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A Novel Fuzzy Identification Approach for Nonlinear Industrial Systems: Eliminating Singularity for Enhanced Control

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Abstract—**The control of nonlinear systems poses significant challenges due to their inherent complexities, limiting the effectiveness of traditional control strategies. This paper presents an improved fuzzy identification and control method for nonlinear industrial systems, using Takagi-Sugeno fuzzy inference to model nonlinear dynamics as an interpolation of multiple linear subsystems. A key improvement of this approach lies in the accurate identification of the nonlinear model, which leads to fewer control system failures. The research contribution is the development of a control strategy that enhances system reliability while simplifying implementation. The method involves minimizing a cost function that optimizes the system's output error, refining the fuzzy identification process for dynamic adaptation to varying operating conditions. The strategy also enables the design of linear controllers for each subsystem and applies Parallel Distributed Compensation (PDC) to regulate the overall nonlinear system. This approach is validated through experimental testing on an aero-pendulum system. The results show that the PDC-based control scheme not only ensures high performance across a wide operational range but also significantly reduces identification errors compared to traditional methods. Given its improved accuracy, reduced complexity, and adaptability, this approach holds significant potential for practical application in industrial environments, where robust and efficient control of nonlinear systems is crucial for operational success.**

Keywords*—*Fuzzy Identificationl; PDC Control; Takagi - Sugeno; Nonlinear Systems; Industrial Control; Aeropendulum

I. Introduction

The control of nonlinear systems has historically posed a significant challenge in both academic and industrial fields due to the inherent difficulties in designing controllers that maintain stability and efficiency across a wide range of operating conditions. Nonlinear systems exhibit complex dynamic behaviors, such as bifurcations and chaos, which make it difficult to apply conventional linear control methods. Over the years, various strategies have been proposed to tackle this problem; however, many of these approaches are either too complex for practical implementation or fail to deliver satisfactory performance over a broad operational range. Some examples of nonlinear control are presented in [1]–[5].

Among the methodologies developed to address nonlinear control, fuzzy systems, particularly those based on Takagi-Sugeno (T-S) models, have gained prominence due to their ability to approximate complex nonlinear systems by interpolating between multiple linear subsystems. Takagi-Sugeno type fuzzy systems were introduced in [6], where a methodology was proposed in which fuzzy rules follow an ”if-then” format, with the particularity that the rule consequents are linear functions (control examples [7]–[13]). Later, in [14], Sugeno and Kang expanded the model by introducing techniques to represent nonlinear functions as interpolation of simpler functions.

The key advantage of this approach is that it allows for the design of linear controllers for each subsystem and the use of Parallel Distributed Compensation (PDC) to regulate the overall nonlinear system. However, the fuzzy identification process, which is used to construct T-S models, still faces significant challenges, especially regarding the issue of singularity. In many cases, fuzzy identification fails to accurately represent the original nonlinear model, which can lead to control system errors and negatively impact its performance.

A challenge with fuzzy identification is how the unit is divided into fuzzy sets, especially with triangular membership functions, which are the most common. This split can cause problems when identifying systems with a batch of data due to uniqueness in the regression matrix.. References [15]–[23] propose ways to handle or avoid this problem. It's also an issue in type-2 fuzzy systems, with solutions using predictive and neuro-fuzzy strategies [24]–[29], but these approaches can be costly in terms of operations and computation.

The primary objective of this work is to propose a novel approach for improving the fuzzy identification of nonlinear systems, specifically focusing on eliminating singularity in the identification process. Singularity in fuzzy identification occurs when the system of equations used to estimate the model parameters does not have a unique or stable solution, resulting in uncertainties in control and an increased likelihood of failures. This problem is particularly acute in industrial applications, where system stability and reliability are critical. By enhancing the identification process, our approach enables a more accurate representation of the nonlinear system, leading to more robust and efficient control.

Moreover, this work introduces a least squares error cost function for the identification process. This optimization criterion minimizes the difference between the predicted and actual outputs of the system, ensuring that the identified model closely fits the real system's dynamic behavior. Minimizing the mean squared error not only improves the model's accuracy but also reduces the likelihood of control system failures (see [30]), which is particularly important in industrial settings where control errors can result in costly downtime or equipment damage.

The state of the art in nonlinear control has seen the emergence of various approaches aimed at improving both identification and control. For instance, recent works have proposed using Linear Matrix Inequalities (LMI) to verify the stability of T-S fuzzy models [31]–[33]. While these approaches have proven effective in validating system stability, they do not directly address the problem of singularity in identification. Additionally, advanced techniques have been proposed for tuning membership functions and automatically generating fuzzy rules from empirical data [32]–[38]. Although these techniques have improved the performance of fuzzy controllers, there remains a need for more robust and practical methods that effectively resolve the singularity problem.

This work positions itself at the intersection of these research lines by proposing a method that not only enhances the accuracy of fuzzy identification but also solves the singularity problem in the regression matrix that arises during the identification process. Our approach, by incorporating an identity matrix scaled by a power factor with initial conditions, ensures that the system of equations is complete and that the model parameters can be estimated stably and accurately. In doing so, it prevents singularity from affecting system performance, significantly improving control robustness.

One of the key contributions of this work is the experimental validation of the proposed methodology through its application to the control of an aero-pendulum system. This system, known for its nonlinear dynamics and complexity in terms of control, serves as an ideal case study for demonstrating the effectiveness of the improved identification and PDC control strategy (see examples [39]–[44]). The experimental results show that the proposed approach not only improves system stability and controllability but also reduces identification error and optimizes overall controller performance. Other control techniques applied to the aeropendulum can be seen in [45]–[54].

The contributions of this research are two. First, the development of an improved fuzzy identification methodology that provides a closer approximation to the original nonlinear model, effectively minimizing control system failures. Second, the integration of this identification technique with PDC control strategies, which has been experimentally validated on an aero-pendulum system, demonstrating its applicability in real-world scenarios. These contributions position this work as a significant advancement in the field of nonlinear system control, with practical implications for industrial environments.

II. Methods

The original fuzzy identification method, applied to dynamic systems, is implemented using an input-output dataset, the dataset to be used will have a sampling time $\Delta t = tk$ seconds, for each $k = 1, 2, 3, \ldots, n$ samples. The input $u[k]$ should have sufficient frequency variations to model a complex plant. High-frequency input signals are optimal for estimating dynamic systems because they can excite a wider range of the system's modes, enabling a more thorough characterization of its behavior. These signals offer greater resolution in the frequency spectrum, helping to avoid issues such as resonance or parameter correlation that may arise with low-frequency signals. Moreover, they enhance the system's observability and controllability, facilitating precise parameter identification and the capture of transient dynamics that might be difficult to detect with lower frequencies.

