q+\dot{\theta}\dot{R}(\theta)R(\theta)^{-1}1v_q \tag{14}$$

Substituting, can obtain the following dynamic equation of FMWR as below:

$$\dot{v}_q=\bar{f}(q)v_q+\bar{g}(q)\tau+\bar{g}(q)d(q,v_q) \tag{15}$$

where

$$\bar{f}(q)=\left[\dot{\theta}\dot{R}(\theta)R(\theta)^{-1}-M^{-1}C\right]$$

$$\bar{g}(q)=R(\theta)M^{-1}B^T$$

$$fd(q,v_q)=-R(\theta)M^{-1}B^T\xi$$

The problem of controlling a robot to follow a set trajectory $q_r(t)$ in a short time with a low cost function. The dynamic model (15) is rewritten as:

$$\dot{x} = f(x) + g(x)\tau + g(x)d\left(q, v_q\right) \tag{16}$$

where

$$\begin{array}{l} x = (q^T, v_q^T)^T, \\ f(x) = (v_q^T, (f(q)v_q)^T)^T, \\ g(x) = (\mathbf{0}_{3\times3}, g(q)^T)^T \end{array} \tag{17}$$

To develop RL technique for time-invariant systems, we reference system expressed by

$$\dot{x}_r = h_r(x_r) \tag{18}$$

We defining the tracking error as $e = x - x_r$

$$\begin{array}{l} \dot{e} = f(x) + g(x)\tau - h_r(x_r) \\ \quad = f(x) - h_r(x_r) + g(x)\tau + g(x)u \end{array} \tag{19}$$

where $u = \tau - \tau_r$,

$$\tau_r(x_r) = g^+(x_r)(h_r(x_r) - f(x_r)) \tag{20}$$

By employing a new concatenated state as $z = (e^T, x_r^T)^T$, then the dynamics of z is formulated by an invariant-time system as

$$\dot{z} = F(z) + G(z)u \tag{21}$$

where

$$F(z) = \begin{pmatrix} f(x) - h_r(x_r) + g(x)\tau_r \\ h_r(x_r) \end{pmatrix}, G(z) = \begin{pmatrix} g(x) \\ \mathbf{0}_{6\times4} \end{pmatrix}$$

III. Controller Design on the Online Actor-Critic Algorithm

A. Controller design

To control the robot with equation (21) to follow the trajectory, we use the HJB function with the following the Hamiltonian function [34] is given by

$$\begin{array}{l} H(z, u, \bigtriangledown V) = \lambda_M^2(z) + z^T\bar{Q}z + u^T(z)Ru(z) \\ -\gamma V(z) + \bigtriangledown V^T(z)(F(z) + G(z)u(z)) \end{array} \tag{22}$$

With the optimal function clearly defined as follows:

$$V^*(z) = \min_{u\in\pi(\Omega)} \int_t^\infty e^{-\gamma(s-t)}(\lambda_M^2(z) + z^T\bar{Q}z + u^TRu)ds \tag{23}$$

The optimal control signals is

$$u^*(z) = \arg\min_{u\in\pi(\Omega)}[H(z, u, \bigtriangledown V^*(z)] = -\frac{1}{2}R^{-1}G^T(z)\bigtriangledown V^*(z) \tag{24}$$

The control system follows the structure of Online Actor - Critic. The signal control from the Actor will be continuously updated and included in the system size when the signal control is optimal as shown in Fig. 3.

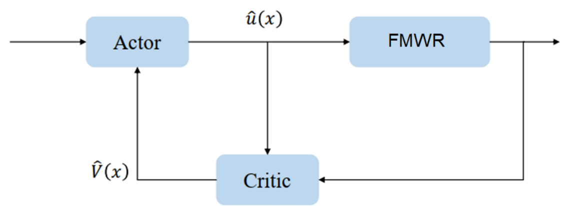


Fig. 3. Control diagram online AC

For optimal control, we use the online actor–critic algorithm to solve the HJB equation using a single-layer NN as

$$V^*(z) = W^T\phi(z) + \varepsilon_v(z) \tag{25}$$

where W is an ideal constant weight vector, the number of neurons is N, $\varepsilon_v(z)$ is the approximation error.

Based on formulas (25) and optimal control, the Actor NNs and Critic NNs approximation for the optimal policy are given by:

$$\hat{u}(z, \hat{W}_a) = -\frac{1}{2}R^{-1}G^T(z)\bigtriangledown\phi^T(z)\hat{W}_a \tag{26}$$

where $\hat{W}_c$ and $\hat{W}_a$ a are the estimates.

