# Bipedal Robots: A Systematic Review of Dynamical Models, Balance Control Strategies, and Locomotion Methods

Ibrahim Al-Tameemi [1*], Oger Amanuel [2]
[1, 2] Department of Mechatronics Engineering, Al Khwarizmi College of Engineering, University of Baghdad, Baghdad, Iraq
Email: [1] Ibrahim.s@kecbu.uobaghdad.edu.iq, [2] Awkar.zia1202a@kecbu.uobaghdad.edu.iq
*Corresponding Author

*Abstract*—**Bipedal robots, designed to replicate human locomotion, face significant balance challenges due to instability and high degrees of freedom. This study examines dynamical models, balance control strategies, and locomotion methodologies. Dynamical models are categorized into simplified, centroidal dynamics, and whole-body dynamics models. Simplified models, such as the Linear Inverted Pendulum Model (LIPM), approximate the robot as a point mass at the Center of Mass (CoM) but neglect upper-body dynamics and complex terrain interactions. Centroidal dynamics models incorporate CoM motion, contact forces, and angular momentum for improved disturbance rejection but require extensive computational resources. Whole-body models achieve high fidelity by integrating joint torques and external forces but are constrained by computational complexity. Balance control methods for standing bipedal robots are classified into joint-specific and whole-body approaches. Ankle and hip strategies address small perturbations but are insufficient for real-world disturbances. Whole-body control utilizes all body segments to modulate contact forces and regulate momentum, enhancing stability against external disturbances, though challenges remain in force modeling and state estimation. Locomotion control is divided into model-based and learning-based approaches. Model-based strategies include LIPM and its extensions-based methods, Zero Moment Point (ZMP)-based methods, which ensure dynamic stability by maintaining moments within the support polygon; Capture Point (CP)-based methods, which predict foot placement to prevent falls; and Divergent Component of Motion (DCM)-based approaches, which adjust footsteps based on CoM divergence. While learning-based methods leverage Reinforcement Learning (RL) and human motion data for adaptive and energy-efficient gait generation. This study highlights challenges in energy efficiency, terrain adaptation, and scalability, proposing sensor fusion, energy-aware RL reward functions, and hierarchical control architectures as potential solutions.**

***Keywords—Bipedal Robots; Balance Control Strategies; Walking Control Methods; ZMP; CoP; LIPM; Push Recovery Control.***

## I. Introduction

Because of their similar structure and capability, bipedal robots are expected to execute human-designated applications. Unlike robotic arms, which are restricted to a fixed workspace, bipedal robots can traverse diverse terrains, giving them a significant advantage in real-world applications. For instance, in industrial settings, they can maneuver through crowded spaces to transport materials or perform inspection tasks. In healthcare, they can assist with patient mobility, rehabilitation, and caregiving duties. Additionally, in search and rescue operations, their ability to navigate hazardous environments allows them to locate and aid individuals in disaster-stricken areas, making them invaluable in emergency response efforts.

One of the main challenges faced by bipedal robotics is their ability to maintain balance while standing and walking in the presence of disturbances. The complexity arises from the inherent instability of bipedal robotics, which lack a stable configuration. Furthermore, the high number of degrees of freedom makes it difficult to synchronize and control them in order to achieve smooth mobility. Additionally, the dynamics of bipedal robots are highly non-linear, resulting in a high degree of sensitivity to external disturbances. During the past two decades, a substantial number of studies have focused on improving the standing stability of bipedal robots, improving their interaction with diverse environments and their ability to interact with their environment, and ensuring their safe and effective locomotion on uneven surfaces. However, existing reviews primarily focus on isolated aspects of balance control, such as specific control strategies or modeling approaches, without providing a comprehensive comparison across methodologies. Additionally, a standardized evaluation framework is also lacking, making it difficult to compare different balance control methods.

The objective of this paper is to bridge these gaps by providing a structure and in-depth review of balance control methods for bipedal robots. Initially, we conduct a thorough examination of the dynamical models used to represent bipedal robotics. This review covers a variety of models, ranging from simplified to comprehensive whole-body dynamic models. Secondly, we analyze and categorize bipedal robots' balance control strategies during environmental interactions and while standing into two groups: joint control strategies, which utilize specific joints such as ankles or hips to maintain equilibrium amid perturbations, and whole-body control strategies, which calculate the necessary motions throughout the entire robot to ensure stability. Third, we explore the various methodologies proposed in the literature to regulate bipedal locomotion. We categorize these into two groups: model-based walking control methods and learning-based walking control methods.

Fourth, this review identifies ongoing challenges in the field, including adaption to varied terrain, scalability issues, and energy consumption. Current bipedal robots struggle with stability on slippery or blind terrain, existing control methods for small robots do not scale efficiently to larger ones, and the energy consumption has not been adequately investigated. Lastly, we offer some specific recommendations for future research, with the aim of enhancing the practical application and development of bipedal robotics. These include using infrared or ultrasonic sensors for real-time surface classification, applying reinforcement learning to improve navigation, developing hierarchical control architectures to accommodate large-scale robots by adapting trajectory planning without compromising control precision, and utilizing techniques like Model Predictive Control to prioritize energy efficiency.

The structure of this paper is meticulously organized to facilitate a comprehensive understanding of the subject matter, as depicted in Fig. 1. Section 2 delves into the dynamical models employed for elucidating the dynamics of bipedal robots. Section 3 critically evaluates balance control strategies essential for standing robots. Section 4 provides a detailed review and classification of walking control methodologies, distinguishing between model-based and learning-based approaches. Section 5 discusses the prevailing challenges within the field and offers pertinent recommendations for future research. The paper concludes with Section 6, which synthesizes the findings and implications of the study.

## II. DYNAMICAL MODELING OF BIPEDAL ROBOTS

Several dynamical models have been suggested for describing bipedal robot behavior in both standing and walking scenarios. This section provides a comprehensive examination of simplified models, such as the Linear Inverted Pendulum Model (LIPM) and its extensions, as well as more complex and accurate models, such as centroidal dynamics and whole-body dynamics.

### *A. Linear Inverted Pendulum Model (LIPM)*

The LIPM simplifies the analysis of a bipedal robot, consisting of multiple rigid bodies, by treating it as a single mass located at the Center of Mass (CoM). Additionally, a leg with no mass connects the CoM to the Center of Pressure (CoP). LIPM also assumes that the CoM's height is constant during motion (see Fig. 2 (a)). This assumption simplifies the process of developing control algorithms and analyzing stability. However, it fails to account for dynamic behaviors such as running, jumping, or walking on uneven terrain, which necessitate vertical movement of the CoM. To guarantee high processing efficiency, the LIPM neglects the angular momentum generated by the CoM. Initially, [1], [2] introduced the LIPM as a method for generating a stable walking pattern for bipedal robots. The dynamics of the LIPM in the sagittal plane ($x - z$ plane) can be described by a simple linear differential equation as follows:

$$\ddot{x}_{CoM} = \frac{g}{z_{CoM}} x_{CoM} \tag{1}$$

Where $x_{CoM}$ is the horizontal displacement of the CoM, $g$ is the gravitational acceleration, and $z_{CoM}$ is the CoM's height.

Other simplified dynamical models have been proposed to enhance the original LIPM and provide it with additional characteristics and capabilities. The following is a summary of these models, with a particular focus on their distinctions from the LIPM.

#### *1) Linear Inverted Pendulum Model with Flywheel LIPM-FW)*

The LIPM-FW is an enhanced version of the original LIPM that dynamically adjusts the robot's angular momentum by incorporating a flywheel [3] (see Fig. 2 (b)). This model is useful for preserving balance while standing or walking on uneven terrain due to the flywheel's response to disturbances.

#### *2) Spring-Loaded Inverted Pendulum (SLIP) Model*

The SLIP model improves the simple inverted pendulum model by representing the legs as massless, elastic springs [4] (see Fig. 2 (c)). As a result of the spring's compression and extension during the stance phase, this model is capable of simulating dynamic motions, including running and walking.

#### *3) Variable Height Inverted Pendulum (VHIP) Model*

The VHIP model expands upon the LIPM by allowing the pendulum's height to change (see Fig. 2 (d)) [5], [6]. This model is beneficial for locomotion tasks that necessitate an adjustment of the CoM's vertical position, such as stair climbing.

