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Synergetic Control Design Based Sparrow Search Optimization for Tracking Control of Driven-Pendulum System

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***Abstract*—This study investigates the performance of designing a Synergetic Control (SC) approach for angular position tracking control of driven-pendulum systems. SC is one of the popular nonlinear control techniques that contributed in a variety of control design applications. This research shows a unique application of the SC for angular position tracking control of driven-pendulum systems. Initially, the equations of motion of the system are developed. Subsequently, the control law of the SC is established. For the stability analysis of the closed loop control system, the Lyapunov Function (L.F) is used. To guarantee optimal performance, a Sparrow Search Optimization (SSO) based approach is presented in order to search for the optimum designing parameters of the controller. For performance comparison, the classical Sliding Mode Control (SMC) is introduced. The simulation's outcomes of the study have been confirmed that the proposed control algorithm is addressed the tracking problem of the angular position of the system successfully. Besides, when an external disturbance is inherited in the simulation, the SC exhibits a robustness performance. Moreover, the performance of the SC is slightly similar as SMC. However, the distinct difference in the performance is that the control signal of the SMC exhibits chattering problem, while this phenomenon is absent in the SC. All computer simulations are carried out using MATLAB software.**

Keywords*—Nonlinear Control Design; Driven-Pendulum System; Tracking Control Problem; Synergetic Control; Sliding Mode Control; Lyapunov Stability; Sparrow Search Optimization.

I. INTRODUCTION

A driven-pendulum system is a motorized propeller with a single degree of freedom that located at the end of a pendulum. The propeller positioned at the pendulum's free end generates torque that moves the pendulum up and down while the axis of rotation remains fixed [1]. The concept of simple pendulum gains wide applications in physical subjects due to its simple harmonic motion [2]. In the context of engineering, the pendulum system is a nonlinear dynamic system which commonly used in academic for in mechanical, control and mechatronics engineering [3]. In terms of applications, driven-pendulum systems can be used in modeling Unmanned Aerial Vehicles (UAVs) and helicopters for the process of take-off and landing [4]. As a result, studying the dynamic performance and investigating of numerous control algorithms for driven-pendulum systems has been attracted a lot of interest. The relevant studies for controlling the propeller-driven system are reviewed as follows.

Kizmaz et al. [5] has applied a robust control to a linearized model of the driven-pendulum system using Sliding Mode Control (SMC). The proposed SMC is evaluated using a computer simulation and implemented experimentally. The performance of the SMC is evaluated for various controller parameters values. The outcome is proven the robustness of the SMC for different amplitudes of step reference inputs. However, the chattering issue has not mentioned in the control law that have been made. The application of Linear Quadratic Regulator (LQR) to the driven-pendulum system was performed by Farmanbordar et al. [2]. Changing the weights in the matrix Q and R of the LQR have been applied while the system was exposed to a step input. Therefore, the linearized model of the system gave the best stability possible. Yoon [6] stabilized driven-pendulum system based on lead compensator controller to a square wave reference input. The linear model of the system was used to design the controller. The linear model was validated experimentally. The parameters of the controller were tuned based on the trial and error. The proposed lead controller was successful stabilization the system with slightly overshoot. However, the closed loop system was suffered from large steady state tracking errors. Günel and Ankarah [7] applied Proportional-Integral-Derivative (PID) controller to a linearized model of the driven-pendulum system. Alternative to classical method of tuning the PID such as Ziegler-Nichols (ZN) and Root Locus, the tuning process of the PID controller has been handled by two optimization techniques: Genetic Algorithm (GA) and Particle Swarm Optimization (PSO). Both algorithms show similar performance when the system is subjected unit step input. However, there is a slightly better improvement in the transit response was observed by the PSO in comparison with the GA.

The system's angular position was first controlled by the SMC for step input reference at the Al-Qassar et al. [4]. However, to address the chattering problem in the classical SMC, a Super-Twisting SMC (STSMC) is proposed. Furthermore, to handle the external disturbance, an Adaptive STSMC (ASTSMC) is introduced. The PSO was employed to tuning the design coefficients of the controller. Hamoudi and Rasheed [3] implemented of an Adaptive

Backstepping Control (ABSC) for position control of driven-pendulum system. The simulation results of the proposed ABSC in comparison with the classical BSC for step input revealed the effectiveness of the proposed controller to compensate the effect of the variation of the system's mass. The tuning process of the controller was performed by the PSO. Ahmed and Al-Khazraji [8] investigated the performance of three control strategies: PID, State Feedback (SF), and Sliding Mode (SM) controllers for angular position tracking to step input. The Gorilla Troops Optimisation (GTO) has been implemented to determine the optimal value of the design variables, therefore enhance each controller. The outcomes indicated that SM exhibits better performance in comparison with the other controllers. Besides, SM demonstrated superior resilience to the external disturbance compared to other controllers.