The T-S fuzzy system is structured with R common if-then rules, where the consequents of the fuzzy model are difference equations. To represent the system dynamics, a common rule can be constructed in the following form:

R_i: If $y[k]$ is A_1^L and $y[k-1]$ is A_2^M and ...$y[k-n+1]$ is A_n^N, then:

$$y^i[k+1] = a_0^i + a_1^i y[k] + a_2^i y[k-1] + \ldots + a_n^i y[k-n+1] + \ldots$$

$$\ldots + b_1^i u[k] + b_2^i u[k-1] + \ldots + b_m^i u[k-m+1]$$

Where a and b are the coefficients of each difference equation. There is a term independent of the system dynamics, which accounts for the lag that the models might perceive.

The crisp output of the fuzzy system when evaluating each of its rules is defined by:

$$y[k+1] = \frac{\sum_{i=1}^{R} \hat{y}^i u_i(y[k], y[k-1], \ldots, y[k-n+1])}{\sum_{i=1}^{R} u_i(y[k], y[k-1], \ldots, y[k-n+1])} \quad (1)$$

Where u_i refers to the degree of firing of each fuzzy rule, referring to the present state of the system and the past states.

How many past states to use depends on the system's complexity, the balance between accuracy and simplicity, and the nature of the process. A more complex system may require more past states to adequately capture its dynamics, while using too many can increase complexity and the risk of overfitting. Typically, the selection is made through experimentation, testing different configurations and choosing the one that best fits the system's behavior.

Whereas that, $\hat{y}^i$ is the clear output of the consequent of each rule. The expression is rewritten as:

$$y[k+1] = q_1 * \xi_1(u_1) + q_2 * \xi_2(u_2) + ... + q_R * \xi_R(u_R) \quad (2)$$

Where ξ_i is called base functions of the fuzzy model, these functions refers to the membership functions that describe how input values are associated with fuzzy sets. These functions, which can take different shapes like triangular, trapezoidal, or Gaussian, define the degree of membership of an input to a specific set, It is calculated by dividing the firing strength of a rule by the total firing strengths of all the rules.

$$\xi_i = \frac{u_i(y[k], y[k-1], ..., y[k-n+1])}{\sum_{i=1}^{R} u_i(y[k], y[k-1], ..., y[k-n+1])} \quad (3)$$

This being a simple value to determine. The output of each fuzzy rule will be:

$$y[k+1] = a_0 + a_1 y[k] + a_2 y[k-1] + ... + a_n y[k-n+1] + \\ b_1 u[k] + b_2 u[k-1] + ... + b_m u[k-m+1]$$

By combining this expression with equation (2), the following is obtained:

$$y[k+1] = (a_0^1 + a_1^1 y[k] + .. + a_n^1 y[k-n+1] + b_1^1 u[k] ... + \\ b_m^1 u[k-m+1])\xi_1 + (a_0^2 + a_1^2 y[k] + ... + \\ a_n^2 y[k-n+1] + b_1^2 u[k] ... + b_m^2 u[k-m+1])\xi_2 + \\ + ... + (a_0^R + a_1^R y[k] + .. + a_n^R y[k-n+1] \\ + b_1^R u[k] ... + b_m^R u[k-m+1])\xi_R$$

It can be expressed in matrix form as:

$$y[k+1] = \phi\theta \quad (4)$$

where:

$$\phi = [a_0^1 \; a_1^1 \; ... \; a_n^1 \; a_0^2 \; a_1^2 \; ... \; a_n^2 \; ... \; a_0^R \; ... \; a_n^R \\ b_1^1 \; ... \; b_m^1 \; b_1^2 \; ... \; b_m^2 \; b_1^R \; b_m^R]$$

The matrix contains the parameters to be estimated, specifically the coefficients of the consequents of each rule. Meanwhile, θ is known as the regressor, representing the matrix formed by the base functions of the fuzzy model and the system states.

$$\theta = [\xi_1 \;\; \xi_1 y[k] \;\; \xi_1 y[k-1] \; ... \; \xi_1 y[k-n+1] \\ \xi_2 \;\; \xi_2 y[k] \;\; \xi_2 y[k-1] \; ... \; \xi_2 y[k-n+1] \; ... \\ \xi_R \;\; \xi_R y[k] \;\; \xi_R y[k-1] \; ... \; \xi_R y[k-n+1]^T$$

Since the estimation will be performed using a least squares regression, the previously obtained dataset will be utilized. A more straightforward interpretation of the matrices θ and ϕ can be achieved by grouping them according to each rule, as follows:

$$\Theta = [\theta_1 \quad \theta_2 \quad .... \quad \theta_R] \; \text{1xR}$$

$$\phi(j) = [\phi_1(j) \quad \phi_2(j) \quad .... \quad \phi_R(j)] \quad \text{1xR}$$

Where, j is the sample number of the batch of data being analyzed, if k samples are available, the result is:

$$\Phi = [\phi_1(1) \quad \phi_2(1) \quad .... \quad \phi_R(1); \\ \phi_1(2) \quad \phi_2(2) \quad .... \quad \phi_R(2); \\ . \\ . \\ . \\ \phi_1(k) \quad \phi_2(k) \quad .... \quad \phi_R(k)] \quad \text{kxR}$$

The batch of system outputs turns out to be:

$$Y = \Phi\Theta \quad (5)$$

The cost function is the minimum square error, so we seek to minimize:

$$V = \frac{1}{2}\sum_{j=1}^{k}[y(j) - \phi(j)\theta^T]^2 \quad (6)$$

To reduce the sum, the combined matrices are used and to solve the square of matrix operations, it is multiplied by the transpose:

$$V = \frac{1}{2}(Y - \Phi\Theta^T)^T(Y - \Phi\Theta^T) \quad (7)$$

The cost function is minimized if: $\frac{\partial V}{\partial \Theta} = 0$

$$\Theta^T = (\Phi^T\Phi)^{-1}\Phi^T Y \quad (8)$$

This method has a clear limitation: if unit partition sets are used in the fuzzy universes, a linear dependence between the columns of the matrix ϕ will be generated. This can be easily demonstrated; let us consider two fuzzy universes, each consisting of two sets, as shown in Fig. 1, if we calculate the basis functions for an arbitrary value of the system states, for example, $y[k] = y[k-1] = 55$, the resulting basis functions are:

$$\begin{gathered} \xi_1 = u_1(y[k]) * u_1(y[k-1]) = (0.8) * (0.8) = 0.64 \\ \xi_2 = (0.8) * (0.2) = 0.16 \\ \xi_3 = (0.8) * (0.2) = 0.16 \\ \xi_4 = (0.2) * (0.2) = 0.04 \end{gathered} \quad (9)$$