#### *4) Hybrid-Linear Inverted Pendulum Model (H-LIPM)*

The H-LIPM enhances the original LIPM by describing the robot's dynamics for both single and double support phases [7]. The H-LIPM uses the same LIPM dynamics for single support phase; however, it assumes that the robot's CoM maintains a constant velocity ($\ddot{x}_{CoM} = 0$) during the double support phase. This model is more accurate than LIPM in capturing the transition between phases, which leads to a more natural locomotion.

#### *5) Reaction Mass Pendulum (RMP) Model*

The RMP model expands upon the LIPM by incorporating an ellipsoidal reaction mass to determine the whole body's generalized inertia projected at the CoM [8], [9] (Fig. 2 (e)). The centroidal moment of inertia, which is the result of limb movements, causes the ellipsoidal reaction mass to undergo changes in shape, size, and orientation. The RMP model is implemented to evaluate a bipedal robot's ability to maintain stability in the presence of disturbances, as it determines the robot's CoM and its interactions with the ground.

#### *6) Virtual-Mass-Ellipsoid Inverted Pendulum (VIP) Model*

In [10], the authors proposed the VIP model to remove both the constant CoM height constraint and the constant centroidal angular momentum constraint. The model has a variable ellipsoid mass at the CoM, and a telescopic leg connecting the pivot point (CoP) and the CoM (see Fig. 2 (f)). In contrast to the RMP model, which encompasses the dynamics of both the robot and the reaction mass and includes multiple degrees of freedom, the VIP model simplifies to fewer degrees of freedom, concentrating on the rotational dynamics of a virtual mass. This simplifies the VIP model and facilitates its analysis; however, it may not effectively represent the impact of actual physical masses on the system's behavior [11].

*B. Centroidal Dynamic Model*

The centroidal dynamic model describes the relationship between the angular momentum, forces, and the CoM's motion. It is necessary to control the CoM's motion, which includes position and velocity, to ensure stability of a bipedal robot that is either standing or walking and is subjected to external forces (such as ground reactions forces or pushing forces). The centroidal dynamic model achieves stability by regulating angular momentum around the CoM and generating opposing forces through the robot's actuators. The CoM dynamic equation of bipedal robots can be formulated as (2).

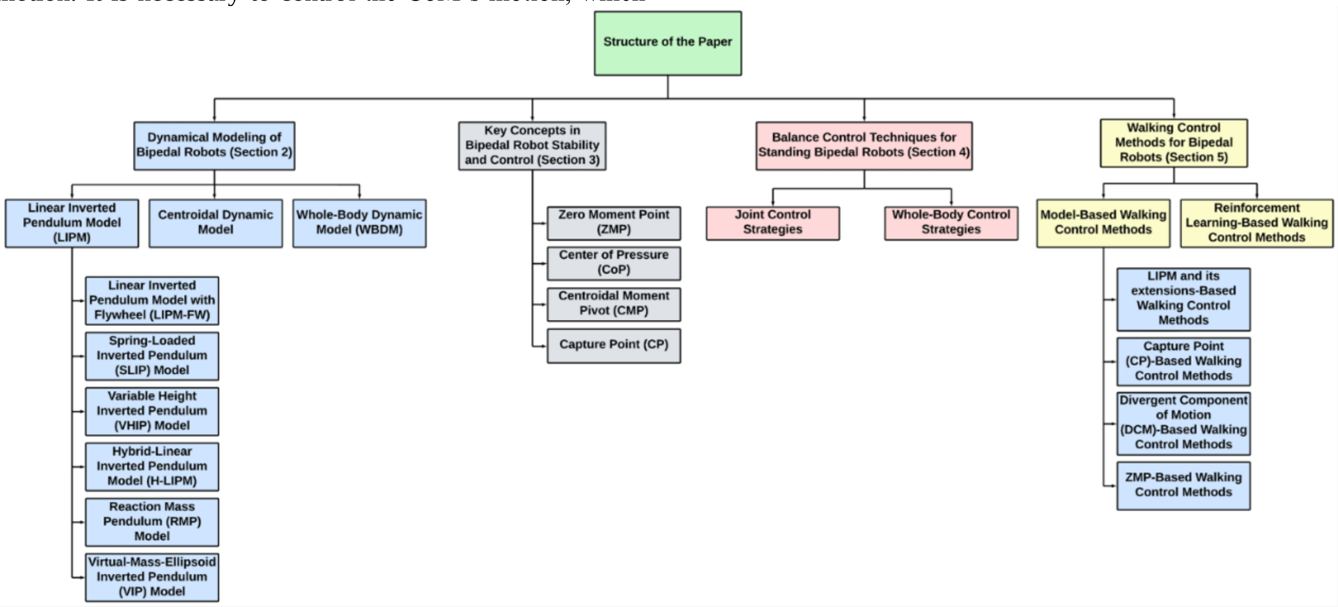


Fig. 1. The paper's structure

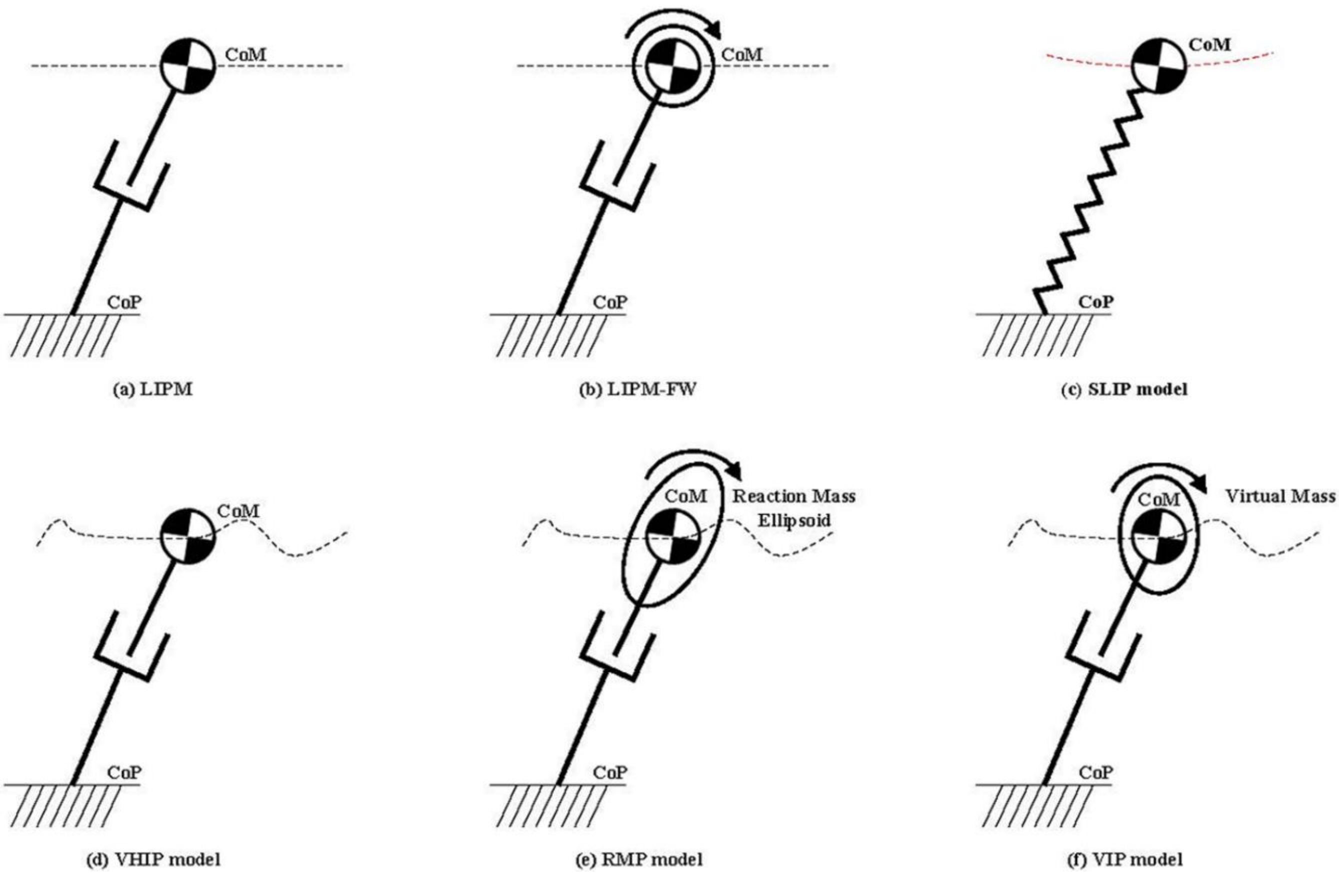


Fig. 2. Simplified models: (a) LIPM, (b) LIPM-FW, (c) SLIP model, (d) VHIP model, (e) RMP model, and (f) VIP model

$$\begin{bmatrix} A_{CoM,1} \\ A_{CoM,2} \end{bmatrix} F_{Contact} = b_{CoM} \tag{2}$$