Based on the aforementioned research, it is clear that a variety of control algorithms are being investigated in order to govern the driven-pendulum system's angular position. While the system's performance has improved somewhat as a result of each of these studies, but searching for better performance has been an ongoing endeavor. Besides, the majority of these studies have been utilized the linearized model to design the controller. In addition, none of these studies have been evaluated the performance of the system when the input is a sinusoidal. The novelty of this study is to introduce the Synergetic Control (SC) mechanism for tracking the sinusoidal angular position of the driven-pendulum system. Compared to the other nonlinear controller strategies, SC has many advantages including high efficiency, fast response and simple implementation [9]. SC is utilized the model of the system to synthesize the control law. For the stability analysis, the Lyapunov Function (L.F) is employed. On the basis of performance evaluation, the classical Sliding Mode Control (SMC) is introduced. SMC has some similarities as SC in terms of designing the control law. For example, both controllers are forced the system to operate on a pre-define manifold. However, SC operates at constant switching frequency and it does not have the chattering problems as SMC. The performance comparison between SC and SMC has been conducted previously for Boost converter [11], Permanent Magnet Synchronous Machines (PMSMs) [12], and pendulum [13]. Besides these studies, this work can be viewed as an addition to these investigations. Sparrow Search Optimization (SSO) is used to pick the controller design parameters without relying on an inefficient trial-and-error approach. The SSO is characterized by fast convergence and its capability to find the global solution [10].

The main contributions of this paper can be stated as follows:

1. A method based on synergetic control is applied to a nonlinear driven-pendulum system for angular position tracking control.
2. The performance of the proposed controller is enhanced by applying the sparrow search optimization.
3. The performance of the proposed controller is compared with the performance of sliding mode control.
4. The stability of the closed system is proved based on Lyapunov stability analysis.

The remainder of the paper is prepared as follows: the mathematical model and equations of motion of the driven-pendulum system are given in Section 2. In Section 3, the procedure to synthesize the control law of SMC and SC are presented. Section 4 describes the concept of the SSO. The simulation results and discussions are reported in Section 5. Conclusion is summarized in Section 6.

II. MATHEMATICAL MODEL OF DRIVEN-PENDULUM

Before proceeding in the control design of the nonlinear driven-pendulum system to meet the required performance, the mathematical model that is mimicked the dynamics of the system is established. The schematic diagram of the driven-pendulum system is depicted in Fig. 1 [8].

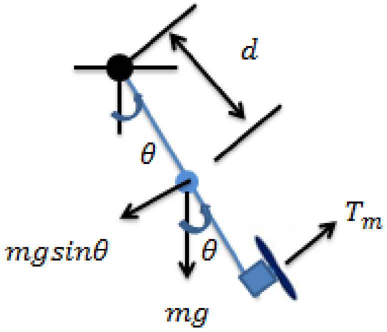


Fig. 1. Driven-pendulum system

The propeller, which is fastened to the end of the arm, powers the pendulum, as illustrated in Fig. 1. A direct current motor powers the propeller. The voltage of the DC serves as the system's control input u, and the angular position θ, which lies between the arm and vertical axis. The DC motor's input voltage causes the propeller to revolve, producing torque T_m that moves the arm with an angular acceleration of $\dot{\theta}$ and an angular speed of $\ddot{\theta}$. Newton's law governs the driven-pendulum system's dynamic motion. Newton's law states that the system's equation of motion is as follows [14][3]:

$$J\ddot{\theta} + C\dot{\theta} + mgd\sin\theta = T_m \tag{1}$$

where the parameters m, J, g, d and C stand for, the propeller's mass, the moment of inertia, the acceleration of gravity, the distance between the suspending point and the mass centre, and the damping coefficient, respectively.

The relationship between the DC motor's input voltage and its produced output torque is shown in the following formula [4]:

$$T_m = K_m u \tag{2}$$

where K_m denotes the DC motor's constant coefficient.