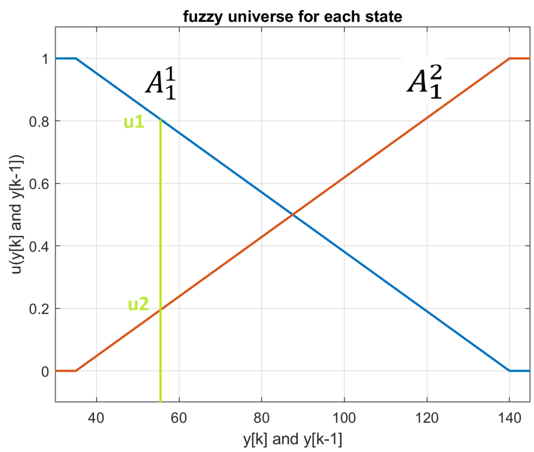

Fig. 1. input fuzzy sets

The basis functions are obtained directly using the T-norm of the fuzzy sets, since the denominator of the equation (3) is one in all cases due to the unit partition of the fuzzy sets.

Therefore, a portion of the matrix θ from equation 4 is as follows:

$$\theta = \begin{matrix}[\xi_1 & \xi_1^* y[k] & .... & \xi_2 & \xi_2^* y[k] \\ & .... & \xi_3 & \xi_3^* y[k] & .... &]\end{matrix} \tag{10}$$

Numerically, the example yields:

$$\theta = \begin{matrix}[0.64 & 0.64 * y[k] & .... & 0.16 * y[k] & .... \\ 0.16 & 0.16 * y[k] & .... &]\end{matrix} \tag{11}$$

From the equation (11), it is observed that there is a linear dependence between the third and fifth terms (of the simplified matrix) for the same value of $y[k]$. In other words, the matrix Φ is not full-rank, and therefore the expression $\Phi^T\Phi$ would not be invertible.

Although the matrix Φ is non-singular, the expression presents an incomplete solution. This is because the matrix $\Phi^T\Phi$ will be square with dimensions corresponding to the reduced rank. Consequently, the expression 8 will yield the number of valid coefficients equal to the rank of the matrix Φ, and it will complete the matrix with values close to singularity at the points of conflict.

A small number of fuzzy sets and rules can lead to singularity in the regression matrix, as it may not provide enough diversity in the inputs or adequate coverage of the input space, causing linearity or redundancy issues in the system. However, the advantage of using fewer sets and rules is the reduction in computational cost, since fewer operations and less memory are required to run the model. Therefore, there is a trade-off between system complexity (more sets and rules to avoid singularity) and computational efficiency, which should be considered when designing the fuzzy system.
Como

Adding an identity matrix multiplied by a factor γ can help prevent singularity in some cases, and this method is commonly known as regularization. It is used to stabilize ill-posed problems or when a matrix is near singular, meaning its determinant is close to zero.

The idea is that by adding γI (where I is the identity matrix and γ is a small positive factor), the eigenvalues of the resulting matrix are shifted away from zero, reducing the likelihood of the matrix being singular or nearly singular. This identity matrix must have dimensions equal to the number of columns of the original matrix Φ.
The cost function is given by:

$$V = \frac{1}{2}\sum_{j=1}^{k}[y(j) - \phi(j)\theta^T]^2 + \gamma^2 \sum_{j=1}^{R}\theta_j^2 \tag{12}$$

And in matrix form, it is expressed as:

$$V = \parallel Y - \Phi\Theta \parallel^2 + \gamma^2 \parallel \Theta \parallel^2 \tag{13}$$

$$V = \parallel Y_a - \Phi_a\Theta \parallel^2 \tag{14}$$

After extending the regression matrix, the number of rows in the matrix increases and, therefore, the number of outputs in the batch of data. Since there is no system output information available for the samples in the extended matrix, the batch of outputs will be filled with a column vector of zeros of the same size as the identity matrix.

$$Y_a = \begin{bmatrix} Y \\ 0 \end{bmatrix} \tag{15}$$

and,

$$\Phi_a = \begin{bmatrix} \Phi \\ \gamma I \end{bmatrix} \tag{16}$$

With this extension, the coefficient matrix Θ is given by:

$$\Theta_a^T = (\Phi_a^T\Phi_a)^{-1}\Phi_a^T Y_a \tag{17}$$

This expression ensures that the matrix Φ_a is full-rank, thereby providing an effective solution. Naturally, this expression minimizes the estimation error as a function of the value of γ. The smaller the perturbation, the lower the error, but it also brings the matrix closer to singularity.

The choice of γ in regularization depends on balancing stability and accuracy. Commonly, it is selected through cross-validation or trial and error, testing different values and choosing the one that minimizes generalization error. It can also be adjusted based on the conditioning of the matrix or using heuristics specific to the problem. In some cases, automatic methods like the Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC) can help determine γ optimally.

A simple way to determine γ to avoid singularity in a fuzzy estimation process is to choose it as a small percentage of the largest eigenvalue of the matrix being regularized. This ensures that the resulting eigenvalues are not close to zero, preventing singularity without introducing excessive bias. A practical approach is to select $\gamma = \epsilon \cdot \lambda_{\max}$, where ϵ is a small factor, such as 0.01. This method is quick and effective for stabilizing the system without significantly affecting accuracy.

To improve the estimation and seek an optimal solution, this work proposes starting from the expression 17 and using the coefficients obtained in the first iteration instead of the zero outputs presented in equation (19), resulting in the following minimization expression:

$$\Theta_b^T = (\Phi_b^T \Phi_b)^{-1} \Phi_b^T Y_b \tag{18}$$

Where:

$$Y_b = \begin{bmatrix} Y \\ \Theta_a \end{bmatrix} \tag{19}$$

Meanwhile, the matrix Φ_b will follow the form shown in (16), with the minimization factor γ.

III. Results

A dynamic model of an aeropendulum will be built to demonstrate the fuzzy identification of Takagi - Sugeno inference applied to a nonlinear system. The aeropendulum is more efficient than an industrial system for testing nonlinear system identification methods due to its simplicity, low cost, and safety. Its simpler structure allows for easier modeling and control, while its operation is more economical and safer compared to the risks associated with industrial systems. Additionally, it enables the quick implementation of experimental techniques and provides a controlled environment for obtaining consistent results. The fuzzy identification method proposed in this work can be applied to any type of system, as long as key factors are considered: using a low number of fuzzy rules and fuzzy sets, and extending the regression matrix through a regularization method to avoid singularity issues.

