Journal of Robotics and Control (JRC)
Volume 6, Issue 3, 2025
ISSN: 2715-5072, DOI: 10.18196/jrc.v6i3.26659

Robust Optimal Tracking Control for Wheel Mobile Robot via Reinforcement Learning

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***Abstract*—This paper aims to address robust optimal tracking control for a wheel mobile robot (WMR) with unknown dynamics. Firstly, the WMR system is considered a nonholonomic system with nonlinearity and input disturbance. Traditional optimal methods typically require solving the Hamilton- Jacobi- Bellman (HJB) equation or Algebraic Riccati equation (ARE), which are related to minimizing a cost function. However, these methods become increasingly difficult or even impossible to implement for high nonlinear systems such as the WMR in practical applications. To overcome this challenge, a Reinforcement Learning (RL) algorithm is designed to learn the solution of the HJB equation by using the input-output system data collected from the WMR during the data collection process. Consequently, the WMR can achieve optimal trajectory tracking without knowledge of the dynamic system. Finally, a simulation built in MATLAB software is given to show the effectiveness of the robust controller for WMR under the influence of uncertainties and input disturbance.**

Keywords*—*Wheel Mobile Robot; Reinforcement Learning (RL); Robust Control; Unknown Dynamics.

I. INTRODUCTION

In recent years, wheel mobile robots (WMR) have been widely used in many fields and have attracted much attention from both academic and industrial areas [1]-[3]. The control problem such as trajectory tracking control for WMR has become an interesting topic [4]-[7]. However, a wheel mobile robot (WMR) is a nonholonomic system and the control design for this system is difficult due to the nonlinearities, external disturbance, and uncertainties in the system. To ensure robustness and trajectory tracking performance, many control methods are used to design control laws for WMR, such as backstepping control [8][9], sliding mode control [10]-[13], fuzzy control [14], neural network-based control [15]-[17], and H_∞ control [18]. The majority of these control methods are to solve the trajectory tracking for the WMR while guaranteeing the stability of the WMR system. Although these control methods can achieve adaptive or robust with nonlinearity, external disturbance, and uncertainty in the system dynamics, the optimal tracking problem has not been discussed and these methods usually require knowing partial or completed dynamic information of the system.

In order to achieve optimal trajectory tracking, it usually requires solving the Hamilton-Jacobi-Bellman (HJB) equation [19][20] or the Algebraic Riccati Equation (ARE) [21], which are difficult to directly solve by only math analysis. The WMR is considered a nonholonomic system subjected to nonlinearity, external disturbance, and uncertainty, the traditional optimal methods are more difficult or impossible to implement in practical application. The problem is to design a robust controller that achieves optimal trajectory tracking without the knowledge of the system dynamics. Reinforcement learning (RL) is a kind of learning method, which has received much attention in learning the optimal control policy without knowing the system dynamics. In particular, the RL approach by Actor-Critic structure [22]-[24], in where, a Critic neural network is employed to approximate the value of the performance function while an Actor neural network is employed to approximate the optimal solution. In studies [21], an online algorithm based on RL was developed for continuous systems to learn optimal control solutions online, but the completed knowledge of the system dynamics must be required. In the studies [22]-[24], RL algorithms are developed to learn the optimal control solution online with partially knowing dynamic information. While the studies in [22][23] achieve optimal regulation, the study in [24] can achieve optimal tracking that makes the output of the system track to the desired reference. In the studies [25][26], a data-based RL algorithm was developed for the nonlinear system to address the optimal tracking control problem without knowing system dynamics by using the input-output system data, but the nonlinear dynamic model did not consider the influence of the external disturbance. In the study [27], a robust offline RL algorithm is developed to find the optimal policy for a nonlinear system subjected to external disturbance with unknown dynamics.

Inspired by the study [27], this paper proposes a robust controller based on reinforcement learning (RL) to address the optimal tracking control for a nonholonomic wheel mobile robot (WMR) with unknown dynamics and input disturbance. With the proposed controller, the optimal control input can be learned without dynamic information by observing the input-output system data and the position tracking errors can converge to zero optimally. The research contributions of this paper are listed as follows.

1. Different from the studies [1]-[18], [31]-80], which use the nonlinear control methods and the Lyapunov theory to analyse the stability of the system. This paper proposed a robust controller for WMR to optimally track a time-varying trajectory under the influence of input disturbance by investing in an optimal control scheme.