Substitute the torque as given in Eq. (2) into Eq. (1) gives:

$$J\ddot{\theta} + C\dot{\theta} + mgd\sin\theta = K_m u \tag{3}$$

Solving Eq. (3) for $\ddot{\theta}$ gives:

$$\ddot{\theta} = \frac{-C\dot{\theta} - mgdsin\theta + K_m u}{J} \tag{4}$$

In the context of control design, let $x_1(t)$ and $x_1(t)$ represent the angular position $\theta(t)$ and the angle velocity $\dot{\theta}(t)$ respectively. As results, the differential equations of the state variables of system are:

$$\dot{x}_1 = x_2 \tag{5}$$

$$\dot{x}_2 = \frac{-Cx_2 - mgdsinx_1 + K_m u}{J} \tag{6}$$

III. CONTROLLER DESIGN

The utilization of control feedback has been proven over the years as a good mechanism that could be used to improve the performance of the system [15]-[21]. Among these controllers and due to the intrinsic feature of Sliding Model Control (SMC) and Synergetic Control (SC) such as insensitivity to parameter variations and external disturbance rejection, these control algorithms are gained a wide application in the control problem design [22]-[24]. For the purpose of controlling design, Eq. (6) can be revised as follows:

$$\dot{x}_2 = f(x_1, x_2) + bu \tag{7}$$

Where

$$f(x_1, x_2) = \frac{-Cx_2 - mgdsinx_1}{J} \tag{8}$$

$$b = \frac{K_m}{J} \tag{9}$$

Besides, let define the tracking error e_t as the difference between the desired angular position x_{1d} and the actual angular position x_1 as follows:

$$e_t = x_{1d} - x_1 \tag{10}$$

Taking the 1st and the 2nd derivatives of the error gives:

$$\dot{e}_t = \dot{x}_{1d} - \dot{x}_1 \tag{11}$$

$$\ddot{e}_t = \ddot{x}_{1d} - \dot{x}_2 \tag{12}$$

Substitute Eq. (7) gives:

$$\ddot{e}_t = \ddot{x}_{1d} - f(x_1, x_2) - bu \tag{13}$$

The procedure to design the SMC and the SC for tracking control problem of the nonlinear driven-pendulum system is presented in the following subsections.

A. *Sliding Mode Control*

The basic principle of the SMC is to manipulate the trajectory of the system's states to pre-define surface and to maintain on it [25]. Using that definition, the procedure of designing SMC is divided into two parts. In the first part, let define the sliding surface (s) as (14).

$$s = \dot{e}_t + a_{smc} e_t \tag{14}$$

where $a_{smc} > 0$ is a tuning parameters.

Taking the 1st derivative of the s gives (15).

$$\dot{s} = \ddot{e}_t + a_{smc}\dot{e}_t \tag{15}$$

Substitute Eq. (13) in Eq. (15) gives:

$$\dot{s} = \ddot{x}_{1d} - f(x_1, x_2) - bu + a_{smc}\dot{e}_t \tag{16}$$

The second part of designing the SMC is to define a desired evolution of the sliding variable called reaching law. Let define the reaching law (σ) as follows [26]:

$$\sigma = -k_{smc} sgn(s) \tag{17}$$

where k_{smc} $(k_{smc} > 0)$ and sgn are an adjusted parameter and sign function respectively. In order for the system to be sliding on the pre-define surface, the reaching law that is given in Eq. (17) and the first derivative of the sliding surface that is given in Eq. (16) must also be equal. This gives:

$$\ddot{x}_{1d} - f(x_1, x_2) - bu + a_{smc}\dot{e}_t = -k_{smc} sgn(s) \tag{18}$$

Solving Eq. (18) for u yields the following:

$$u_{smc} = \frac{1}{b}\left(\ddot{x}_{1d} - f(x_1, x_2) + a_{smc}\dot{e}_t + k_{smc} sgn(s)\right) \tag{19}$$

The schematic diagram of SMC is shown in Fig. 2.

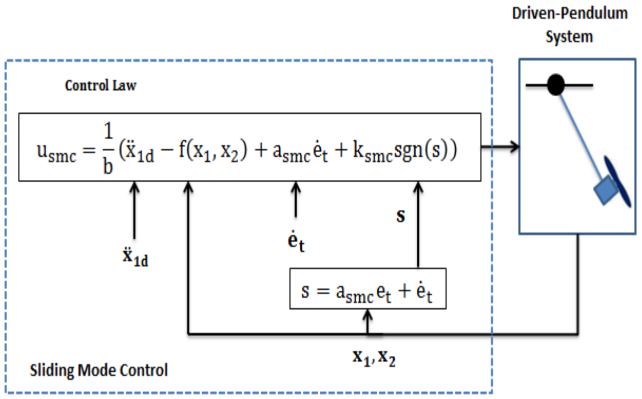


Fig. 2. SMC for driven-pendulum system

Proof. The L.F is selected as:

$$V = \frac{1}{2}s^2 \tag{20}$$

Taking 1st derivate gives

$$\dot{V} = \dot{s}s \tag{21}$$

Substitutes Eq. (117) into Eq. (21) gives:

$$\dot{V} = -k_{smc}|s| \tag{22}$$

As can be seen from Eq. (22), the stability SMC is guaranteed due to the negative derivative of the L.F.■