It begins with obtaining the non-linear model of the system to obtain identification data. According to Newton's second law for rotational movements, the following equation is derived, which models the aeropendulum.

$$I\ddot{\theta} = -mgl\sin(\theta) - b\dot{\theta} + T \tag{20}$$

where:

- $mgl\sin(\theta)$ is the torque due to gravity.
- $B\dot{\theta}$ is the torque due to friction.
- T is the torque generated by the propeller, which can be expressed as $T = k_m u$, where δ represents the duty ratio of the PWM signal sent to the motor ESC.

As can be seen, there is a dependent variable as part of a coefficient of the model, which generates the nonlinearity.

Since the motor used in the aeropendulum is a brushless motor and knowing that its control is carried out through an ESC, the input signal was adapted, where k_m, which was initially a constant of proportionality, now turns out to be a simple function of the relationship of work (δ) sent to the ESC.

$$T = 8\delta - 8 \tag{21}$$

This expression relates the pulse width of the PWM signal generated by the controller to the torque of the model, enabling a direct identification between the control action and the angle of the aeropendulum.

This model was used to obtain the training data, based on an input with high dynamic content. In Fig. 2 you can see the input data, referring to the pulse width of the signal sent to the motors (values between 1 and 2 for brushless motor control) and as output (θ) the pendulum angle.

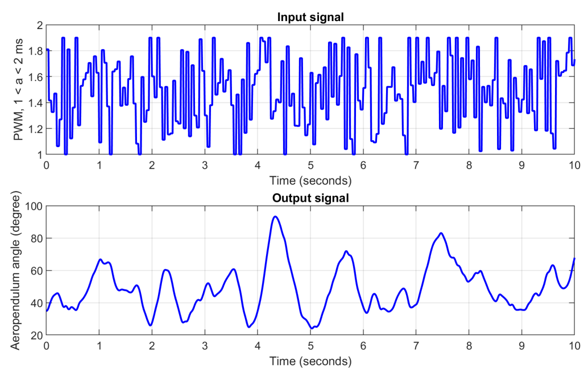

Fig. 2. Trainig data

In this work, only two state values are used: the current state and the previous one. As mentioned in the methods section, this choice is arbitrary and directly impacts the algorithmic complexity of the fuzzy system. Using two states results in a minimum of 4 rules, while three states would require at least 6 rules, and so on.

The physical limits of the system generate the identification domain of the model, in this case the aeropendulum can move between 35 and 135 degrees. Each state is associated with two fuzzy sets with unit partitions as seen in Fig. 1.

Similarly, the smallest possible number of fuzzy sets is chosen to reduce algorithmic complexity. Using 2 fuzzy sets per state results in 4 rules, 3 sets per state leads to 9 rules, and so on. A lower number of states and sets increases the likelihood of achieving uniqueness in the identification process.

The input sets will be identical for each state of the system, giving rise to 4 fuzzy rules:

R1: If $y[k]$ is A_1^1 and $y[k-1]$ is A_2^1 then:

$$y[k+1] = a_0^1 + a_1^1 y[k] + a_2^1 y[k-1] + b_0^1 u[k]$$

R2: If $y[k]$ is A_1^1 and $y[k-1]$ is A_1^2 then:

$$y[k+1] = a_0^2 + a_1^2 y[k] + a_2^2 y[k-1] + b_0^2 u[k]$$

R3: If $y[k]$ is A_1^2 and $y[k-1]$ is A_2^1 then:

$$y[k+1] = a_0^3 + a_1^3 y[k] + a_2^3 y[k-1] + b_0^3 u[k]$$

R4: If $y[k]$ is A_1^2 and $y[k-1]$ is A_2^2 then:

$$y[k+1] = a_0^4 + a_1^4 y[k] + a_2^4 y[k-1] + b_0^4 u[k]$$

From the gathered rules, it can be observed that there are 16 coefficients to estimate. Once the regressor is constructed and the mean squared error is minimized by applying equation (8), the following consequents are obtained:

R1: $y[k+1] = -0.51 + 1.94y[k] - 0.94y[k-1] + 0.51u[k]$

R2: $y[k+1] = -0.06 + 1.86y[k] - 0.89y[k-1] + 0.56u[k]$

R3: $y[k+1] = -0.94 + 2.01y[k] - 0.99y[k-1] + 0.39u[k]$

R4: $y[k+1] = -0.36 + 1.92y[k] - 0.92[k-1] + 0.51u[k]$

By applying the extended cost function 17, the following consequent functions were obtained:

R1: $y[k+1] = 0.38 + 1.59y[k] - 0.62y[k-1] + 0.48u[k]$

R2: $y[k+1] = -0.09 + 0.99y[k] - 0.90y[k-1] + 0.55u[k]$

R3: $y[k+1] = -0.05 + 2.15y[k] - 0.24y[k-1] + 1.05u[k]$

R4: $y[k+1] = 0.02 - 0.68y[k] + 1.68[k-1] - 0.60u[k]$

In this work, the value of γ was selected by taking the maximum eigenvalue of the matrix and multiplying it by a scaling factor. This approach ensures that the regularization term is appropriately adjusted to prevent singularity, while maintaining the balance between system stability and accuracy in the identification process.

Finally, by applying the proposed function with a double identification iteration, the following functions for the consequents of the fuzzy system were obtained:

R1: $y[k+1] = -0.63 + 1.94y[k] - 0.94y[k-1] + 0.53u[k]$

R2: $y[k+1] = 4.54 + 1.78y[k] - 0.97y[k-1] + 0.37u[k]$

R3: $y[k+1] = -6.37 + 1.91y[k] - 0.72y[k-1] + 0.72u[k]$

R4: $y[k+1] = -0.73 + 1.77y[k] - 0.78[k-1] + 0.44u[k]$

In the vast majority of the estimated coefficients across the three applied methods, no significant variations are observed, except in certain values where the singularities of the original Takagi-Sugeno estimation method would clearly be found.

The three systems were tested with a unit step input to verify their response and proximity to the original nonlinear model.

Fig. 3 shows that only the model estimated using the approach proposed in this work aligns with the nonlinear model in aspects such as the system's static gain K_e and the settling time t_s.

The start-up of the nonlinear system is delayed due to its physical limitations, as the nonlinear model exhibits a characteristic dead zone. In contrast, the identified models begin at the same time as the transient response, since they are an interpolation of linear models, which typically do not have dead zones except in the case of zero input. Despite the difference in the dead zone of the model, the rise time matches well for both the extended simple model and the approach proposed in this work.