Where

$$A_{CoM,1} = [I_{3\times3} \quad I_{3\times3} \quad 0_{3\times3} \quad 0_{3\times3}] \tag{3}$$

$$A_{CoM,2} = [(p_R - c) \quad (p_L - c) \quad I_{3\times3} \quad I_{3\times3}] \tag{4}$$

$$F_{Contact} = \begin{bmatrix} F_R \\ F_L \\ M_R \\ M_L \end{bmatrix} \tag{5}$$

$$b_{CoM} = \begin{bmatrix} m(g + \ddot{c}) \\ \dot{H} \end{bmatrix} \tag{6}$$

$p$ is the position of the foot, $F$ is the foot contact force, $M$ is the foot contact moment, $H$ is the CoM angular momentum, $c$ is the position of the CoM. The subscript $L$ represents the Left foot and $R$ represents the Right foot [12].

### C. Whole-Body Dynamic Model (WBDM)

The whole-body dynamics of a floating base bipedal robot, which involve calculating the robot's dynamic behavior and balance by considering joint torques, CoM, and external forces, are computed using Roy Featherstone's algorithm [13].

The whole-body dynamics equation of general biped robots is:

$$M(q)\ddot{q} + N(q, \dot{q}) = B\tau_q + J(q)^T F_f \tag{7}$$

where $q \in R^{n+6}$ is generalized coordinates ($q = [q_{base}, q_{joints}]$), $q_{base} = [x, y, z, \emptyset, \theta, \varphi]$ representing the position and orientation of the robot's base, $M(q) \in R^{(n+6)\times(n+6)}$ is the rigid body dynamic inertial matrix, $N(q, \dot{q}) \in R^{n+6}$ is the generalized force vector of all modeled forces including coriolis force, centrifugal force and gravity, $\tau_q \in R^n$ is the driving torque vector, $B = [0_{n\times6} \quad I_{n\times n}]$ is select matrix, $I_{n\times n}$ is $n$-dimensional identity matrix, $J(q) \in R^{k\times(n+6)}$ is force Jacobian matrix, $F_f \in R^k$ is foot contact force, $n$ is the number of joints of the robot [12].

This model is crucial for planning and controlling complex, coordinated movements in bipedal robots, such as walking, running, and performing tasks that require precise manipulation.

Table I shows the comparison between the dynamical models, including their computational complexity, accuracy, applicability, advantages, and limitations.

## III. Key Concepts in Bipedal Robot Stability and Control

This section provides a concise overview of four stability metrics, including the Zero Moment Point (ZMP), Center of Pressure (CoP), Centroidal Moment Pivot (CMP), and Capture Point (CP).

### A. Zero Moment Point (ZMP)

The ZMP is the point on the ground where the total horizontal inertia force and gravity force have zero moment [14]-[17]. The ZMP equation for the multi-body model is available in [17], and for a simplified inverted pendulum model is:

$$x_{ZMP} = x_{CoM} - \frac{z_{CoM}}{g}\ddot{x}_{CoM} \tag{8}$$

$$y_{ZMP} = y_{CoM} - \frac{z_{CoM}}{g}\ddot{y}_{CoM} \tag{9}$$

where $x_{ZMP}$ and $y_{ZMP}$ represent the ZMP position in $x$ and $y$ axis, respectively.

The ZMP is primarily utilized as a stability metric to guarantee that the bipedal robot remains stable, with the ZMP kept within the boundaries of the foot support region.

### B. Center of Pressure (CoP)

The CoP is the point on the support surface where the total sum of the vertical reaction forces acts. Ankle joints can be equipped with force/torque sensors to measure ground reaction forces and torques. These measurements are used to determine the position of the CoP as follows:

$$x_{CoP} = \frac{\tau_{Ly} + p_{Lz} f_{Lz} + \tau_{Ry} + p_{Rz} f_{Rz}}{f_{Lx} + f_{Rx}} \tag{10}$$

$$y_{CoP} = \frac{\tau_{Lz} + p_{Lz} f_{Ly} + \tau_{Rz} + p_{Ry} f_{Ry}}{f_{Lx} + f_{Rx}} \tag{11}$$

where $f_L$= $[f_{Lx}, f_{Ly}, f_{Lz}]$ and $\tau_L$=$[\tau_{Lx}, \tau_{Ly}, \tau_{Lz}]$ represent the forces and torques measured by the left ankle sensor located at position $p_L$=$[p_{Lx}, p_{Ly}, p_{Lz}]$. Similarly, $f_L$=$[f_{Lx}, f_{Ly}, f_{Lz}]$ and $\tau_R$=$[\tau_{Rx}, \tau_{Ry}, \tau_{LRz}]$ are the forces and torques measured by the right ankle sensor located at position $p_R$=$[p_{Rx}, p_{Ry}, p_{Rz}]$ [18].

The CoP is commonly used to analyze balance in both static and dynamic scenarios in bipedal robotics, providing a direct measure of how and where ground reaction forces are distributed under the feet. It is worth mentioning that when the bipedal robot is standing still with no external disturbances or accelerations (inertial effects are minimal), the ZMP and CoP coincide inside the support polygon.

TABLE I. Comparison of Dynamical Models

| Model | Computational Complexity | Accuracy | Applicability | Limitations |
|---|---|---|---|---|
| LIPM | Low | Moderate for flat terrain | Simple tasks, flat terrain | Limited to flat terrain, ignores upper body dynamics |
| LIPM Extensions | Moderate | High for uneven terrain | Dynamic tasks, uneven terrain | Increased complexity |
| Centroidal Dynamics | High | High | Complex tasks requiring momentum control | Computationally expensive, required detailed system parameters |
| Whole-Body Dynamics | Very high | Very High | Highly dynamic tasks, precise control | Extremely complex, requires significant computational resources and accurate models |

### C. Centroidal Moment Pivot (CMP)

The CMP denotes the point at which the ground reaction force vector must act so as to produce no torque about the CoM [19]. As a result, the rate of change of angular momentum is strongly correlated with the distance between CMP and ZMP. When the moment about the CoM is zero, the CMP coincides with the ZMP. However, when the CoM moment is nonzero, the extent of separation between the CMP and ZMP is equal to the magnitude of the horizontal component of the moment about the CoM, divided by the normal component of the ground reaction force. The CMP position in $x$ axis is given by:

$$x_{CMP} = x_{ZMP} - \frac{\dot{H}_{CoM}}{mg} \tag{12}$$

where $\dot{H}_{CoM}$ is the rate of change of angular momentum about the CoM.

The CMP is useful in dynamic scenarios where angular momentum plays a significant role, such as in high-speed walking, or when the upper body is actively moving.

### D. Capture Point (CP)

The CP is the point on the ground where the robot must step to maintain balance, considering its current state of motion. [20] first derived the CP based on orbital energy concept and using simplified inverted pendulum model as follows:

$$x_{CP} = x_{CoM} - \frac{\dot{x}_{CoM}}{\sqrt{\frac{g}{z_{CoM}}}} \tag{13}$$

where $x_{CP}$ is the capture point.

The CP plays a vital role in emergency situations, particularly when the robot encounters an abrupt external impact. Through the process of determining the CP, the control system is capable of making precise adjustments to the robot's step, so restoring its stability.

To provide a clear understanding of the interrelationships between the stability metrics, Table II provides their definitions, strengths, weakness, and applications. This comparison highlights how each metric contributes to stability control in bipedal robots and underscores the importance of selecting appropriate metric based on the specific task and environments.