B. *Synergetic Control*

Synergetic control utilizes the model of the system for developing the control law [27]. Let define the marco-variable φ as a function of the tracking error as (23).

$$\varphi = \dot{e}_t + a_{sc} e_t \tag{23}$$

Taking the 1st derivative of the $\varphi(e)$ gives:

$$\dot{\varphi} = \ddot{e}_t + a_{sc}\dot{e}_t \tag{24}$$

where $a_{sc}(a_{sc} > 0)$ is a scalar designing parameter

To guarantee the stability (i.e. ensure the state trajectories moves towards the desired manifolds and remain on it), lets define the following:

$$\dot{\varphi} + k_{sc}\varphi = 0 \tag{25}$$

where $k_{sc2}(k_{sc2} > 0)$ is an adjustable parameter represents the rate of convergence towards the desired manifolds.

Applying the result of Eq. (24) in Eq. (25) gives:

$$(\ddot{e}_t + a_{sc}\dot{e}_t) + k_{sc}\varphi = 0 \tag{26}$$

Substitute Eq. (13) in Eq. (26) gives:

$$\ddot{x}_{1d} - f(x_1, x_2) - bu + a_{sc}\dot{e}_t + k_{sc}\varphi = 0 \tag{27}$$

Solving Eq. (27) for u yields the following:

$$u_{sc} = \frac{1}{b}(\ddot{x}_{1d} - f(x_1, x_2) + a_{sc}\dot{e}_t + k_{sc}\varphi) \tag{28}$$

The schematic diagram of CSC is shown in Fig. 3.

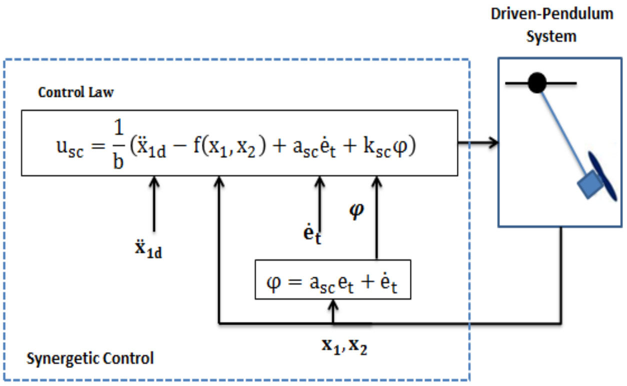


Fig. 3. SC for driven-pendulum system

Proof. The L.F is selected as:

$$V = \frac{1}{2}\varphi^2 \tag{29}$$

Taking the 1st derivate gives

$$\dot{V} = \dot{\varphi}\varphi \tag{30}$$

Substitutes Eq. (25) into Eq. (30) gives:

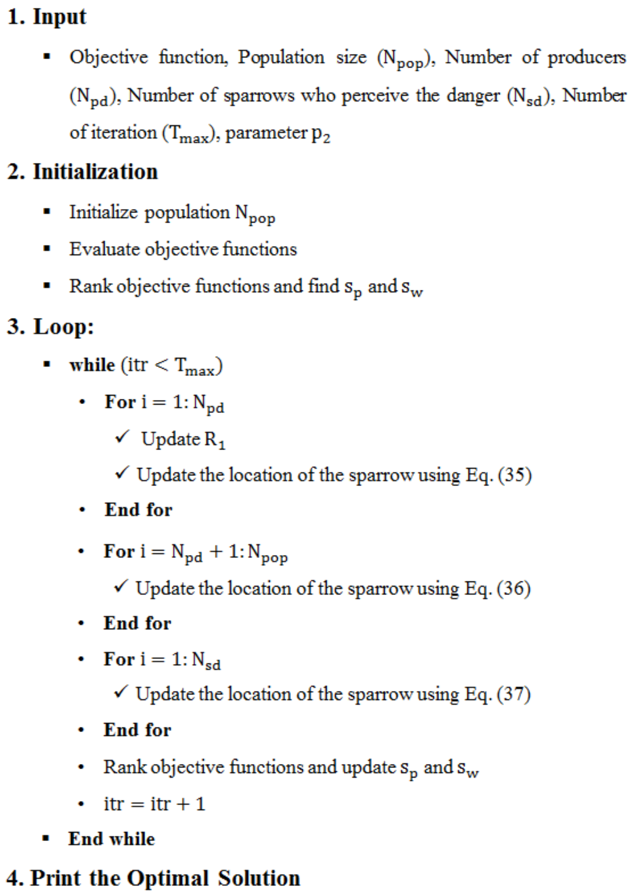
$$\dot{V} = -k_{sc}\varphi^2 \tag{31}$$

As can be seen from Eq. (31), the stability of the SC is guaranteed due to the negative derivative of the L.F.■