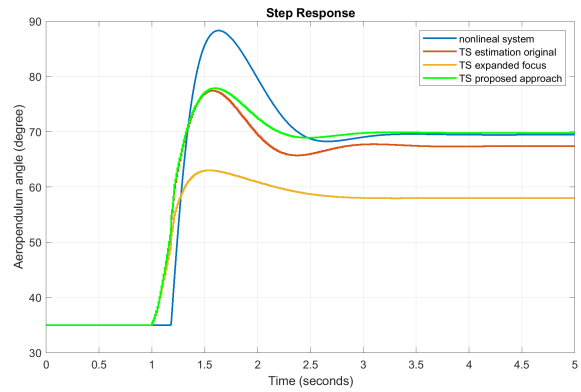

Fig. 3. Testing of the systems with a unit step input

Fig. 4 illustrates the error between each estimated fuzzy model and the nonlinear model of the aero-pendulum. The approach proposed in this work achieved a lower RMSE compared to the other two evaluated methods, indicating that the model identified using this method provides a better fit to the real data of the nonlinear system. This demonstrates that the proposed approach captures the system's dynamics more accurately, reducing prediction errors and offering a more precise estimation compared to the other methods.

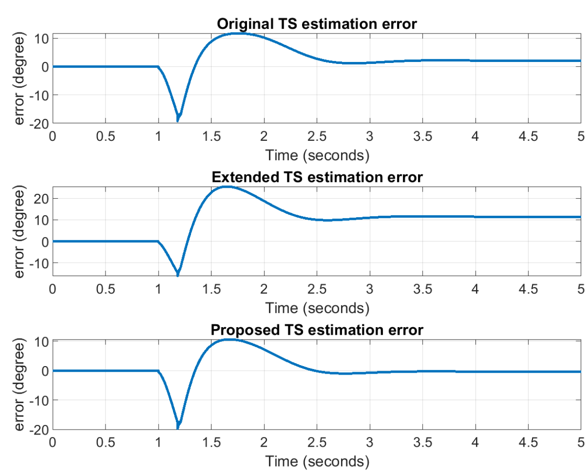

Fig. 4. a) Original TS estimation rmse = 5.34, b) Extended TS estimation rmse = 12.23, c) Proposed TS estimation error = 4.63

The ANOVA analysis shows statistically significant differences between the errors of the three models ('Original', ‘Extended‘, and ‘Proposed‘), with an extremely low p-value (3.39×10^{-303}), which is far below the significance level of 0.05. This indicates that at least one of the models differs significantly from the others in terms of error (see Fig. 5). The high F-statistic value ($F = 939.28$) reinforces this conclusion, demonstrating that the variability between the errors of the different models is considerably greater than the variability within each group of errors. In summary, the results suggest that the errors between the models are not equal, and further comparisons are needed to identify which ones are significantly different.

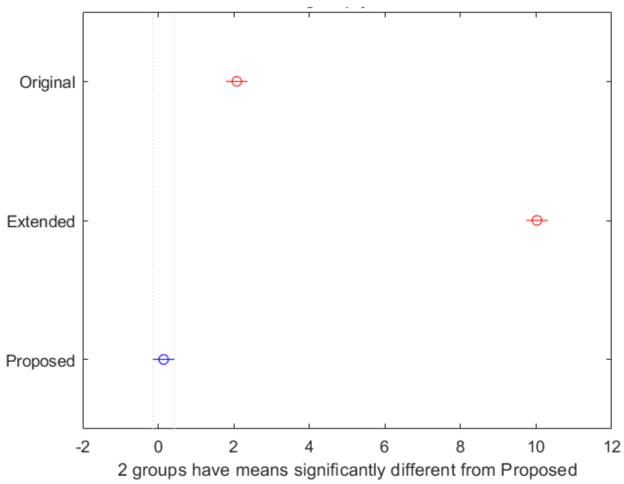

Fig. 5. Boxplot of the error comparison between the Original, Extended, and Proposed models, illustrating the variability and distribution of errors across the three models based on the ANOVA analysis.

A second test was conducted to verify the validity of the models, this time generating step signals of various values over a period of 20 seconds, with more than two thousand samples. The system responses can be seen in Fig. 6.

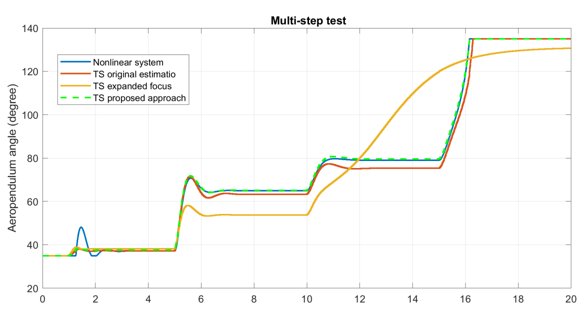

Fig. 6. Testing the models with multiple step inputs

In the multi-step test graph, the responses of the real non-linear system are compared with three estimation models: the original Takagi-Sugeno (TS) model, the expanded TS model, and the proposed approach. The nonlinear system serves as a reference to assess the accuracy of these models.

The proposed approach (green dashed curve) shows the best alignment with the nonlinear system, accurately following the step changes and capturing the transitions effectively. The original TS model (red curve) also performs well, but it exhibits slight delays and overestimations in some transitions, particularly around 7 and 15 seconds.

On the other hand, the expanded TS model (orange curve) shows the largest deviations from the real system, with slower rise times and a response that remains significantly below the reference during the steps. In conclusion, the proposed approach provides the best approximation to the dynamics of the nonlinear system, outperforming the other two evaluated models.

Fig. 7 again shows the evolution of the error for each model relative to the nonlinear model. In this case, the similarity of the model estimated with the proposed approach is even more evident, particularly when the system reaches instability.

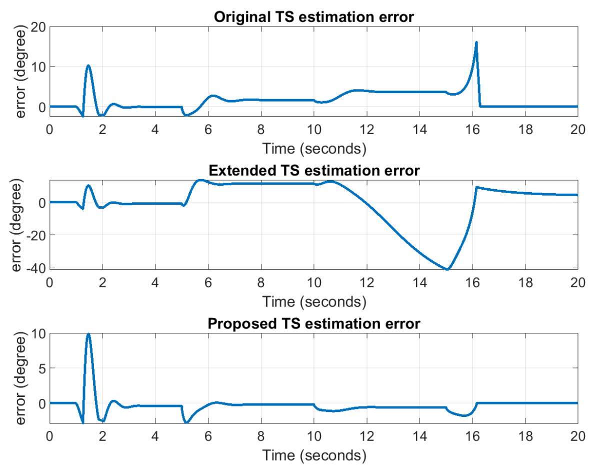

Fig. 7. a) Original TS estimation rmse = 2.818, b) Extended TS estimation rmse = 13.8, c) Proposed TS estimation error = 1.28

Finally, a test was conducted with a sinusoidal input at a frequency of 15 rad/s to verify the response speed of the models. The results can be seen in Fig. 8.