2. By using the RL approach, the optimal control policy can be learned without the dynamic information by observing the input-output system data of the WMR.

The rest of the paper is organized as follows. The dynamic model of the wheel mobile robots is introduced in Section 2. A robust controller-based RL method is designed in Section 3. Finally, a simulation result is given in Section 4, and a brief conclusion is contained in Section 5.

Notations: Let $\mathrm{I_n} \in \mathrm{R}^{\mathrm{n}\times\mathrm{n}}$ indicate a unit matrix and $0_{\mathrm{n}\times\mathrm{m}} \in \mathrm{R}^{\mathrm{n}\times\mathrm{m}}$ a zero matrix.

II. Dynamic Model

A wheel-mobile robot is a nonholonomic system consisting of a vehicle body connected with wheels: two rear driving wheels and one free front wheel as shown in Fig. 1. The front wheel prevents the robot from tipping over and two rear driving wheels with two DC motors as actuator can be controlled to generate forces for robot move on a plane. It is assumed that the coordinate of the mass center of the WMR is located in the middle of the rear driving wheels.

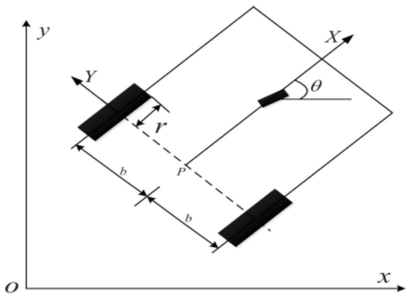

Fig. 1. Two-wheel mobile Robot

According to [28], the dynamic model of WMR is described as

$$\begin{aligned} \dot{q} &= S(q)\eta \\ M(q)\ddot{q} + F(\dot{q}) + \tau_d &= B(q)\tau - A^T(q)\lambda \end{aligned} \tag{1}$$

where $q = [x, y, \theta]^T \in \boldsymbol{R}^3$ is a vector of the positions and orientation of the robot, $\eta = [v, w]^T \in \boldsymbol{R}^2$ is a vector of linear and angular velocities of the robot. The control torque vector $\tau = [\tau_r, \tau_l] \in \boldsymbol{R}^2$ consist of the torques of the right and left rear wheels respectively. The torque disturbance vector $\tau_d \in \boldsymbol{R}^2$ is unknown and bounded. $M(q)$ is a symmetric and positive definite inertia matrix, $F(\dot{q})$ is the surface friction vector, $B(q)$ is the input transformation matrix. Matrix $S(q), M(q), F(\dot{q}), B(q)$, and $A(q)$ are described as

$$S(q) = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix}, M(q) = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J \end{bmatrix}$$

$$B(q) = \frac{1}{r}\begin{bmatrix} \cos(\theta) & \cos(\theta) \\ \sin(\theta) & \sin(\theta) \\ b & -b \end{bmatrix},$$

$$A^T(q) = \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \\ 0 \end{bmatrix}$$

where b is the distance between the rear wheel and the mass center of robot, r is the wheel's radius, m is the mass of robot, J is the inertia moment. According to [29], the nonholonoic constrant is $A(q)\dot{q} = 0$. Following Eq. (1a), we have

$$\ddot{q} = S(q)\dot{\eta} + \dot{S}(q)\eta \tag{2}$$

Subtituting into Eq. (1b)

$$\begin{aligned} M(q)\big(S(q)\dot{\eta} + \dot{S}(q)\eta\big) + F(\dot{q}) + \tau_d \\ = B(q)\tau - A^T(q)\lambda \end{aligned} \tag{3}$$

Mutipling both sides of Eq. (3) with $S^T(q)$

$$\begin{aligned} S^T(q)M(q)\big(S(q)\dot{\eta} + \dot{S}(q)\eta\big) + S^T(q)F(\dot{q}) \\ + \quad \mathrm{S^T(q)\tau_d} \\ = S^T(q)B(q) - S^T(q)A^T(q)\lambda \end{aligned} \tag{4}$$

From the definitions of $S(q)$, $M(q)$, and $A(q)$, we have

$$S^T(q)A^T(q) = 0, \qquad S^T(q)M(q)\dot{S}(q) = 0 \tag{5}$$

One has

$$\begin{aligned} \dot{q} &= S(q)\eta \\ \overline{M}\dot{\eta} &= \overline{B}\tau + \Delta(t) \end{aligned} \tag{6}$$

where $\overline{M} = \begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix}$, $\overline{B} = \frac{1}{r}\begin{bmatrix} 1 & 1 \\ b & -b \end{bmatrix}$, and $\Delta(t) = -S^T(q)(F(\dot{q}) + \tau_d)$ as unknown and bounded disturbance.