The signal control is determined as follows: (20), the tracking controller τ of FWMR can be obtained as

$$\tau = -\frac{1}{2}R^{-1}G^T(z)\bigtriangledown\phi^T(z)\hat{W}_a + g^+(x_r)(h_r(x_r) - f(x_r)) \tag{27}$$

Using the approximations $\hat{u}$ and $\hat{V}$:

$$\begin{array}{l} \hat{H}(z, \hat{W}_c, \hat{W}_a) = \lambda_M^2(z) + z^T\bar{Q}z + \frac{1}{4}\hat{W}_a^TD_1\hat{W}_a \\ -\gamma\hat{W}_c^T\phi + \hat{W}_c^T\bigtriangledown\phi(F - \frac{1}{2}GR^{-1}G^T\bigtriangledown\phi^T\hat{W}_a) \end{array} \tag{28}$$

Define the Bellman error as $\delta = \hat{H} - H^*$,

$$\begin{array}{l} \delta(z, \hat{W}_c, \hat{W}_a) = \lambda_M^2(z) + z^T\bar{Q}z + \frac{1}{4}\hat{W}_a^TD_1\hat{W}_a \\ +\hat{W}_c^T(\bigtriangledown\phi(F + G\hat{u}) - \gamma\phi) \end{array} \tag{29}$$

Training critic NN to minimize the integral error, The tuning law for the actor NN is developed as

$$\dot{\hat{W}}_a = -\eta_{a1}(\hat{W}_a - \hat{W}_c) - \eta_{a2}\hat{W}_a + \frac{\eta_a}{4}D_1\hat{W}_a\frac{\bar{\sigma}^T}{m_\sigma}\hat{W}_c \tag{30}$$

where $\eta_{a1} > 0$, $\eta_{a2} > 0$ and $\eta_a > 0$ are tuning parameters

B. Stability Analysis

Considering the candidate Lyapunov function for the stability of the system

$$V(t) = V^*(t) + \frac{1}{2\eta_c}\tilde{W}_c^T(t)\Gamma^{-1}(t)\tilde{W}_c(t) + \frac{1}{2\eta_a}\tilde{W}_a^T(t)\tilde{W}_a(t) \tag{31}$$

In which the optimal cost function has the following derivative:

$$\dot{V}^*t = W^T \bigtriangledown \phi^F - \frac{1}{2} W^T D_1 \hat{W}_a + \bigtriangledown \varepsilon_v^T (F \\ - \frac{1}{2} G R^- 1 G^T \bigtriangledown \phi^T \hat{W}_a) \tag{32}$$

From the HJB equation (III-A), we can rewrite the optimal cost function determined can be rewritten as:

$$\begin{aligned} \dot{V}^*(t) = & -\lambda_M^2(z) - z^T \bar{Q} z + \gamma W^T \phi + \frac{1}{4} W^T D_1 W \\ & + \frac{1}{2} \tilde{W}_a^T D_1 W + \frac{1}{2} \tilde{W}_a^T \bigtriangledown \phi D_2 \bigtriangledown \varepsilon_v \\ & + \frac{1}{4} \bigtriangledown \varepsilon_v^T D_2 \bigtriangledown \varepsilon_v + \gamma \varepsilon_v(z) \end{aligned} \tag{33}$$

into the derivative of $V(t)$ yields

$$\begin{aligned} \dot{V}(t) = & -\lambda_M^2(z) - e^T Q e - \frac{1}{4} W^T D_1 W - \frac{\tilde{W}_c^T \bar{\sigma}}{m_\sigma} \varepsilon_H \\ & -\tilde{W}_c^T A_1 \tilde{W}_c - \tilde{W}_a^T A_2 \tilde{W}_a + \tilde{W}_a^T A_3 \tilde{W}_c + \tilde{W}_a^T B_1 + B_2 \end{aligned} \tag{34}$$

where

$$A_1 = \frac{1}{2} \bar{\sigma} \bar{\sigma}^T + \frac{\beta}{2} \Gamma^{-1}, A_2 = \frac{(\eta_{a1} + \eta_{a2})}{\eta_a} I - \frac{1}{4} D_1 \frac{\bar{\sigma}^T}{m_\sigma} W$$

$$A_3 = \frac{1}{4} D_1 W \frac{\bar{\sigma}^T}{m_\sigma} + \frac{\eta_{a1}}{\eta_a} I$$

$$B_1 = \frac{1}{2} D_1 W + \frac{1}{2} \bigtriangledown \phi D_2 \bigtriangledown \varepsilon_v - \frac{1}{4} D_1 W \frac{\bar{\sigma}^T}{m_\sigma} W + \frac{\eta_{a2}}{\eta_a} W$$

$$B_2 = \gamma W^T \phi + \frac{1}{4} \bigtriangledown \varepsilon_v^T D_2 \bigtriangledown \varepsilon_v + \gamma \varepsilon_v(z)$$