This test allows for evaluating the model’s response at different frequencies, checking its ability to handle smooth and continuous variations, and verifying the system’s linearity and robustness. Additionally, by varying the frequency of the signal, different dynamic behaviors can be analyzed, providing a more comprehensive validation of the model across a wide range of operating conditions.

The error comparison is presented in Fig. 9, where it is observed that the lowest mean squared error is exhibited by the model with the extended identification. This is due to the significant oscillation in the response around the equilibrium point. The lower RMSE of the extended model in the sinusoidal test suggests that this model is better suited for capturing the system dynamics under oscillatory or frequency-specific signals. Although the proposed model also performs well, its intermediate RMSE indicates that it strikes a good balance but may not be as specialized in handling sinusoidal inputs as the extended model. The original model, having the highest RMSE, appears to struggle more with accurately following the sinusoidal variations. This highlights the importance of evaluating models with different input types to fully understand their behavior and strengths.

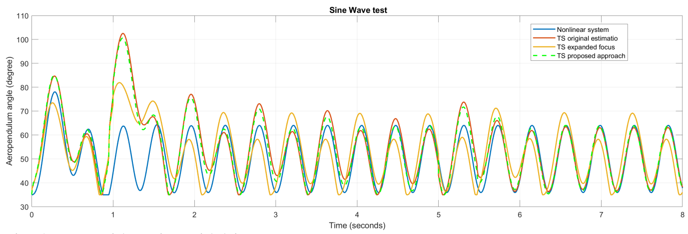

Fig. 8. Test with a sinusoidal input

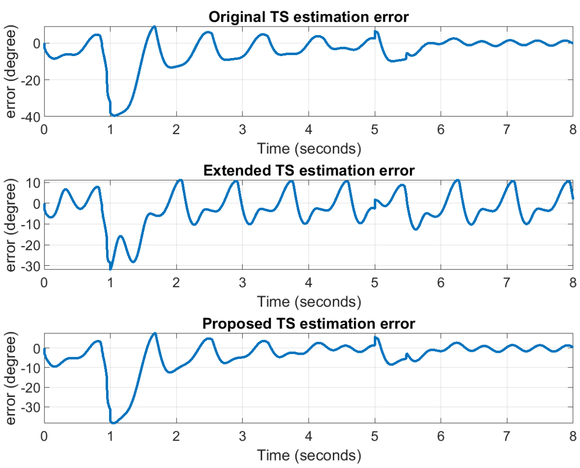

Fig. 9. a) Original TS estimation rmse = 9.98, b) Extended TS estimation rmse = 8.72 , c) Proposed TS estimation error = 9.15

A final validation of the proposed model was carried out using a step input with added noise. This test was designed to evaluate the robustness of the model under noisy conditions, ensuring that it can maintain accurate performance even when the input signal is subject to perturbations (see Fig. 10).

The Fig. 11 shows the error evolution of the three identified models. The RMSE results demonstrate that the proposed model significantly outperforms both the original and extended TS models. With an RMSE of 4.21, the proposed model provides the most accurate estimation, while the original TS model, with an RMSE of 6.57, performs moderately well. In contrast, the extended TS model shows the highest error, with an RMSE of 31.65, indicating that it is the least effective in capturing the system's dynamics. These results confirm the superiority of the proposed model in terms of accuracy and robustness.

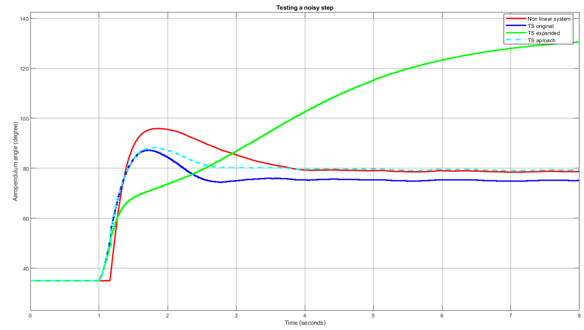

Fig. 10. Response of the proposed model to a noisy step input, demonstrating the model's robustness and ability to accurately estimate the system dynamics under noisy conditions.

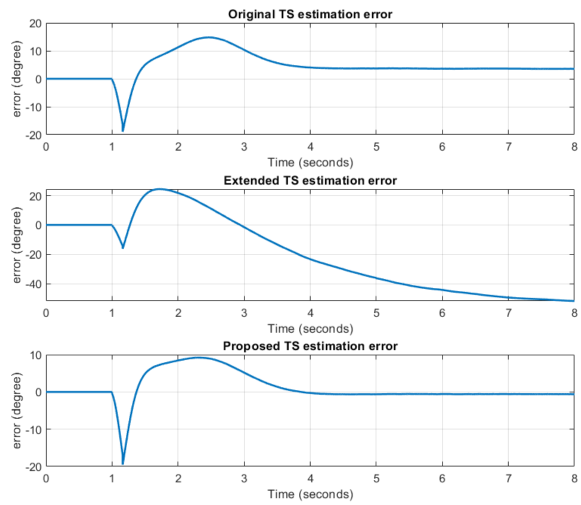

Fig. 11. a) Original TS estimation rmse = 6.57, b) Extended TS estimation rmse = 31.65 , c) Proposed TS estimation error = 4.21

A. PDC Control

To design the nonlinear fuzzy controller, we used the consequents of the rules, thereby converting these difference equation expressions into discrete state models. An example is shown for the consequent of the first rule:

$$y[k+2] + 0.63 - 1.94y[k+1] + 0.94y[k] = 0.53u[k] \quad (22)$$

The expression in discrete state space is:

$$\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} = \begin{bmatrix} -0.63 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.94 & 1.94 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + \begin{bmatrix} 0.53 \\ 0 \end{bmatrix} u[k]$$

It can be observed that the systems include a term without dynamics, referred to as the affine term, which is responsible for interpolating the discrete linear systems of the fuzzy model. If tuning a regulator is desired, this offset value of the models would need to be compensated. However, since this is a servomechanism aiming to maintain the system at a reference value, such compensation is not necessary.

Once all the discrete models are obtained from the consequent estimations, the next step is to tune as many controllers as there are rules in the system. The simplest approach is to apply the Riccati optimal controller, adjusting the weights as needed, without considering the affine terms, as they do not exhibit any dynamics. Additionally, an integrator stage should be applied to ensure that the controlled variable remains at the reference value.

Using a tuning method based on the Riccati equation provides an optimal, stable, and robust solution for controller design in complex systems. Its ability to balance precision and control effort, its applicability to multivariable systems, and the flexibility in defining control objectives make it a preferred choice over other tuning methods, especially in applications where stability and efficiency are crucial.

The general structure of the servomechanism is shown in Fig. 12, and this controller is designed for each rule of the fuzzy system.