Control objective: This paper aims to develop a robust optimal control scheme for a wheel-mobile robot with input disturbances. Generally, this paper focuses on solving some problems as

1. To optimally track a time-varying trajectory for the wheel mobile robot under the influence of input disturbance by investing in an optimal control scheme.
2. By using the RL approach, the optimal control policy can be learned by observing the wheel-mobile robot's input-output system data. This means that the proposed controller achieves optimal tracking control with unknown dynamic information.

III. Control Design

In this section, a robust optimal controller based on the RL method and optimal control theory is proposed for the wheel-mobile robot with input disturbance.

Firstly, this is an assumption that the reference trajectory $q_d = [x_d, y_d, \theta_d]^T \in \boldsymbol{R}^3$ is generated by the system.

$$\dot{q_d} = f_d(q_d) \tag{7}$$

in where $f_d(q_d)\colon \boldsymbol{R}^3 \to \boldsymbol{R}^3$ is an unknown and bounded function, which is known as the Lipchitz continuous function. To ensure the tracking objective, a trajectory tracking error $e_q(t) = q - q_d \in \boldsymbol{R}^3$, we have

$$\dot{e}_q = S(q)\eta - \dot{q}_d \tag{8}$$

Define $\eta_d \in \boldsymbol{R}^2$ as virtual control input for the subsystem in Eq. (8), which ensures its asymptotic stability.

$$\eta_d = S(q)^+(\dot{q}_d - \alpha_q e_q) \tag{9}$$

where $S(q)^+ = [S(q)^T S(q)]^{-1} S(q)^T$ is the pseudo-inverse matrix and $\alpha_q \in R^{3\times3}$ is positive define matrix. To ensure the trajectory tracking in the inner loop, we define the tracking error $e_\eta = \eta - \eta_q \in \boldsymbol{R}^2$. The dynamic model can be rewritten as

$$\dot{\eta} = [\overline{M}]^{-1}\overline{B}\tau + [\overline{M}]^{-1}\Delta \tag{10}$$

Define $X = [e_\eta^T, e_q^T, q_d^T]^T \in \boldsymbol{R}^8$ as an argument state vector, the argument system is constructed as

$$\begin{aligned}\dot{X} &= F(X) + Gu + D\Delta \\ e &= CX\end{aligned} \tag{11}$$

where $e = [e_\eta^T, e_q^T]^T \in \boldsymbol{R}^5$ as tracking error, $u = \tau$ as control input, and $F(X), G, E$ are

$$F(X) = \begin{bmatrix} 0 \\ S(e_q + q_d)^+(\dot{q}_d - \alpha_q e_q) \\ f_d(q_d) \end{bmatrix}, \quad G = \begin{bmatrix} [\overline{M}]^{-1} \\ 0 \\ 0 \end{bmatrix}, \quad D = \begin{bmatrix} [\overline{M}]^{-1} \\ 0 \\ 0 \end{bmatrix}, \quad C = [I_5, 0_{3\times3}]$$

To achieve the robustness and optimal trajectory tracking for the wheel-mobile robot under the influence of unknown disturbance $d(t)$, the disturbance attenuation condition is first defined as

$$\int_t^\infty e^{-\beta(\tau-t)}(e^T Q e + u^T R u)d\tau \le \gamma \int_t^\infty e^{-\beta(\tau-t)}(\Delta^T\Delta)d\tau \tag{12}$$

where $\beta > 0$ is a discount factor, $\gamma > 0$ is a positive constant, $Q \in \boldsymbol{R}^{5\times5}$ and $R \in \boldsymbol{R}^{2\times2}$ are positive defined matrices. Following Eq. (12), the influence of input disturbance Δ on the tracking performance can be attenuated by at least to γ, the performance function as

$$V(e,u,\Delta) = \int_t^\infty e^{-\beta(\tau-t)} r(e,u,\Delta)d\tau \tag{13}$$

where $r(e,u,\Delta) = (e^T Q e + u^T R u - \gamma\Delta^T\Delta)$. The performance function $V(e,u,\Delta)$ is considered a two-player zero-sum game with the minimizing player u and the maximizing player Δ. This is to say that the performance function is constructed to maximize energy efficiency and minimize the impact of input disturbance.