IV. SPARROW SEARCH OPTIMIZATION

Swarm optimization techniques have advantages of strong global search capability, not restricted by the differentiability of the objective function, self-organized, simplicity of execution and fast convergence among other optimization techniques [28]-[31]. Therefore, these algorithms are drawing attention from the scientific to be applied to solve a wide range of optimization-related problems [32]-[43].

Sparrow Search Optimization (SSO) is a swarm optimization method proposed by Xue and Shen in (2020) [44] inspired by social living behaviors of sparrows. Unlike other species of birds, sparrows are highly intelligent and have a good memory. Sparrows are switching between two different strategies to find their food and avoid predators. While the scroungers steal food from the producers, the producers actively seek out food sources and direct other sparrow for finding a foraging area and avoid predators [45]. Fig. 4 displays the algorithm's pseudo code. The following is an explanation of the algorithm: Inside the specified problem's search space, a random initialization of N_{pop} sparrows is performed. Next, each sparrow's objective function is evaluated according to its position. Then, an evaluation of the objective function of each sparrow based on its position is performed. Then, the population is separated into producers and scroungers. The producers are the sparrow that are seek out food sources in the meanwhile the scroungers follow the producer to find foraging areas are.



```
1. Input
    ▪ Objective function, Population size ($N_{pop}$), Number of producers
      ($N_{pd}$), Number of sparrows who perceive the danger ($N_{sd}$), Number
      of iteration ($T_{max}$), parameter $p_2$
2. Initialization
    ▪ Initialize population $N_{pop}$
    ▪ Evaluate objective functions
    ▪ Rank objective functions and find $s_p$ and $s_w$
3. Loop:
    ▪ while ($itr < T_{max}$)
        • For $i = 1{:}N_{pd}$
            ✓ Update $R_1$
            ✓ Update the location of the sparrow using Eq. (35)
        • End for
        • For $i = N_{pd} + 1{:}N_{pop}$
            ✓ Update the location of the sparrow using Eq. (36)
        • End for
        • For $i = 1{:}N_{sd}$
            ✓ Update the location of the sparrow using Eq. (37)
        • End for
        • Rank objective functions and update $s_p$ and $s_w$
        • $itr = itr + 1$
    ▪ End while
4. Print the Optimal Solution
```

Fig. 4. Framework of SSO

The position of the producers (N_{pd}) is updated in two directions. In the first direction, they move in a broad range searching for food. However, once there is a detection of a predator, all sparrows move quickly to other safe areas. These two mechanisms of movement can be modeled mathematical as (32).

$$s_i(itr + 1) = \begin{cases} s_i(itr)e^{\left(\frac{-itr}{k_1 T_{max}}\right)}, & if\ p_1 < p_2 \\ s_i(itr) + k_2, & if\ p_1 \geq p_2 \end{cases} \tag{32}$$

where s_i, itr, T_{max} and i are the position of the sparrow, index for iteration, the maximum number of iteration and index for population. The coefficient k_1 ($k_1 \in [0,1]$) is random value. The coefficient $p_2 (p_2 \in [0.5,1])$ represents safety threshold selected by the user. The coefficient p_1 ($p_1 \in [0,1]$) is random value. The coefficient k_2 is a random value which follows normal distribution. Based on Eq. (32), the condition $p_1 < p_2$ means there is no fear in the surrounding area where the sparrows can fly searching for food. However, if $p_1 \geq p_2$, this indicated that there are predators and the predators should moves quickly to a safe area [10]. The position of the remaining scroungers ($N_{pop} - N_{pd}$) is updated as follows:

$$s_i(itr+1) = \begin{cases} k_2 e^{\left(\frac{s_w(itr)-s_i(itr)}{itr^2}\right)}, & if\ itr > \frac{N_{pop}}{2} \\ s_p(itr) + |s_i(itr) - s_p(itr)|A^+, & otherwise \end{cases} \tag{33}$$