The boundness ofε_v and $\bigtriangledown \varepsilon_v$, positive constants $\kappa_1, \kappa_2, \kappa_3$ such that $||B_1|| \leq \kappa_1, ||B_2|| \leq \kappa_2, ||(1/4) D_1 (\bar{\sigma}^T / m_\sigma) W|| \leq \kappa_3$. Using the Young's inequality, (34) becomes

$$\begin{aligned} \dot{V}(t) \leq & -\underline{q} ||e||^2 - \frac{\beta}{4\alpha_2} ||\tilde{W}_c||^2 - \frac{\eta_{a1} + \eta_{a2}}{2\eta_a} ||\tilde{W}_a||^2 \\ & - \left(\frac{\beta}{4\alpha_2} - \frac{1}{2\epsilon} \left(\frac{\eta_a 1}{\eta_a} + \kappa_3 \right) \right) ||\tilde{W}_c||^2 \\ & - \left(\frac{\eta_{a1} + \eta_{a2}}{2\eta_a} - \kappa_3 - \frac{\epsilon}{2} \left(\frac{\eta_{a1}}{\eta_a} + \kappa_3 \right) \right) \\ & \times ||\tilde{W}_a||^2 + \frac{\varepsilon_h}{\sqrt{\upsilon \alpha_1}} ||\tilde{W}_c|| + \kappa_1 ||\tilde{W}_a|| + \kappa_2 \end{aligned} \tag{35}$$

where $\underline{q} = \lambda_{min}(Q)$. Choose $(\beta / 4\alpha_2) \geq (1/2\epsilon)((\eta_{a1}/\eta_a) + \kappa_3)$ and $((\eta_{a1} + \eta_{a2}/2\eta_a) \geq \kappa_3 + (\epsilon/2)((\eta_{a1}/\eta_a) + \kappa_3)$ (35) becomes

$$\dot{V}(t) \leq -\bar{\omega}_1 ||\zeta||^2 + \bar{\omega}_2 ||\zeta|| + \kappa_2$$

Applying (23) and (26) gives

$$u^* - \hat{u} = -\frac{1}{2} R^{-1} G^T (\bigtriangledown \phi^T(z) \tilde{W}_a + \bigtriangledown \varepsilon_v)$$

According one can get

$$||u^* - \hat{u}|| \leq \frac{1}{2\lambda_{min}(R)} b_g (b_{\phi z} b_\zeta + b_{\varepsilon z}) = b_u \tag{36}$$

where b_u is a positive constant Therefore, the tracking error dynamics of the FMWR (21) is eventually bounded uniformly, which further shows that the state $q(t)$ can track the desired trajectory $qr(t)$.

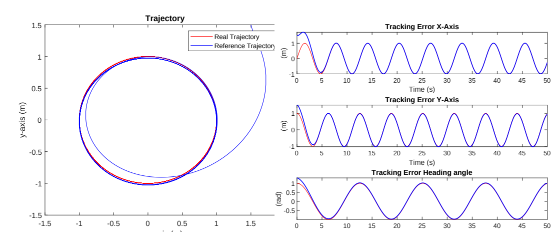
IV. Simulation and Results

In this section, we present a simulation for the Four Mecanum robot model based on MATLAB Simulink environment. The parameters of FMWR: m_b = 12 kg, m_w=0.313kg, a=0.2 m, b=0.3 m, r = 0.0508 m, I = 0.5 $kg.m^2$, I_w = 4.0378 x 10^{-4} $kg.m^2$, g=9.8 m/s^2 And the controller parameters are chosen as

$$x(0) = \begin{pmatrix} 1.5 & 1.5 & 1.4 & 0 & 0 & 0 \end{pmatrix}^T$$
$$\hat{W}_c(0) = 600 \times \mathbf{1}_{51x1}, \hat{W}_a(0) = 6 \times \mathbf{1}_{51\times1}, \Gamma(0) = 10 \times \mathbf{I}_{51}.$$

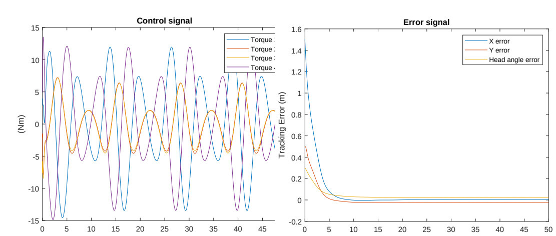
we choose $Q = \mathbf{I}_6$ and $R = \mathbf{I}_3$ the control gains are $\eta_c = 2, \eta_{a1} = 2, \eta_{a2} = 0.001, \eta_a = 0.001, \beta = 0.001, \gamma = 0.0001, \nu = 0.002$. The reference trajectory $q_r = [\sin(t) \cos(t) \cos(0.5t)]^T$.