Remark 1: Similar to [30], the positive discount factor β has an important role in the performance function $V(e,u,\Delta)$, which indicates that the current rewards contribute more significantly to the performance function than future rewards and ensures the performance function is bounded.

Remark 2: Generally, there is no analytical method to determine the smallest achievable disturbance attenuation level (γ). Consequently, γ is typically chosen as a sufficiently large predefined value in practice. The minimum value of γ that satisfies the required condition corresponds to the optimal robust control solution.

Following the argument system in Eq. (11), the static Bellman equation can be achieved by the Dynamic Programming (DP) principle as

$$V^*(e,u,\Delta) = \min_u \max_\Delta V(e,u,\Delta) \tag{14}$$

The Hamiltonian function iss formulated by taking the time derivative of the static Bellman equation in Eq. (14) as

$$\frac{d}{dt}V^*(e,u,\Delta) = \frac{\partial V^*}{\partial X}\frac{dX}{dt} = \frac{\partial V^*}{\partial X}(F(X) + Gu^* + D\Delta^*) \tag{15}$$

By computing the static Bellman equation at instantaneous time t by DP principle as

$$V^*(t) = \int_t^{t+T} e^{-\beta(\tau-t)} r(e,u^*,\Delta^*)d\tau + e^{-\beta T}V^*(t+T) \tag{16}$$

Hence

$$\begin{aligned}\frac{V^*(t) - V^*(t+T)}{T} &= \frac{1}{T}\int_t^{t+T} e^{-\beta(\tau-t)} r(e,u^*,\Delta^*)d\tau \\ &+ \frac{e^{-\beta T} - 1}{T}V^*(t+T)\end{aligned} \tag{17}$$

Once $T \to 0$, the Bellman equation can be obtained as

$$\begin{aligned}H(V^*,u^*,\Delta^*) &= r(e,u^*,\Delta^*) - \beta V^* \\ &+ \nabla V^*(F(X) + Gu^* + D\Delta^*) = 0\end{aligned} \tag{18}$$

where $\nabla V^* = \frac{\partial V^*}{\partial X}$. By setting $\frac{\partial H(V^*,u,\Delta)}{\partial u} = 0$ and $\frac{\partial H(V^*,u,\Delta)}{\partial \Delta} = 0$, the optimal policies can be obtained as

$$\begin{aligned}u^* &= -\frac{1}{2}R^{-1}G^T\nabla V^* \\ \Delta^* &= \frac{1}{2\gamma^2}D^T\nabla V^*\end{aligned} \tag{19}$$

Substituting into the Eq. (18) the modified Hamiltonian function as

$$\begin{aligned}e^T Q e - \beta V^* &- \frac{1}{4}[\nabla V^*]^T G R^{-1} G^T \nabla V^* \\ &+ \frac{1}{4\gamma^2}[\nabla V^*]^T D D^T \nabla V^* \\ &+ [\nabla V^*]^T F(X) = 0\end{aligned} \tag{20}$$

Remark 3. Similar to [27], the argument system in Eq. (11) using the optimal control input in Eq. (19) satisfies the disturbance attenuation condition in Eq. (12) and is asymptotically stable if $\Delta = 0$ and $\beta \le 2(||PQ||)^{1/2}$ where $O = GR^{-1}G^T + DD^T/\gamma^2$.

Define V_i, u_i, and Δ_i as the updated policy in the i^{th} iteration. To achieve model-free optimal tracking control for the wheel mobile robot, the argument system is rewritten as

$$\dot{X} = F(X) + Gu_i + D\Delta_i + G(u - u_i) + D(\Delta - \Delta_i) \tag{21}$$

Different the performance function in Eq. (13) using Eq. (18) and Eq. (19) as

$$\begin{aligned}\dot{V}_i = \beta V_i - r(e_i,u_i,\Delta_i) &- 2u_{i+1}^T R(u - u_i) \\ &+ 2\gamma^2\Delta_{i+1}^T(\Delta - \Delta_i)\end{aligned} \tag{22}$$