where $s_w(itr)$ refers to the position of the worst sparrow, $\mathrm{s_p(itr)}$ refers to the position of the best producer. A^+ is randomly assigned either 1 or -1. In Eq. (33), the scroungers follow the location of the best sparrows. Nevertheless, when $itr > \frac{N_{pop}}{2}$, sparrows moves towards the sparrows with the worse objective function for survival [46]. Finally, it assume that there are a 10% to 20% of the population are detected a danger (N_{sd}). In that case, the sparrows will quickly move to a safe place to find a better position. According to this, the position of the sparrows, which are aware of the danger, is updated as follows [47]:

$$\begin{aligned} & s_i(itr+1) \\ & = \begin{cases} s_g + k_4|s_i(itr) - s_g(itr)|, & if\ f_{s_i(itr)} > f_{s_g} \\ s_i(itr) + k_5\left(\frac{|s_i(itr) - s_w(itr)|}{\left(f_{s_i(itr)} - f_{s_w(itr)}\right) + k_6}\right), & if\ f_{s_i(itr)} = f_{s_g(itr)} \end{cases} \end{aligned} \tag{34}$$

where $s_g, f_{s_g}, f_{s_i(itr)}\ and\ f_{s_w(itr)}$ are the position of the global optimal value, the global optimal value, the objective function of the current sparrow and the worst objective function respectively. k_4 is a random value which follows normal distribution. k_5 ($k_5 \in [-1,1]$) is a random number. k_6 is a small number to avoid the division by zero. When $f_{s_i(itr)} > f_{s_g}$, the sparrow moves towards the best location. Whereas, if $f_{s_i(itr)} = f_{s_g(itr)}$, the sparrow moves closer to the others.

V. NUMERICAL SIMULATION OF CONTROL SYSTEM

In this part, computer simulation is performed based on MATLAB to evaluate the effectiveness of the proposed controllers to the angular position tracking control of the driven-pendulum systems. The equations of motion that are given in Eq. (5) and Eq. (6) of the system have been coded in the m-files of the MATLAB. The MATLAB simulation parameters used for solving the nonlinear differential equation of the controlled system are the solver (ode45), the step size (0.01 s) and the simulation time (10 s). The controlled system is subjected to a periodic unity-sin-wave input. The initial value of the system's states (x_1 and x_2) is set to zero. The numerical values of the parameters of the driven-pendulum system are given in Table I [4]. To get high performance quality of each controller, the SSO has been utilized for tuning the design variables according to the Integral Absolute Error (IAE) index. The equation (35) of IAE index is given by [48]-[52]:

$$IAE = \int_0^{t_{sim}} |e|\, dt \tag{35}$$

where t_{sim} denotes to the simulation time and e is the deviation the output (angular position) x_1 and the desired value (x_{1d}). It must be pointed out that, the input voltage of the DC motor is saturated by ±30v. Given this, the optimization problem of tuning process of the controllers based on the SSO can be formulated as follows [53]:

$$\begin{gathered} \text{minimize IAE(var)} \\ \text{s.t} \\ -30 \geq \text{u} \geq 30 \end{gathered} \tag{36}$$

where IAE is the objective function that needs to be minimized. The decision vectors (var) are the SMC adujstable gains (a_{smc} and k_{smc}) as given in Eq. (19) and the SC adjustable gains (a_{sc} and k_{sc}) as given in Eq. (28). The saturated of the control input u impose as a constraint in the optimization problem. The parameters of the SSO are provided in Table II. The best setting of the SSO are selected after repeated the simulation several times with different values until the quality solution of is improved. The convergence of SSO is shown in Fig. 5. Table III presents the optimal setting of SMC and SC based on SSO.

TABLE I. NUMERICAL VALUES OF SYSTEM PARAMETERS

Parameter	Value and Unit
Motor coefficient ($\mathrm{K_m}$)	0.0296
Moment of the inertia (J)	0.0106
Mass of the propeller (m)	0.36
Viscous damping coefficient (C)	0.0076
Distance from suspending point to the mass center (d)	0.03
Acceleration due to the gravity (g)	9.81

TABLE II. SSO'S PARAMETERS

Parameter	Value
Number of iterations ($\mathrm{T_{max}}$)	25
Population size (N)	25
Number of producers($\mathrm{N_{pd}}$)	15
Number of sparrows who perceive the danger ($\mathrm{N_{sd}}$)	5
Coefficient value ($\mathrm{p_2}$)	2

The simulation response of the close loop driven-pendulum system based on the SMC and the SC to a unity-sin-wave input is depicted in Fig. 6. The corresponding control actions deduced by the SMC and the SC are shown in Fig. 8. The performance is measured by the IAE index and reported in Table IV. Concerning to the saturated voltage input, it can be observed based on Fig. 8 that the control actions of the SMC and the SC are within the acceptable range. Besides, no chattering phenomena have been noticed in the SC's control law whereas the control signal exhibits chattering behavior in the case of SMC. The stability analysis of the two control methods is done theoretically via L.F and it is proven through the simulation. However, by looking at zoom-out picture of Fig. 6 in Fig. 7, it can be noticed that the SC leads to slightly better tracking control than the SMC. Furthermore, the amplitude of the

IAE is reduced from 0.414 in the case of the SMC to 0.303 in the case of the SC.