Simulation results are shown in Figs. 4-6. The system state continues to approach reference trajectory at 5s. We can see simulation results, the neural network weights W_a and W_c converge. The robot's real trajectory also tends to follow the set trajectory at 5s. The corresponding control torques of the four Mecanum wheels are shown in Fig. 5(b) maximum is τ_i = [11.6 6.15 6.15 11.6] Nm, In Fig. 6, the first ten critic and actor weights finally converge to W_a=(-226.6 560.3 578.7 973.9 328.5 81.9 666 501.1 422.9 647.3) and W_c= (-226.5 560.1 578.5 973.6 328.4 81.8 665.8 501 422.8 647) It can be seen from Fig. 5(b) The robot's moving trajectory followed the trajectory set with the reinforcement learning algorithm, deviating from the trajectory set at 16 seconds is within the error range [0.00262 -0.0274 0.0218].



(a) (b)

Fig. 4. (a) Tracking trajectory. (b) Tracking performance of x, y, and θ



(a) (b)

Fig. 5. (a) Control signal. (b) the tracking error

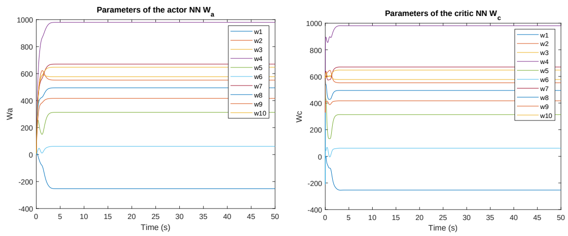
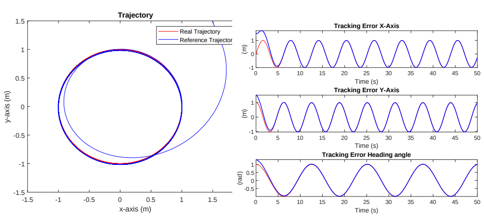


Fig. 6. Weights of the critic and the actor networks

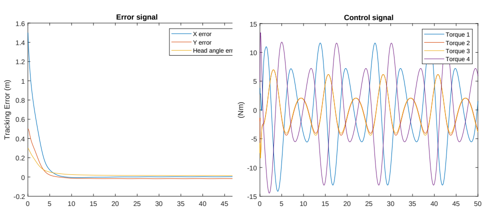
In the simulation, we consider the platform having eccentricity with $d_1 = d_2 = 0.02m$, and the mass has variation $\Delta m_b = 3kg$. Second simulation results are shown in Figs. 7–9.

From Fig. 8(a) we know that the undesired displacement along the x-axes and y-axes are can be kept within 0.0041 0.0052 m and -0.00385 -0.00371 m, respectively, and the orientation θ error is within 0.0312 0.0321 rad. In Fig. 9, the first ten critic and actor weights finally converge to W_a=(-223.1 558.6 577.8 967.1 343.4 81.4 665.7 506.7 423.1 646.6) and W_c=(-223.2 558.8 578 967.4 343.5 81.45 665.98 506.85 423.3 646.8).



(a) (b)

Fig. 7. (a) Tracking trajectory. (b) Tracking performance of x, y, and θ



(a) (b)

Fig. 8. (a) Control signal. (b) the tracking error $q(t)$ - $q_r(t)$

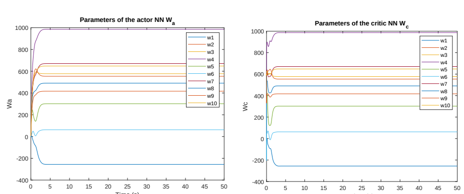


Fig. 9. Weights of the critic and the actor networks

The corresponding control torques shown in Fig. 8(b) maximum is τ_i = [12.06 6.45 6.45 12.06]^T Nm, the control torque is lower compared to the results from the [61], [62] study, this reflects the optimality of the designed method. Indicating that the algorithm is practical and energy saving in real-world applications. Control system for FMWR ensures minimal deviation from the preset trajectory, even in the presence of mass eccentricity and dynamic uncertainties.

V. Conclusion

This study proposes a robust optimal tracking control approach for a Four Mecanum Wheeled Robot (FMWR) in the presence of mass eccentricity and friction uncertainty using an online actor-critic synchronous RL algorithm. The completed mathematical model of mecanum robot is established through Lagrange's equation. Moreover, the modified Hamiltonian function is generated under the discount factor influence. By minimizing its square, the training laws of actor and critic NNs are proposed to obtain RL algorithm. Moreover, the tracking problem and the convergence of learning algorithm are guaranteed by Lyapunov stability theory. Simulation experiments demonstrated the efficacy of the proposed method, significantly improving trajectory tracking accuracy and robustness against disturbances. The results revealed that our control system ensures minimal deviation from the preset trajectory, even in the presence of mass eccentricity and dynamic uncertainties. Future investigation should involve the extension to the experiment systems.

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