Multiplying both sides with $e^{-\beta t}$ and then integrating both sides $(t,\ t+T)$ of Eq. (20) with T is time interval, the Bellman equation as

$$\begin{aligned} e^{-\beta T}V_i(t+T) - V_i(t) &= \int_t^{t+T} e^{-\beta(\tau-t)} r(e_i,u_i,\Delta_i)d\tau \\ &- \int_t^{t+T} 2e^{-\beta(\tau-t)}u_{i+1}^T R(u - u_i)d\tau \\ &+ \int_t^{t+T} 2\gamma^2 e^{-\beta(\tau-t)}\Delta_{i+1}^{\mathrm{T}}(\Delta - \Delta_i)d\tau \end{aligned} \quad (23)$$

Remark 4. According to [25-26], the optimal policies found by solving the Bellman equation in Eq. (20) and the Bellman equation in Eq. (23) are equivalent. Different from the Bellman equation in Eq. (20), the Bellman equation in Eq. (23) does not require knowledge of the system dynamics, which is difficult to obtain accurately in the practical application.

A. Model-free based RL Control Algorithm

1. Initialization

Applying control policy $u_0 = u_s + u_e$ with the stabilizing input u_s and the exploring noise input satisfying the persistence of excitation (PE) condition u_e to the wheel mobile robot to collect data of the state, the control input, and the disturbance. Initializing any control policy u_0 and disturbance policy Δ_0.

2. Update policy

For u_i and Δ_i, solving the Bellman equation in Eq. (21) to find the performance function V_i, the updated control policy u_{i+1}, and the updated disturbance policy Δ_{i+1}.

If convergence $||u_{i+1} - u_i|| \le \epsilon_u$ and $||\Delta_{i+1} - \Delta_i|| \le \epsilon_\Delta$, stop.

Else $i = i + 1$ and go to 2.

Remark 5: The Persistence of Excitation (PE) plays a critical role in ensuring the stability and convergence of reinforcement learning (RL) algorithms, particularly in model-free control systems. When the collected input-output data does not satisfy the PE condition, the excitation of the system dynamics becomes insufficient, leading to inaccurate estimation of value functions or control policies. This may result in convergence to suboptimal solutions, overfitting to noise, or even instability during the control process. Moreover, many theoretical results on stability and convergence in RL [19,21,22,24,26,27], fundamentally rely on the assumption of PE to guarantee the effectiveness of the learning process.

The convergence of Algorithm is shown in [27]. To approximate the optimal policies, three neural networks are employed as

$$\begin{aligned} \hat{V}_i &= \widehat{W}_{v,i}^T\phi_v(X) \\ \hat{u}_i &= \widehat{W}_{u,i}^T\phi_u(X) \\ \hat{\Delta}_i &= \widehat{W}_{d,i}^T\phi_d(X) \end{aligned} \quad (24)$$

Where $\hat{V}_i, \hat{u}_i$, and $\hat{\Delta}_i$ are the approximated values of V_i, u_i and Δ_i, respectively. $W_v \in \boldsymbol{R}^{lv\times 1}, W_u \in \boldsymbol{R}^{lu\times 2}$, and $W_d \in \boldsymbol{R}^{ld\times 3}$ are weight matrixes at i^{th} iteration, $\phi_v \in \boldsymbol{R}^{1\times lv}, \phi_u \in \boldsymbol{R}^{1\times lu}$, and $\phi_d \in \boldsymbol{R}^{1\times ld}$ are action functions with lv, lu and ld are the number of neurons. Substituting Eq. (22) to Eq. (21), the Bellman error $e_{B,i}$ as

$$\begin{aligned} e_{B,i} &= e^{-\beta T}\widehat{W}_v^T\phi_v(X(t+T)) - \widehat{W}_v^T\phi_v(X(t)) \\ &- \int_t^{t+T} e^{-\beta(\tau-t)} r(e_i,u_i,\Delta_i)d\tau \\ &+ 2\sum_{k=1}^{2} r_k \int_t^{t+T} e^{-\beta(\tau-t)}\widehat{W}_{u,i+1}^T\phi_u(X)\sigma_{u,k}d\tau \\ &- 2\gamma^2\sum_{k=1}^{2} \int_t^{t+T} e^{-\beta(\tau-t)}\widehat{W}_{d,i+1}^T\phi_d(X)\sigma_{d,k}d\tau \end{aligned} \quad (25)$$

Where $R = diag([r_1, r_2])$, $\sigma_u = [\sigma_{u,1}, \sigma_{u,2}]^T = u - u_i$ and $\sigma_d = [\sigma_{d,1}, \sigma_{d,2}]^T = \Delta - \Delta_i$. By using least-squares methods to bring the Bellman error $e_{B,i}$ to 0 under the PE condition.