## IV. Balance Control Techniques for Standing Bipedal Robots

This section examines and discusses a variety of balance control techniques for standing bipedal robots. These methods can be classified into two groups: those that rely solely on specific joints, such as the ankles or hips, and those that employ the entire body, including the legs and arms and legs, to maintain standing stability.

### A. Joint Control Techniques for Standing Stability: Ankle and Hip

Biomechanical studies have revealed that humans usually tend to use either their ankle or hip joints to maintain standing stability in the presence of external disturbances [21]. Several studies have utilized this bio-inspired behavior to allow bipedal robots to prevent falling when experiencing pushing forces. The ankle strategy involves utilizing only the ankle joints to generate an opposing force that brings the body into an upright posture, with the CoM positioned within the area of support (see Fig. 3 (a)). Given that only the ankle joint is in motion, the LIPM can be employed to examine this particular strategy [22]. The strategy's limitation is its ability to only control relatively minor disturbances, as the torque generated at the ankle joint influences the ZMP/CoP's position, which is restricted to the foot's geometry. The hip method is capable of effectively managing more disturbances compared to the ankle strategy through rapid rotation of the hip joints in response to the presence of disturbances (see Fig. 3 (b)).

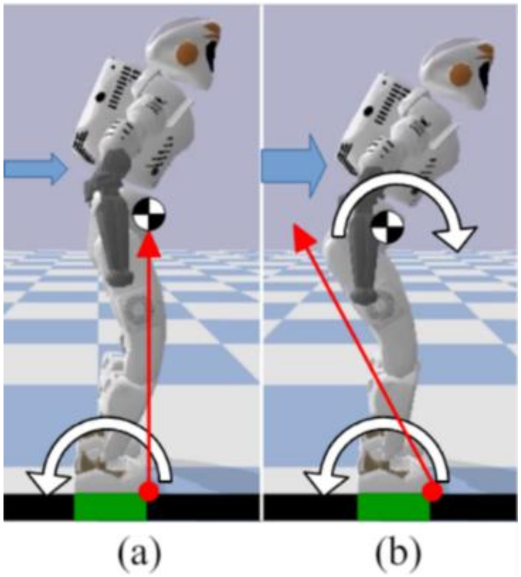


Fig. 3. (a) ankle strategy, (b) hip strategy [23]

The hip joint's rotation is critical for creating angular momentum around the CoM, which in turn increases ground reaction forces while keeping the CoP within the stability region [24]. In order to evaluate this strategy, the LIPM-FW can be used.

TABLE II. Comparison of Stability Metrics for Bipedal Robots

| Metric | Description | Strengths | Weaknesses | Applications |
|---|---|---|---|---|
| **ZMP** | Point where the net moment of inertia equals zero | Simple to compute, widely used in static and dynamic walking | Assume flat ground, less effective for highly dynamic motions | Static balance, trajectory planning for flat terrain |
| **CoP** | Point where the resultant ground reaction | Directly measurable, useful for real-time feedback | Sensitive to noise, limited to contact points | Real-time stability monitoring |
| **CMP** | point where the ground reaction force aligns with the centroidal moment | Captures rotational dynamics, useful for dynamic balance | Computationally intensive, requires accurate system modeling | Dynamic motions, such as running or jumping |
| **CP** | Point where the robot can step to regain balance | Effective for fall prevention | Requires precise foot placement, limited to specific recovery scenarios | Fall prevention, reactive balance control |

In ref. [25], [26] suggest an adaptive ankle impedance model that incorporates a damper, spring, and mass to preserve standing stability. The desired ankle joint torque is determined by combining the torque derived from the LIPM dynamics and the torque computed from the impedance model. The impedance gains are dynamically modified in [25] using the experience-based fuzzy rule interpolation approach in accordance with the angular position and velocity of the ankle joint. In [26], the authors employ an artificial muscle activation model to adjust the impedance gains. [27] suggests three distinct hip techniques to simultaneously manage standing balance and body posture when facing various push force locations. The first approach is the Symmetrical Bang-Bang (STB) control, which maintains the CoM's location while keeping the upright posture unchanged. This method is effective when the applied force passes directly through the CoM, resulting in only horizontal motion without any impact on the upright posture. Asymmetrical Bang-Bang (ATB) control is a secondary technique that corrects posture errors without affecting the CoM's motion. Applying an external torque to the CoM causes a change in posture without affecting horizontal motion, making this method effective. Universal Bang-Bang (UTB) control is employed to effectively regulate both the movements of the CoM and maintain an upright posture. The simulation results of a simpler two-link robot, with a mass concentrated at the hip joint, demonstrate the efficacy of maintaining standing stability.

The ankle and hip strategies are integrated in [28] to maintain standing balance, with emphasis on using three specific forms of torques: feed-bias torque, stiffness torque, and intermittent feedback torque. The feed-bias torque counteracts the joint torques caused by gravity, while the stiffness torques represent the viscoelastic characteristics of the muscles in the joints. The intermittent feedback torques are calculated using sensor data and are only engaged upon detection of a disturbance to guarantee efficient and responsive corrections. In [29], a balance control framework is proposed that determines the most effective strategies (e.g., ankle or hip) based on the external force's magnitude. A Virtual Model Control (VMC) is implemented for the ankle strategy to generate the necessary horizontal force to return the robot to its initial configuration. This VMC includes a virtual spring and damper that are attached to the CoM. A Proportional-Derivative (PD) controller, coupled LIPM-FW, is designed to create the necessary hip rotation for mitigating disturbances.

In [22] controls both ankle and hip joints to limit the ZMP position to be within the support polygon. An integral control algorithm is designed to minimize the error between the desired ZMP and actual ZMP. In [30], a PD controller is employed to modify the robot's upper posture in order to counteract disturbances and shift the ZMP towards the zero position by manipulating the angular position of the ankle joint. A simple change of variables is used to define an augmented CoM in [31]. The augmented CoM is unaffected by the angular accelerations of the upper body and follows the dynamics of LIPM. This results in preserving of standing balance by rotating the upper body using the hip joint and translating the CoM using the ankle joint, all while maintaining the ZMP's position. To determine the most effective strategy, the study in [32] presents a partition-aware push recovery controller. This controller observes the robot's CoM state to decide between using ankle strategies for swift adjustments and hip strategies for enhanced stability, optimizing the robot's reaction to perturbations.

*B. Whole-Body Control for Standing Stability*

Whole-body balance control, as illustrated in Fig. 4, maintains standing balance by moving all body parts, including the arms, legs, and head, rather than relying solely on specific joints, such as the ankles or hips. Two types of whole-body balance control techniques include force control strategies and momentum control strategies. Momentum control techniques address the regulation of a robot's linear and angular momentum to maintain balance, whereas force control techniques concentrate on modulating joint torques and contact forces to ensure standing stability.

Fig. 4. Whole-body balance control for standing robot [37]

A convex formulation of Model Hierarchy Predictive Control (MHPC), which was initially proposed in [33], is employed in [34] to effectively maintain standing balance and execute dynamic motions. In order to decrease computational costs and enhance model accuracy, MHPC incorporates whole-body dynamics in the short horizon and simplified dynamics in the long horizon, as described in Fig. 5. The control problem is formulated as quadratic programming to optimize the joint torques and contact forces, ensuring that the constraints of feasibility and stability are satisfied. In [35], Dynamic Balance Force Control (DBFC) is introduced, which calculates the whole body's torques that enable a humanoid robot to execute various tasks while maintaining its balance and handling external disturbances. The optimal contact forces required to accomplish the desired CoM position and velocity are determined by employing centroidal dynamics in a constrained optimization problem. Additionally, VMC is implemented to account for non-contact forces that are essential for posture control. [36] creates a balancing control system to stabilize the CoM's position and the trunk's orientation in a compliant manner. The necessary wrench (force and torque) needed to achieve balance is calculated and then distributed as forces at predetermined locations. A multi-objective constrained problem is formulated to distribute the force while minimizing the joint torques and Euclidean norm of the contact forces.