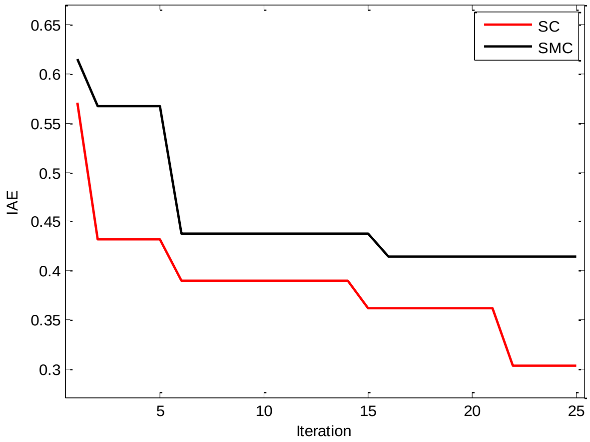


Fig. 5. SSO's convergence

TABLE III. OPTIMAL DESIGN VARIABLES VALUE OF SMC AND SC BASED ON SSO

Controller	Parameter	Value
SMC	a_{smc}	8.7
	k_{smc}	47
SC	a_{sc}	250
	k_{sc}	656

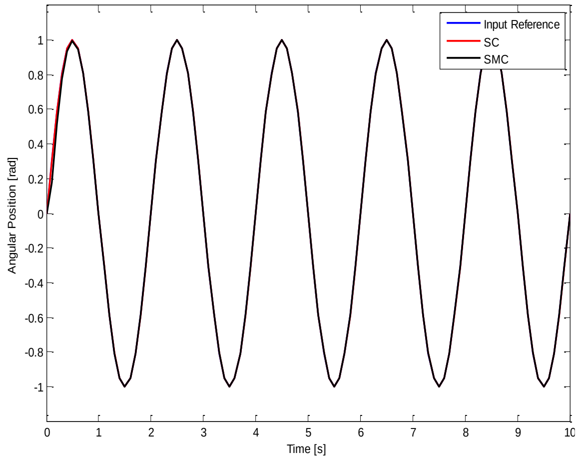


Fig. 6. Response of the system

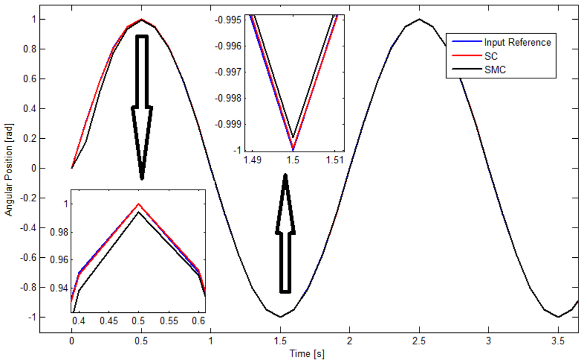


Fig. 7. A zoomed-out view for response of system

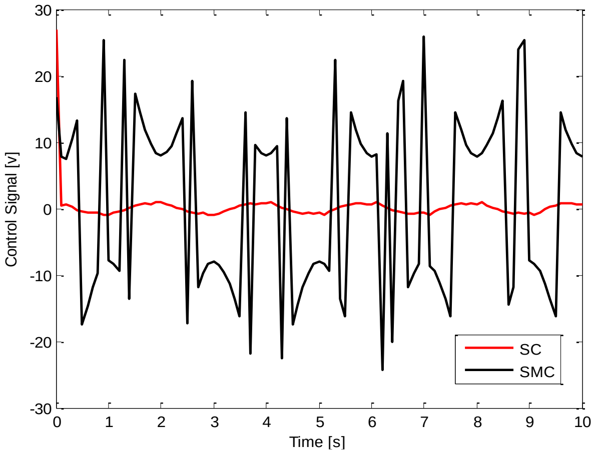


Fig. 8. Control signals

TABLE IV. IAE EVALUATION

Controller	IAE
SMC	0.414
SC	0.303

Regarding the disturbance rejection, Eq. (5) of the system is rewritten as:

$$\dot{x}_1 = x_2 - x_{dis} \tag{37}$$

where x_{dis} an external disturbance injected into the system with amplitude of 0.5 and width of 0.4 s. It can be observed from Fig. 9 that the effect of this disturbance to system at 5 s. The corresponding control actions deduced by the SMC and the SC with the disturbance are shown in Fig. 11. It can be noticed based on Fig. 11 that the control actions of the SMC and SC are within the acceptable range. The robustness of the system is evaluated by the IAE index. The numerical value of the measured performance with the disturbance is given in Table V.