IV. Simulation Result

This section presents a simulation result built on Matlab software to verify the effectiveness of the proposed controller. In this simulation, the parameters of the wheel mobile robot are chosen as: $b = 0.5\ (m)$, $r = 0.2\ (m)$, $m = 5\ (kg)$, and $J = 4\ (kg.m\hat{}2)$. The torque disturbance are set as

$$\tau_d = \begin{bmatrix} 0.1\dot{v} + 0.1v + 0.2w + 0.2\cos^2(t)\sin(2t) \\ 0.2\dot{w} + 0.1v + 0.1w + 0.2\,sin^2(t)\cos(3t) \end{bmatrix} \quad (26)$$

The desired reference trajectory is chosen as

$$\begin{aligned} x_d &= \sin(0.2t) \\ y_d &= -\cos(0.2t) \\ \theta_d &= 0.2t \end{aligned} \quad (27)$$

The initial conditions are initialized $q(0) = [x(0), y(0), z(0)]^T = [0,0,0]^T$. The time interval $T = 0.05$, the RL term is chosen as $\beta = 0.05$, $\gamma = 2$, $Q_1 = I_2$, and $Q_2 = \begin{bmatrix} 100I_2 & 0_{2\times 6} \\ 0_{6\times 2} & 0_{6\times 6} \end{bmatrix}$. The action functions of three neural networks are chosen as multiple polynomials with even orders. The exploration noise input is chosen as a sum of sinusoidal functions in the following form $u_e = 0.1\sum_{i=1}^{10}\sin(2\pi f_i t)$, with $f_i \in (0,20)$ Hz. The convergence of the RL Algorithm is shown in Fig. 2. The tracking errors of the wheel mobile robot are shown in Fig. 3 and Fig. 4. It is easy to observe that the tracking errors converge to zero after about 2 seconds. The position tracking performance is shown in Fig. 5. To verify the tracking performance of the wheel mobile robot, the 2D trajectory tracking is shown in Fig. 6.

This simulation only aims to give a simulation to verify the optimal control policies that are learned by the RL Algorithm to achieve optimal trajectory tracking for wheel mobile robots. Therefore, it should be noted that this paper does not compare the performance of the proposed controller with other control methods. The advantage of this paper is to propose an RL Algorithm to learn optimal control policies using input-output data collection from the wheel mobile robot system. By using RL methods, the control policies can be learned without knowledge of quadrotor dynamics.