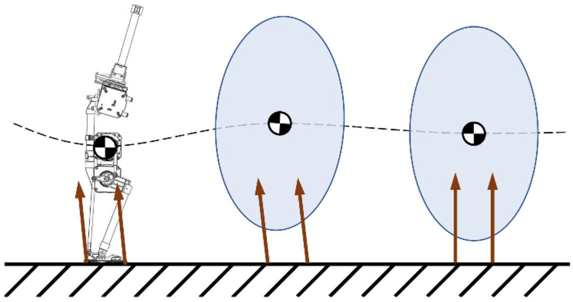

Fig. 5. MHPC incorporates full-body dynamics in the short horizon and simplified dynamics in the long horizon [34]

Integrating a Force/Torque (F/T) sensor onto small or inexpensive robots is challenging due to its high cost and large size. Hence, the researchers in [12] develop an external force observer that employs centroidal dynamics to precisely measure the external forces exerted on the robot. Subsequently, the foot contact force is used to counterbalance the estimated external forces through the implementation of whole-body torque control. In [38], a control algorithm for force-position balance is developed to ensure standing stability while subjected to external pushing forces or standing on slopes with variable inclinations. A quadratic programming optimization problem is formulated to determine the optimal forces and moments exerted by the foot in order to achieve both the desired ZMP and the necessary vertical force.

In [39] introduces the resolved momentum control, which generates the whole-body motion of a humanoid robot to maintain standing stability based on the desired linear and angular momenta. In resolved momentum control, the inertia matrix is derived to represent the relationship between joint velocities and momenta, with the desired linear momentum specified in terms of CoM position and velocity through a PD controller, while the angular momentum is assumed to be zero. In [40], the required joint accelerations, which correspond to the desired rate of momentum changes, are obtained by computing the time derivative of the inertia matrix presented in [39] and using Moore-Penrose pseudo-inverse. The desired change in angular momentum over time is defined as the difference between the positions of the CoP and the CoM. In [41], a multi-objective optimization problem is developed to obtain the joint accelerations required to accomplish the desired momenta and track the reference motion. The inverse dynamics is then used in [41] to calculate the whole-body joint torques by inputting the optimal joint accelerations with ground reaction forces. Instead of employing linear proportional control for linear momentum, in [42] a variable power reaching law for sliding mode control is developed and evaluated to robustly regulate linear momentum. The desired rate of change of angular momentum is defined in terms of the ground reaction forces, CoP, and the desired rate of change of linear momentum. Additionally, in [42], a bipedal robot name HURON efficiently handles multiple successive forces by using the null-space method to execute both posture recovery control and momentum control simultaneously.

A new momentum-based balancing control method is introduced in [43], which uses the admissible values of momenta rate of change. The permissible ground reaction force, which guarantees the friction limit to prevent slipping, and the permissible CoP, which assures that the CoP position is within the robot's support ground, are initially calculated. Afterwards, the admissible values of ground reaction force and CoP are used to recalculate the acceptable rate of change of momentum (see Fig. 6). The Centroidal Momentum Matrix (CMM) is then used to compute the necessary joint accelerations based on the desired momenta. [33] introduces a novel push recovery method for humanoid robots, which uses the rotational dynamics of the system. The algorithm initiates by computing the Centroidal Angular Momentum (CAM) reference in real-time, utilizing the magnitude and direction of the pushing forces. Subsequently, a quadratic optimization problem is formulated, incorporating the CAM reference as an input. This problem aims to generate feasible whole-body motion by optimizing the adjusted velocities, including the linear velocity of the CoM, the angular velocity of the hip, and the translational and rotational velocities of the right and left foot. The final torque required for the robot's desired motion is determined using the passivity-based whole-body controller provided in [37], [44], [45] in the last stage. The experiments demonstrate the humanoid robot's potential to maintain balance and prevent falling when standing on either one or two legs.

Our analysis of whole-body control methods indicates that force-based approaches effectively stabilize bipedal robots by managing joint torques and contact forces. However, these methods demand accurate force modeling and can be computationally intensive. In contrast, momentum-based approaches regulate robot motion by controlling linear and angular momentum, enhancing adaptability to dynamic disturbances. Yet, their real-time implementation remains challenging due to the complexity of state estimation and control computations.

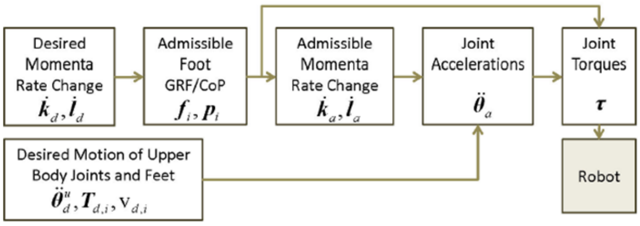


Fig. 6. The admissible momentum-based balance control system [43]

## V. Walking Control Methods for Bipedal Robots

### A. Model-Based Walking Control Methods

We examine several model-based walking control methods and categorize them into four groups: (1) LIPM and its extensions-based walking control methods; (2) CP-based walking control methods; (3) Divergent Component of Motion (DCM)-based walking control methods; and (4) ZMP-based walking control methods.

#### 1) LIPM and its Extensions-Based Walking Control Methods

The 3D-LIPM model was originally used by the authors of [2] to describe the dynamics of a biped robot during the

Single Support Phase (SSP) and to generate a walking pattern. The work presented in [2] is enhanced by [46] through the integration of the robot's dynamics during the Double Support Phase (DSP). This integration enables a smooth transition between various gait types, such as standing to walking, stopping walking, and speed switching. [47] improves walking stability and energy efficiency by analyzing the potential energy variations of LIPM in respect to different foot positions and CoM trajectories.

Walking patterns developed using LIPM are only suitable for flat terrain. Therefore, it is crucial to modify foot placements or to enhance the dynamics of the LIPM to ensure adaptability on uneven terrains. [48] uses the LIPM-based walking pattern generator presented in [49] for regular walking without any external pushing force. Once the push happens, [48] proposes switching to a push recovery walking generator, which is composed of an acceleration phase (when the velocity of the CoM increases) and a deceleration phase (when the velocity of the CoM decreases). The transition between phases occurs when the leg is landed promptly, which prevents further acceleration of the CoM and leads to the creation of a new CoM and foot positions. [50] suggests a novel dynamical model, the Virtual Force Linear Inverted Pendulum Model (VFLIPM), that allows a bipedal robot to modify its locomotion pattern parameters in response to external disturbances. To modify the VFLIPM parameters, such as the step length, lean angle, and virtual mass, a fuzzy controller is developed after the push is detected by measuring the deviation between the natural ZMP reference presented in [51], [52] and the actual ZMP.

Numerous studies use the SLIP model, which extends the LIPM by representing the legs as springs, to develop walking patterns for bipedal robots. Modeling the legs as springs allows the robot to walk naturally like a human and execute other dynamic tasks like hopping and running. Prior to [4], researchers assumed that the SLIP model only applied to running. In [4], the authors highlight the significance of incorporating a compliant spring to obtain the basic walking mechanics. This indicates that the SLIP model produces a more natural walking pattern than other simplified models, such as the LIPM. In [53], a 3D dual-SLIP with bio-inspired leg actuation is used to produce locomotion that closely resembles human walking. The capability to achieve stable walking on uneven terrain has been enhanced through the implementation of an optimization formulation [54]. The foot placements and CoM trajectory are updated to create human-like walking over uneven terrain through the multiple-shooting optimization [54]. In [55], the 3D dual-SLIP model with an updated swing leg trajectory adjusted to the terrain is proposed for the purpose of navigating blind terrain in the absence of perception. The swing leg trajectory retracts and extends towards the final stage of the swing phase, an action that exhibits human-like characteristics when walking in blind terrain. Several extensions of the classical SLIP model have been proposed to accommodate a broader range of environmental conditions as described in Table III.

*2) Capture Point (CP)-Based Walking Control Methods*

As previously stated in section 3.4, the CP is the point on the ground at which the robot must step to achieve a complete stop, considering its current motion state [56]. While the CP provides a one-step solution for instant stability, N-step capturability takes this concept a step further by estimating if a bipedal robot can achieve balance by performing a sequence of N steps. Theoretical analysis of N-step capturability is performed on three distinct dynamic models: LIPM with a point foot, LIPM with a finite size foot, and a reaction mass in [20].