It is evident from zoom-out in Fig. 10 that in the external disturbance scenario, the SC performs better as a disturbance rejection than the SMC. For instance, Table V shows that the IAE index is reduced from 0.589 in the case of the SMC to 0.312 in the case of the SC.

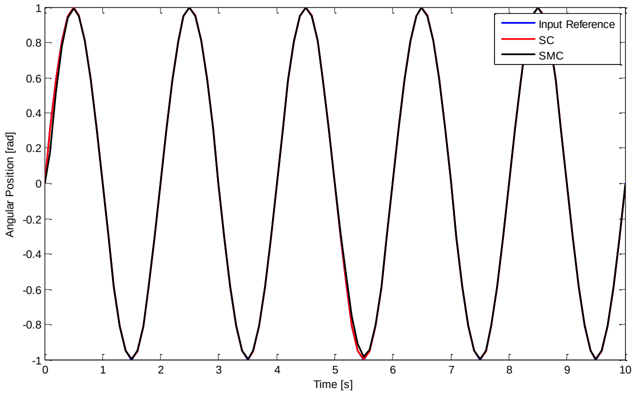


Fig. 9. Response of the controlled systems with disturbance

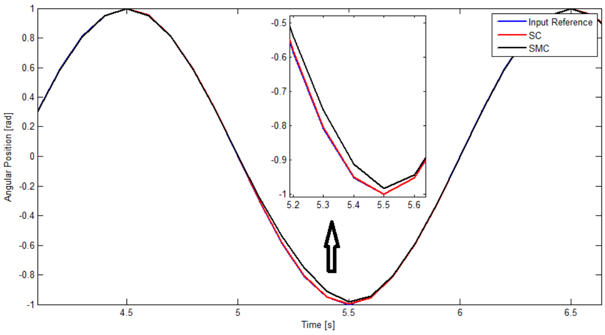


Fig. 10. A zoomed-out view for response of the system under disturbance

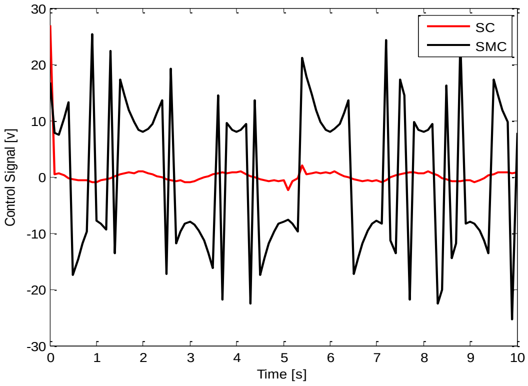


Fig. 11. Control signals with disturbance

TABLE V. TRANSIENT EVALUATION PARAMETERS WITH DISTURBANCE

Controller	IAE
SMC	0.589
SC	0.312

VI. CONCLUSION

Many systems exhibit nonlinear behavior, which makes it difficult to analyze and control. To address this problem, the performance comparison of the Sliding Mode Control (SMC) and the Synergistic Control (SC) methods to tracking control problem of the angular position of the nonlinear driven-pendulum system was presented in this study. To ensure a suitable basis of comparison, the two different angular position control methods are optimized by a Sparrow Search Optimization (SSO) to search for the best designing parameters of the controllers. Besides, the two controllers are evaluated the same sin-wave input and disturbance. The stability analysis of the two control methods is done theoretically via Lyapunov function. Based on the computer simulation on MATLAB, the angular position for both controllers follows perfectly reference input. Nonetheless, the numerical comparison results revealed that the SC preforms slightly better by achieving a good speed tracking performance. Furthermore, tracking performances under disturbance was also slightly improved for the SC approach when compared to the SMC approach. However, the main remark of this study is that SC achieves similar performance as SMC without its inconvenient phenomenon: chattering in the control law. Based on this study, the potential future research directions could be using different types of control systems, exploring hybrid optimization strategies, or integrating adaptive control mechanisms to further enhance performance under varying conditions.

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