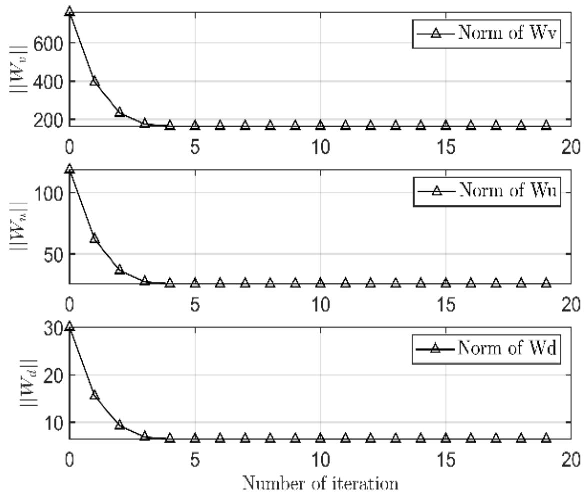

Fig. 2. Convergence of weights of the RL Algorithm

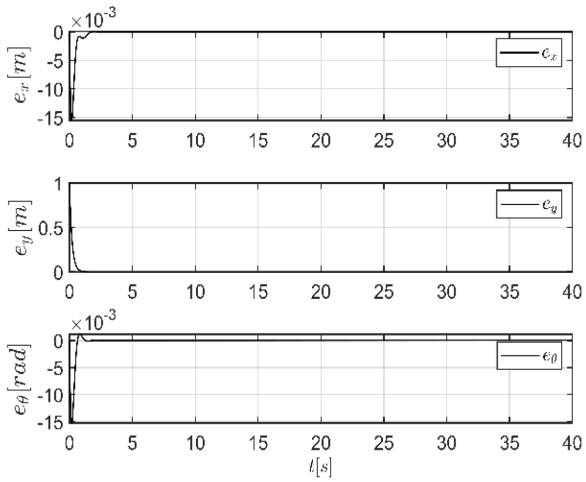

Fig. 3. The positions and orientation tracking errors

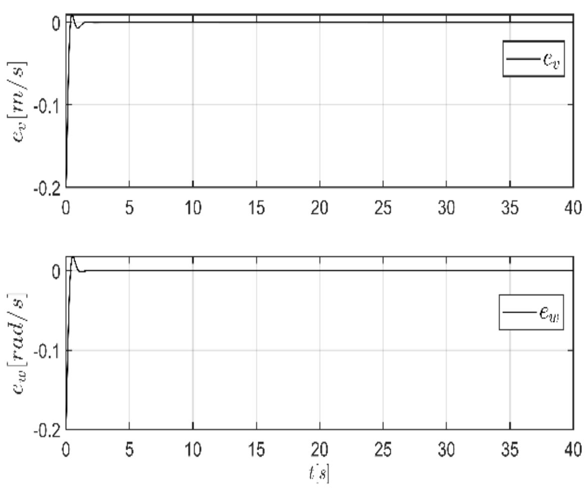

Fig. 4. The linear and angular velocity tracking errors

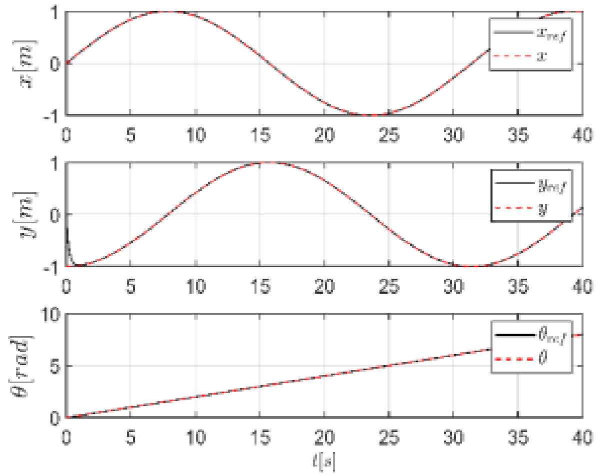

Fig. 5. The positions and orientation responses

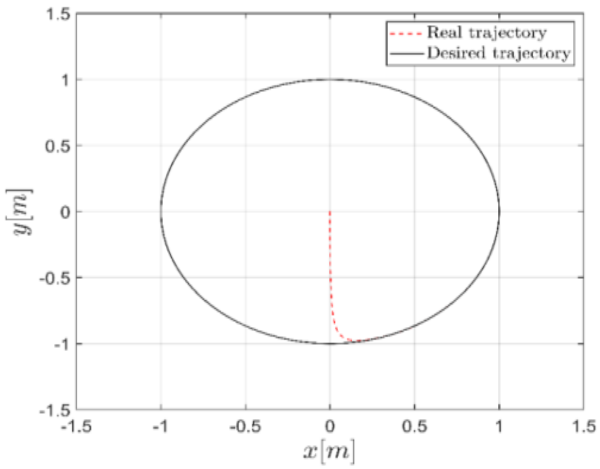

Fig. 6. The 2D trajectory of the wheel mobile robot

V. CONCLUSION

This paper focuses on addressing the robust optimal trajectory-tracking problem for a nonholonomic WMR with unknown dynamics. It is well known that traditional optimal methods become increasingly difficult, or even impossible, to implement for highly nonlinear systems such as WMRs. Furthermore, the WMR system is considered under the influence of bounded disturbance inputs. To overcome this challenge, a model-free RL algorithm is designed to learn the optimal policy using only input-output system data. To verify the effectiveness of the proposed controller, a simulation built in MATLAB software is provided. The simulation results show that the mobile robot with the proposed controller achieves optimal trajectory tracking following a predefined reference trajectory.

ACKNOWLEDGMENT

This research was supported by the Research Foundation funded by the Thai Nguyen University of Technology.

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