TABLE III. EXTENSIONS OF THE CLASSICAL SLIP MODEL

| Model | Description |
|---|---|
| SLIP model with variable leg stiffness (V-SLIP) [57] | This model has the ability to efficiently handle external disturbances while walking by adjusting the stiffness of the legs |
| SLIP model with Swing Legs (SLIP-SL) [58] | This model enhances the SLIP model by adding the dynamics of the passive swing leg |
| Bipedal Trunk SLIP (BTSLIP) [59] | This model incorporates a trunk to the classical SLIP model to accurately depict the dynamics of the upper body |
| decoupled actuated SLIP (aSLIP) [60] | This model incorporates a virtual linear actuator into the SLIP model to independently modify the walking dynamics without affecting the spring system |
| Variable SLIP with Finite-Size Foot (VSLIP-FF) [61] | This model incorporates the contribution of the ankle joint in the walking dynamics |
| Flywheel SLIP (FSLIP) [62] | Flywheel is added to capture the centroidal angular dynamics |

In [63] develops real-time CP trajectory optimization to stabilize dynamic walking in bipedal robots. By optimizing the CP trajectory, the control input keeps the ZMP near the center of the support polygon, thereby preventing the robot from falling while performing dynamic movements like sudden stop and fast walking. [64] improves the work presented in [63] by optimizing both CP and CoM trajectories in real-time. This results in better ZMP control and reduced errors in CP tracking, enabling the robot to walk stably even with reduced foot size and under dynamic walking commands. In [65], a push recovery control system that integrates a CoM angular momentum controller and a steeping controller is created. First, the CP concept [62] is implemented to determine the required foot position to preserve walking balance. Then, the CoM trajectory is modified using the gait dataset in accordance with the desired foot step. The CMP criterion is employed in [65] to regulate the angular momentum around the CoM, thereby producing a natural and robust walking motion.

In [66] proposes the implementation of walking balance control, which involves the modification of landing position and timing based on capturability. First, the authors generate the walking pattern using the foot-guided agile control proposed in [67]. Subsequently, they employ the CP to adjust the landing position and timing in accordance with the magnitude and direction of the perturbation. The concept of CP is also employed in [66] to detect falling, which is crucial for allowing sufficient time to transition to the fall pose. The authors of [5] extend the analysis of capturability from the LIPM to the Variable-Height Inverted Pendulum (VHIP) model. By eliminating the constraint of a constant CoM height, this addition facilitates the development of walking patterns on uneven terrains. [11] enhances CP stability for bipedal walking by incorporating the angular momentum control through a Virtual-mass ellipsoid Inverted Pendulum (VIP) model. This model eliminates the constraints of constant CoM height and constant centroidal angular momentum present in the classical LIPM. The proposed method in [11] optimizes the CP trajectory and incorporates real-time adjustments in angular momentum to maintain stability, even on uneven terrains and under disturbances.

### 3) *Divergent Component of Motion (DCM)-Based Walking Control Methods*

The dynamics of the LIPM consists of two components: one that is stable and another that is unstable. The DCM is an extension of the LIPM that specifically deals with the unstable part of the CoM motion in order to produce a stable walking pattern [68], [69]. The DCM is expressed based on the current state of CoM (position and velocity), and it reflects the part of motion that diverges from the support point and must be regulated in order to maintain stability. Therefore, it provides a prediction of the future position of the CoM, which is valuable for determining foot placement and timing. [70], [71] use a QP-based trajectory optimization which aims to compute the next foot location and timing by minimizing the deviation between the desired DCM and the actual DCM at end of the step. [72] suggests an online planning method for bipedal walking trajectories using DCM. The first step planner generates the nominal location and timing of the footstep, which are then used as inputs for the DCM planner. The DCM planner generates the nominal DCM trajectory and foot position. The step adapter module integrates nominal values (foot and DCM trajectories) with measurements (actual DCM) to evaluate the adapted feet trajectories, footstep position, and time.

### 4) *ZMP-Based Walking Control Methods*

ZMP is primarily used in bipedal locomotion to ensure that dynamic motions like walking are stable, with the ZMP kept within the foot support region. There are numerous studies that generate bipedal walking patterns to satisfy the ZMP reference trajectory. The preview control of ZMP is employed to generate a bipedal walking pattern [73]. The ZMP reference trajectory is supposed to be placed in the middle of the support foot during the Single Support Phase (SSP), and a cubic spline is developed to ensure smooth transitions during the Double Support Phase (DSP). The preview control in [73] is derived from the concept of preview optimization control, which was introduced in [74]. It is designed to find the ZMP reference for N steps in the future at each time step. A control action is calculated and inputted into a cart-table model to generate the desired CoM trajectory, based on the ZMP error. [75] enhances the preview control of ZMP by shifting the ZMP reference trajectory backward (behind the foot) instead of defining it to be in the midpoint of the support foot. The acceleration of the CoM trajectory generated is reduced as a result of the modification in ZMP. [76] improves the preview control of ZMP by incorporating a push recovery generator which adjusts the swing leg trajectory and trunk flexion to counteract the momentum generated by the pushing force. It is important to note that the preview control of ZMP in [73] can only produce a walking pattern with a constant CoM height. Conversely, [77] implements a preview control framework that utilizes the virtual plane method to produce a walking pattern with a variable CoM height. The ZMP preview control in [73] employs a cart-table model that fails to represent the whole-body dynamics of bipedal movement, resulting in an inaccurate ZMP trajectory. In contrast, the ZMP preview control in [78] uses the whole-body dynamics, which takes into account the angular momentum around the CoM and the change in CoM height, resulting in a more precise ZMP trajectory.

In contrast to the optimal preview control employed in [73], which results in poor tracking performance when dealing with uncertain systems, [79] suggests the use of optimal preview integral Sliding Model Control (SMC) to produce a stable and robust walking pattern. Investigations into the natural gait of humans have demonstrated that the ZMP trajectory does not remain beneath the support foot. In order to accomplish a natural walking pattern, Fourier series approximation techniques are implemented in [73] to generate a human-like ZMP trajectory for the LIPM equations. [80] initially defines the reference ZMP trajectory, which is subsequently fed into the LIPM-FW to produce the desired CoM trajectory. To improve the stability when navigating uneven terrains, [80] incorporates the ankle and hip strategy into the control system. Fig. 7 provides an overview of model-based walking control methods.

## B. *Reinforcement Learning-Based Walking Control Methods*

Reinforcement Learning (RL) is a machine learning technique that determines the optimal control actions to accomplish specific objectives by learning them based on future rewards. In situations where the system's dynamics are unknown, or when dealing with highly complex or nonlinear systems, RL is effective. RL-based methods have gained significant interest in bipedal robot locomotion control. This is attributed to RL's ability to develop robust and stable walking with a high degree of adaptability to environments. Model-based RL approaches use an explicit model of the environment or system dynamics to make decisions, while model-free RL approaches learn policies from interactions with the environment without the need for models.

In [81] creates a controller that is inspired by human movement and integrates both learned and unlearned movements to ensure stable walking. Initially, a high-level trajectory planner is developed to produce desired trajectories for the controllable points (trunk and ankles). The required feed-forward joint angles are subsequently determined by inputting these trajectories into the Inverse Kinematics (IK) solver. The feedback policy, responsible for adjusting the joint angles in response to disturbances, is trained using Proximal Policy Optimization (PPO) with the same trajectories of the ankles and trunk. [82] introduces the Behavior-Based Locomotion Controller (BBLC) to regulate the motion of a 3D bipedal robot in the presence of disturbances, applying Behavior-Based Control (BBC) to specify numerous control behaviors and synchronize them, thereby creating a more complicated controller. BBLC consists of three layers: The task layer defines a variety of task-space motions, such as the CoM trajectory, the swing leg trajectory, and the torso rotation, using a variety of methods.

A set of reference joint trajectories is computed by coordinating these tasks. Different methods of planning each task-space trajectory are defined in the behavior layer. The RL layer, which employs the Q-learning algorithm, is responsible for identifying the action vectors that generate the optimal balancing strategies.

In [83] introduces an RL-based walking control method that learns the optimal policy for stable walking by controlling the ankle joints or updating the swing leg foot placement, effectively overcoming a variety of disturbances, including uneven ground and pushing forces. [84] proposes the use of the Q-learning technique to facilitate stable walking of a bipedal robot, without requiring any prior knowledge of the robot's dynamics. The agent performs an action to ensure that the ZMP remains within the limit of the foot. [85] suggests a model-free reinforcement learning technique to develop a policy that allows a bipedal robot (Cassie) to execute different agile actions by following reference motions. Instead of applying a model-based walking controller to generate reference motion, the training system employs a variety of parameterized motions that are generated based on Hybrid Zero Dynamics (HZD), a concept originally introduced in [86]. In [87], the authors suggest the application of a model-free reinforcement learning approach to develop feedback control policies for bipedal walking without relying on prior reference trajectories. The policy structure leverages physical insights derived from the Hybrid Zero Dynamics (HZD) framework to streamline the design of neural networks by minimizing the number of trainable parameters. This methodology enables a 3D bipedal robot to acquire stable and robust gaits independently of reference trajectories.