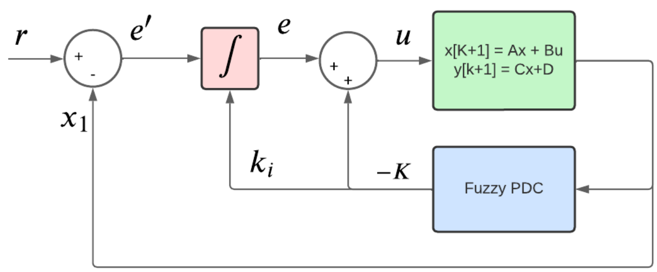

Fig. 12. Diagram of the structure of a controller

The integrative gain is obtained by extending the state model matrices in the following form:

$$A_{ext} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}; \qquad B_{ext} = \begin{bmatrix} B \\ 0 \end{bmatrix} \tag{23}$$

When tuning the controller using the Riccati optimal method, the feedback gains and the integrator gain are obtained as follows:

$$K = \begin{bmatrix} k_1 & k_2 & -k_i \end{bmatrix} \tag{24}$$

Therefore, the control law is ultimately:

$$u[k] = -Kx + k_i e \tag{25}$$

Finally, the PDC fuzzy controller has the same background as the identified model, the difference is that now the consequents are the state feedback gains that will be used to improve and regulate the dynamics of the plant.

R1: $u[k] = -38.97x_1 - 28.81x_2 + 89.48e$

R2: $u[k] = -43.5x_1 - 30.67x_2 + 91.74e$

R3: $u[k] = -35.4x_1 - 27.5x_2 + 86.97e$

R4: $u[k] = -41.2x_1 - 30.2x_2 + 90.8e$

The PDC controller associated with the original nonlinear model of the aero-pendulum was tested with a step input to bring it to a 90-degree position relative to the vertical, yielding the following result, shown in Fig. 13:

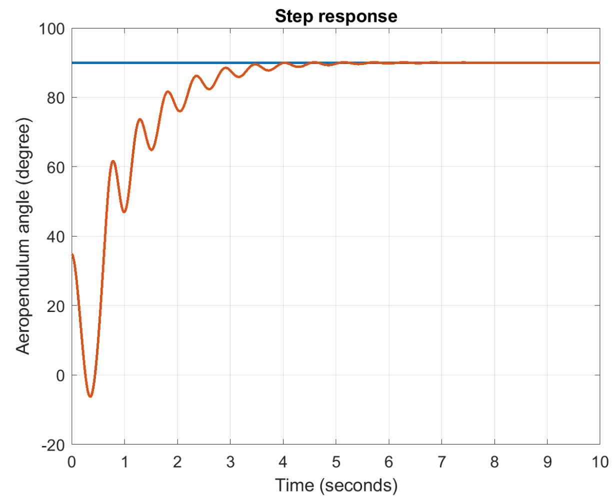

Fig. 13. Step Test, ts aprox = 4.2 s, Ep = 0%

The control law (see Fig. 14) exhibits the expected behavior for a system with moderate damping and overshoot, achieving long-term stability. However, it could benefit from adjustments to reduce the overshoot and improve the stabilization time.

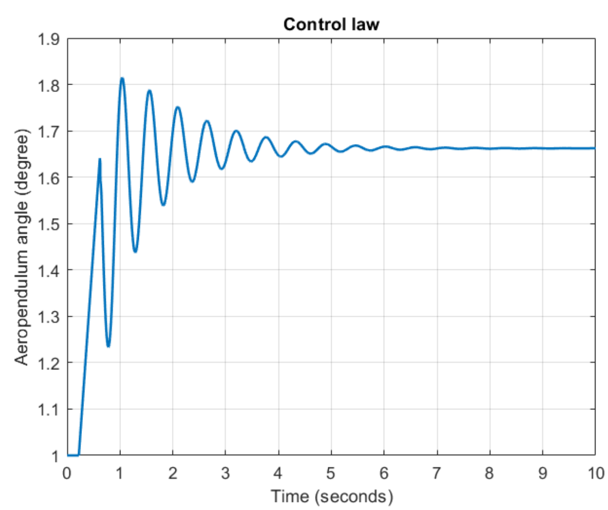

Fig. 14. Control law response for the aeropendulum system using Parallel Distributed Compensation (PDC), showing moderate damping and overshoot, with long-term stability.

The curve exhibits a damped oscillatory behavior that begins with a rapid increase (possible overshoot) and then oscillates with progressively decreasing amplitude until it stabilizes at a pulse width close to 1.7 milliseconds.

The system appears to reach stability around 5 seconds, which is a good indicator that the controller successfully stabilizes the system.

The curve shows that the highest energy consumption occurs during the initial oscillations, when the controller applies significant effort to correct the overshoot and stabilize the system. As the system stabilizes, energy consumption decreases, reducing to the minimum required in the steady state.

A second test was performed by adding white noise to the system output to verify compensation. The curve (see Fig. 15) shows the output response of a system controlled by PDC (Parallel Distributed Compensation) with added noise. In the first few seconds, an oscillatory behavior with overshoot is observed, indicating that the system initially exceeds the target value before stabilizing. These oscillations are typical in systems affected by noise and responding to a step input. The overshoot could be an area for improvement if smoother transients are required, as the system momentarily reaches an angle higher than the desired one before correcting its behavior.

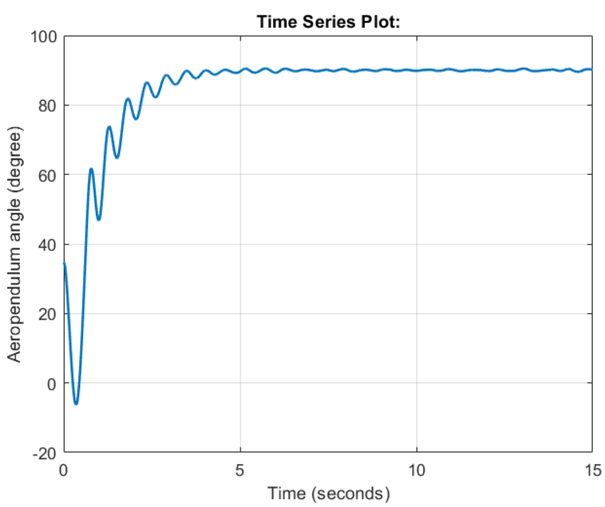

Fig. 15. Output response of the aeropendulum system controlled by PDC with added noise, showing initial overshoot and oscillations before stabilizing around 100 degrees.

Despite the noise, the system stabilizes around 90 degrees, with stability becoming visible around 10 seconds. However, small persistent oscillations in the steady state remain due to the noise, suggesting that while the PDC controller is robust, it is still slightly affected by perturbations.