One method that has been proposed in literature to achieve stability and naturalness in bipedal robots is to use human motion data to produce walking. This method includes the collection and analysis of movements using technologies such as motion capture systems, which capture a variety of parameters, including joint angles and locomotion patterns. Nevertheless, this method is limited by the fact that the direct transfer of observed human motions to the robot's motions leads to unreliable motion as a result of the differences between human and humanoid kinematics and dynamics. Therefore, [88]-[91] suggest the use of model-free RL to track reference motions that are predetermined based on human motion data. [88] introduces a control system that employs model-free RL to transform human motions captured by a low-cost camera into dynamically stable robot motion. Initially, a prioritized controller is suggested, with the primary objective of maintaining the ZMP within the support polygon and the secondary objective of reproducing the movements. Next, RL is implemented to reduce the discrepancy between the observed human motions and the robot's movements by modifying the parameters of dynamic movement primitives (DMPs) using Weighting Exploration with the Returns (PoWER). [89] integrates motion re-targeting and domain randomization techniques to the RL process to reduce the discrepancies in joint configurations between the movements of human and bipedal robot. [90] addresses the challenge of transferring complex human movements into robotic actions by incorporating a reward function that focuses on dynamic balance and smooth transitions between steps, in addition to pose accuracy. Using PPO guarantees reliable and efficient learning, enabling the neural networks to dynamically adjust the robot's walking behavior to closely resemble human gait patterns. [91] addresses the issue by segmenting human motion into meaningful components and incorporating a balancing controller to guarantee the dynamic stability of the robot’s motion. Fig. 8 provides an overview of RL applications in bipedal locomotion.

## VI. Current Challenges and Recommendations

Although there have been advancements in modeling, standing balance control, and robust locomotion, bipedal robots still face problems that limit their efficient performance in real-world applications. We have pinpointed the challenges that require attention.

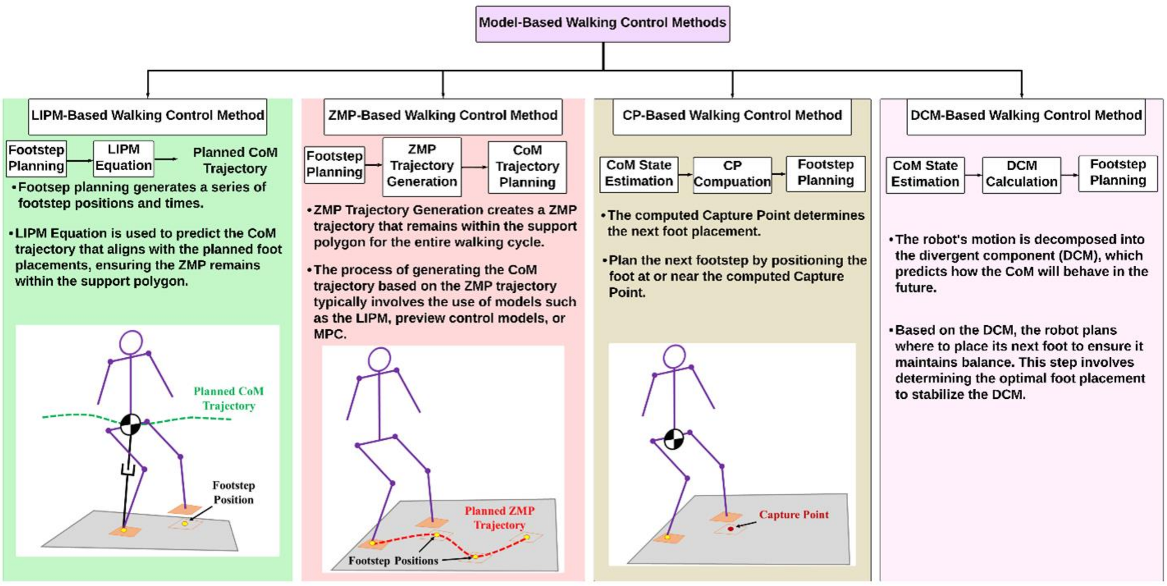


Fig. 7. Overview of model-based walking control methods

## A. *Adaptation to Slippery or Blind Terrain*

Previous studies have suggested several robust locomotion methods, but research on slippery or blind terrain is lacking. Current locomotion methods rely on the assumption that there is sufficient friction between the feet and the ground. However, this assumption does not hold true on slippery surfaces. Adaptive control approaches are used in many locomotion methods to update the next footstep depending on the terrain. These locomotion methods become less effective in the blind terrain, where the environmental feedback is limited.

To enhance bipedal mobility on slippery or concealed surfaces, we recommend the integration of infrared and ultrasonic sensors with reinforcement learning-driven adaptive control. Infrared sensors identify differences in surface materials through thermal emissivity, whereas ultrasonic sensors evaluate surface roughness and indicate potential dangers like water or unstable ground. These sensors provide adaptive alterations in locomotor techniques by delivering real-time terrain classification. RL can enhance flexibility by training policies to modify footstep positions, joint torques, and locomotion patterns based on sensor inputs. The effectiveness of these approaches in actual applications is evidenced by several case studies, including the deployment of ultrasonic sensors in Boston Dynamics' Spot robot and infrared sensors in autonomous cars.

## B. *Energy Efficiency*

Many studies have concentrated on the balance control of bipedal robots, neglecting the significant obstacle of energy consumption. Bipedal robots face challenges when attempting to carry out practical activities in the real world due to their limited operational time. Using techniques such as MPC, one potential solution is to create locomotion methods that prioritize both energy consumption and stability. Additionally, integrate energy consumption metrics into the robot's decision-making process to enable the robot to select actions and paths that minimize energy consumption while still achieving its objectives. Current research investigates methodologies that incorporate energy-aware control systems, enabling robots to utilize energy-related feedback for performance optimization. Our laboratory is presently refining a reward function in reinforcement learning that has a component aimed at minimizing energy usage during task execution.

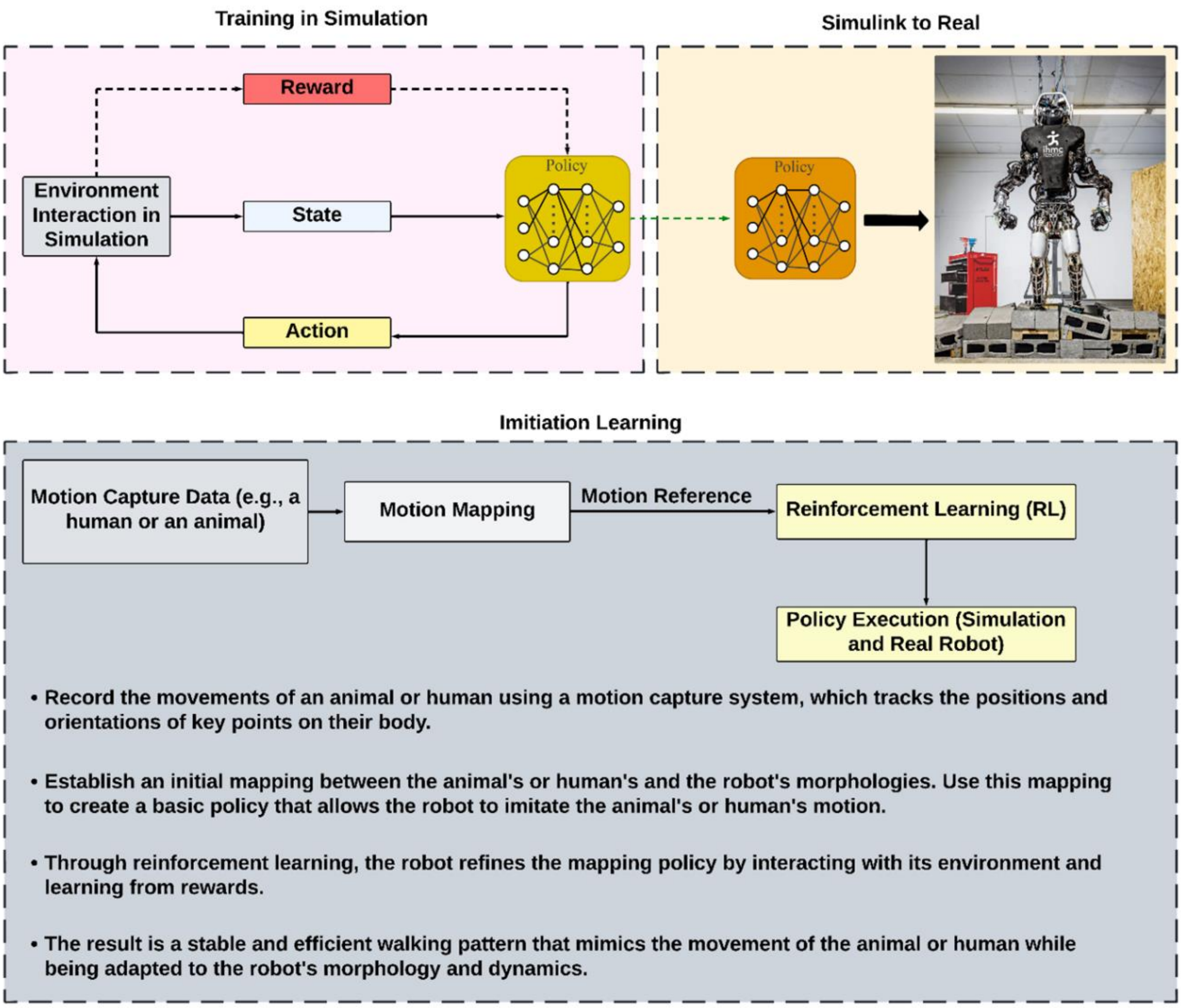


Fig. 8. Overview of RL applications in bipedal locomotion, including (1) training the RL model in a simulated environment (Simulink), (2) transferring the learned policy from simulation to real-world application (Sim-to-Real), and (3) utilizing RL to generate walking patterns that imitate motion data captured

### C. Scalability Issues

Balance control methods developed for small bipedal robots are ineffective when applied to large robots due to differences in dynamic parameters such as inertia, CoM, and joint torques. Large robots are more sensitive to disturbances, necessitating the implementation of robust balance control systems than small-scale robots. Implementing hierarchical control systems, comprising a low-level layer responsible for controlling joint movements and a high-level layer responsible for generating planned trajectories, can help address the scalability problem. Within hierarchical control systems, when the robot's size increases, the high-level layer adapts to the planned trajectory, while the low-level layer remains unchanged. This independence allows the control system to be adjusted to accommodate larger robots.

## VII. Conclusion

This paper presents a systematic review of dynamical models, standing balance control strategies, and walking control methods for bipedal robots. In conclusion, the review highlights a clear trade-off between computational complexity and model accuracy across different dynamic modeling approaches. The LIPM serves as a low-complexity solution ideal for simple, flat terrain tasks but falls short in accounting for upper body dynamics. Extensions to the LIPM offer enhanced accuracy on uneven terrains, albeit with increased computational demands. Centroidal dynamics provide high accuracy and are well-suited for complex tasks requiring precise momentum control, yet they necessitate detailed system parameters and significant computational power. At the extreme, whole-body dynamics deliver the highest fidelity and control for highly dynamic tasks, but their implementation is hampered by substantial computational expense and model complexity. Ultimately, the optimal choice depends on the specific application requirements and the available computational resources, underscoring the need for a balanced approach in selecting the appropriate dynamic model.

In addition, we categorized balance control strategies for standing bipedal robots into two main categories: joint control strategies and whole-body control strategies. Joint control strategies are limited to specific joints, such as ankles or hips, to maintain standing stability when subjected to external disturbances like pushing forces. These strategies utilize simplified models and control systems to manage ankle or hip joints for achieving stability. However, they are inadequate in managing high-level disturbances due to their exclusion of whole-body dynamics. Consequently, other researchers have developed whole-body balance control strategies that involve all body parts, including the arms, legs, and head, to achieve standing stability. Our analysis of whole-body control methods reveals that these strategies either compute the desired contact forces or joint torques, or regulate both linear and angular momentum to stabilize the robot in an upright posture. Force-based approaches effectively stabilize bipedal robots by managing joint torques and contact forces, though they demand accurate force modeling and can be computationally intensive. In contrast, momentum-based approaches regulate robot motion by controlling linear and angular momentum, enhancing adaptability to dynamic disturbances. However, their real-time implementation remains challenging due to the complexity of state estimation and control computations. To conclude, whole-body control strategies are more effective than joint control strategies in absorbing greater disturbances and producing more natural responses. However, the application of these strategies in real physical robots is limited by the computational demands of whole-body dynamic model or centroidal dynamic model.

This paper also reviewed walking control methods, distinguishing between model-based and learning-based methods. Model-based walking control methods were classified into four groups: The first group includes the LIPM-based methods and their extensions, which use simplified models such as the LIPM or the SLIP model to generate the desired CoM trajectories, ensuring that the ZMP remains within the support polygon. LIPM-based methods are generally suitable for flat terrains, whereas SLIP-based approaches can handle uneven surfaces. The second group, CP-based methods, utilizes the concept of the capture point to update the foot placements in response to disturbances such as uneven terrain. The third group, DCM-based methods, employs the Divergent Component of Motion to inform strategic adjustments in foot placement, ensuring balance when the stability point diverges from the stable region. The fourth group, ZMP-based methods, which plans the ZMP trajectory to stay within the support polygon, using techniques such as MPC or preview control to generate the planned CoM trajectory, facilitating precise foot position updates for stable locomotion. In contrast, learning-based walking control methods use RL algorithms to train bipedal robots in simulation environments for natural walking. Some studies integrate human motion data, applying RL algorithms to minimize kinematic and dynamic discrepancies between the real robot and human motion capture. These methods tend to produce more natural and energy-efficient walking patterns compared to model-based methods. However, they require extensive training time to enable robots to learn robust and natural walking over rough terrains.

Over the past two decades, there has been a significant increase in research on bipedal robots, which has resulted in significant advancements in areas such as learning algorithms, stability, and motion control. Despite the progress made in the field, there are still challenges to be addressed. Generating walking patterns over slippery or blind terrains remains a significant challenge that impedes the real-world application of bipedal robots. Traditional walking methodologies often assume adequate friction between the feet and the ground, utilizing terrain information as feedback to adjust subsequent foot placements. However, these methods fail to handle slippery or blind terrains where such assumptions are invalid. To address this, infrared or ultrasonic sensors can be employed for real-time detection of surface conditions. This real-time data can then be used to classify the type of terrain, a crucial step in determining optimal walking parameters to achieve stable locomotion. Furthermore, RL algorithms can enhance flexibility by training policies to modify footstep positions, joint torques, and overall locomotion patterns based on sensor inputs, paving the way for more robust and stable locomotion in challenging environments.

It is evident that bipedal robots exhibit short operational time. To enhance their efficiency, it is recommended to employ optimization algorithms that focus not only on stabilizing movement but also on minimizing energy consumption. Additionally, integrating energy consumption metrics into the decision-making process of these robots could enable the selection of actions and paths that optimize energy usage while still fulfilling the intended objectives. This integrated approach could significantly extend the operational effectiveness of bipedal robotic systems in various applications. In addition to the challenges previously outlined, balance control methods developed for large robots do not efficiently translate to small-scale robots. Large robots, being more sensitive to disturbances, necessitate robust balance control mechanisms. Implementing hierarchical control systems, which consist of a low-level layer for controlling joint movements and a high-level layer for generating planned trajectories, may mitigate scalability issues. Within such systems, the high-level layer adapts the planned trajectory as the robot's size increases, while the low-level layer remains consistent. This structural independence enables the control system to be effectively adjusted for larger robots, facilitating scalable application across different robot sizes. Future research should focus on sensor fusion, efficient control algorithms, and scalability, paving the way toward more versatile, energy-conscious robots capable of handling complex, unstructured environments.

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