A final test with multiple reference points demonstrates the efficiency of the control system shown in Fig. 16.

The provided curve shows the system's response to different step inputs. In the first step, the system exhibits a typical damped response with an initial overshoot and minor oscillations before stabilizing around 50 degrees. The system's stabilization time is relatively fast, indicating that the controller effectively adjusts the output to match the desired value. However, the presence of the initial overshoot suggests that the controller could be fine-tuned to reduce this effect and improve energy efficiency.

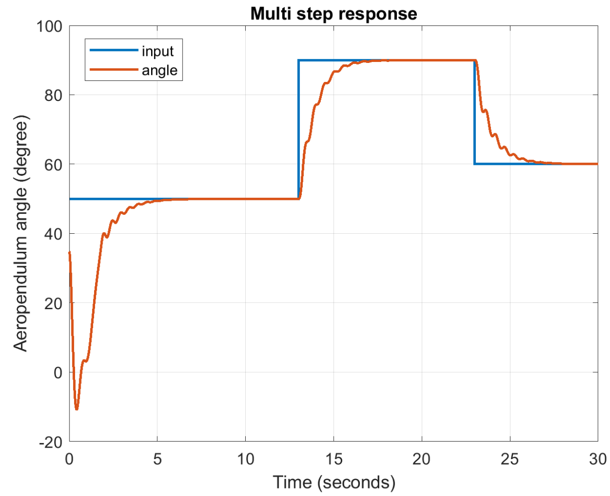

Fig. 16. Multi-step test

In the second and third step changes, the system shows similar behavior, with small overshoots and oscillations before stabilizing. After the second step, the angle reaches around 90 degrees, and when the input decreases, the system experiences a slight undershoot before stabilizing near 60 degrees.

IV. Discussion

The main objective of this work was to present an improved approach for the identification of nonlinear dynamic systems, with a specific focus on enhancing system identification techniques. The addition of the Parallel Distributed Compensation (PDC) control to the aeropendulum was used as a practical application to validate the effectiveness of the proposed identification method. Through the results, it was observed that the identification technique successfully captured the nonlinear behavior of the aeropendulum, as demonstrated by the accuracy of the control response in various test scenarios, including noisy step inputs. The Root Mean Square Error (RMSE) values indicated that the proposed approach for system identification offered superior performance compared to traditional methods like the Takagi-Sugeno (TS) models, both in terms of accuracy and dynamic response.

Specifically, the proposed identification approach achieved an RMSE of 4.21, outperforming the original TS estimation with an RMSE of 6.57, and significantly better than the extended TS estimation, which had an RMSE of 31.65. Additionally, the control tests, which served as an application of the identified model, confirmed that the PDC controller could stabilize the aeropendulum effectively. The system was able to manage external noise while maintaining stability, proving that the model identification process had successfully captured essential system dynamics.

In comparison with previous research, such as [55]–[60], the present study demonstrated similar effectiveness in controlling complex, nonlinear dynamics. However, the main contribution of this study lies in the improved identification process, which enhances the control system's response. Previous studies have typically focused on control performance without placing as much emphasis on the underlying system identification, which is where this study makes a unique contribution.

Furthermore, studies such as [61]–[67] that employed the Takagi-Sugeno models for system identification showed reasonable results, but as the RMSE values in our study demonstrate, the proposed identification approach clearly outperformed the TS models, offering a much more accurate representation of the system's behavior. The success of the PDC controller in stabilizing the system, particularly in the presence of noise, also aligns with findings from other studies that demonstrate the robustness of PDC controllers in nonlinear control applications.

The key implications of this study highlight the importance of accurate system identification as a foundation for effective control. The improvements in RMSE values and the robust handling of noise suggest that the proposed identification approach can be highly beneficial for a variety of nonlinear systems where accurate dynamic modeling is critical. By providing a more accurate model of the aeropendulum's behavior, the controller is better equipped to predict and manage the system's responses, leading to improved stabilization and overall performance.

The application of PDC control to the identified model further confirmed that the enhanced identification process directly benefits practical control applications. The system's ability to handle noise while maintaining stability demonstrates the practical applicability of the proposed identification technique in real-world scenarios, where external disturbances are inevitable. This suggests that the proposed method can be extended to other systems with similar nonlinear dynamics, where robust control and accurate identification are necessary.

One of the strengths of this study lies in its focus on system identification as the primary contribution. While many studies focus primarily on control techniques, this research emphasizes the crucial role of accurate identification in enhancing control performance. The successful application of PDC control serves as a practical validation of the identification process, showing that it can be effectively used in real systems like the aeropendulum.

V. Conclusion

The proposed approach for the identification of nonlinear dynamic systems using fuzzy models has proven to be an innovative and effective solution, addressing critical issues such as matrix singularity. By combining Takagi-Sugeno models with least squares identification techniques, it has achieved a precise representation of complex systems. The elimination of singularity in the identification process represents a significant breakthrough, allowing for greater confidence in the results and more accurate control.

This approach is not only robust in representing nonlinear systems, but when integrated with an optimal controller, it dynamically adjusts system parameters based on operational conditions. This adaptability makes it a powerful solution, capable of effectively responding to environmental changes and system variations, thus enhancing both system performance and stability in real-time applications.

Despite the advances achieved, several future research directions are proposed to further explore and expand the potential of this approach. First, it is essential to compare this method with other novel identification approaches, such as neural networks and optimization algorithms, which have shown great promise in modeling and controlling complex systems. A comparative analysis will provide insight into the advantages and limitations of each method in terms of accuracy and applicability.

Additionally, verifying and comparing the algorithmic complexity of this method against other approaches will be crucial. Algorithmic efficiency is a key factor for practical implementation, particularly in real-time nonlinear systems. A detailed study of computational complexity will help identify optimization opportunities and potential improvements in performance.

Another important area for future study is the evaluation of hardware requirements needed for the efficient implementation of this method. Comparing it with other techniques will help determine the processing and storage demands, ensuring that the system can be deployed on hardware with limited resources, such as embedded devices or real-time systems.

Finally, it is critical to study the system's response in real-time control applications. The ability to quickly adapt to changing operational conditions is vital in industrial, robotic, or energy systems, where response time and stability are paramount. Assessing the system's behavior in real-world environments will validate its applicability and robustness.

In conclusion, the proposed approach not only resolves key identification challenges in nonlinear systems but also provides an adaptable and accurate framework for control across a wide range of applications. However, to maximize its impact and efficiency, it will be essential to continue investigating and comparing this method with other emerging solutions, evaluating its algorithmic complexity, hardware requirements, and real-time performance.

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