A Robust Fuzzy Fractional Order PID Design Based On Multi-Objective Optimization For Rehabilitation Device Control

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Abstract—Gait rehabilitation robots show great promise in assisting people to reform their lower limbs with injuries or disabilities. Certainly, providing an accurate and customized aid specifically to children necessitates an effective control strategy. Such strategy should ensure robust and adaptive control. In this context, Fuzzy Fractional Order Proportional Integral Derivative (FOPID-FLC) controllers are emerged as efficient approaches due to their flexibility and ability to handle nonlinearities and uncertainties. This paper proposes the use of a FOPID-FLC controller for a two-degree-of-freedom (2-DOF) lower limb exoskeleton. Our proposal is based on an enhanced control approach that combines fuzzy logic advantages and fractional calculus benefits. Contrary to popular existing methods, that use the FLC to tune the FOPID parameters, the FLC in this work is used to generate the system torque depending on patient morphology. Indeed, our fundamental contribution is to design and implement an enhanced FOPID-FLC that achieves an adequate optimal control based on system rules composed of optimal torques and input data. The fractional calculus is approximated using successive first order filters. Next, a multi-objective optimization is established for the tuning of each FOPID parameters. Finally, the FLC is used to adjust the torque depending on the kid’s age. The effectiveness of the proposed controller in various scenarios is validated based on numerical simulations. Extensive analyses prove that the FOPID-FLC outperforms the FOPID with a 90% of improvement in terms of error performance indices and 20% of improvement for the control action. Moreover, the controller exhibits improved robustness against uncertainties and disturbances encountered in rehabilitation environments.

Keywords—Rehabilitation robot; Fractional order PID controller; Fuzzy logic controller; Optimization; Genetic algorithm; Robustness.

I. INTRODUCTION

Lower limb dysfunction refers to impairments, limitations, or abnormalities in the lower limb’s function, including the hip, knee, ankle, and foot [1]. It is caused essentially by injuries, diseases affecting the musculoskeletal, neurological, or vascular systems [2]. Such dysfunction has significant implications for public health, healthcare systems, and individuals. Indeed, it causes reduced mobility, increased risk of falls, limitations in daily activities, loss of independence, and decreased overall well-being [3].

These symptoms’ severity depends on many factors including age, lifestyle, and health conditions [4]. Clinical studies prove that they are more severe in children. In particular, children affected by Cerebral Palsy (CP) risk to develop musculoskeletal degeneration, reduced muscle strength, joint stiffness, and balance impairments [5]. Rehabilitation and interventions targeting lower limb dysfunction play a crucial role in managing these symptoms, improving functional abilities, and enhancing the quality of life for affected individuals [6].

The physiotherapy treatment helps patients to restore their limited range of motion, restrengthen weak muscles, recover dynamic equilibrium, and thus progressively restore their movement ability [7]-[8]. Such treatment requires a lengthy, repetitive, and strict rehabilitation process [9]. Furthermore, due to time, effort and resource constraints, traditional rehabilitation cannot provide sufficient training frequency and intensity [10].

To substitute conventional training and its long-term process, assistive technologies are proposed as an efficient alternative to the traditional physiotherapy treatment [11]-[12]. Besides, rehabilitation robots aim to address the specific needs of individuals with lower limb impairments [13]-[14]. The latter are machines that are designed to assist people with disabilities in regaining their physical or cognitive abilities [15]-[18].

Indeed, we distinguish robots designed to help children with specific physical tasks, such as walking or grasping objects [19][20]. Another category of robots is used to help children with cognitive tasks, such as memory and attention [21]. In [22], authors design and implement "ArmeoSpring,” a robotic arm is used to help children with CP to improve their upper
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Tuning all of these FOPID parameters with the fuzzy logic controller is a challenging task since it requires accurate experimentation. Hence, a multi-objective optimization approach is adopted to adjust them to fit with the specificities of the intended age. Besides, the (Fuzzy Logic) FL controller is deployed to guarantee an adaptive and robust torque. Such a controller uses fuzzy logic-based rules to ensure a dynamic switching between the optimal FOPID controllers providing that high versatility and accuracy.

It is worthwhile to note that our proposal is not applied only to patients affected by cerebral palsy but it can be applied also to other cases of lower limb impairments (stroke, spinal cord injury, and epilepsy). The proposed controller is tested on a 2-DOF (degrees of freedom) gait rehabilitation robot, where the joint angle trajectories are tracked in real-time. The proposal is further compared with a traditional FOPID controller. Simulation results show that the FOPID-FLC controller outperforms the legacy FOPID controller in terms of tracking accuracy, adaptability and robustness.

The followings are the fundamental contributions of this research work:

- The design of an efficient and robust fuzzy logic-based control strategy namely the FOPID-FLC for the lower limb exoskeleton for rehabilitation applications. This control strategy combines both the flexibility of the fractional FOPID and the adaptability of the FLC. It depends on a set of rules basically depending on the patient age. The decision is an optimized controller.
- A mathematical model of 2-degrees of freedom (DOF) nonlinear and coupled lower-limb exoskeleton is developed with Simulink/MATLAB, taking into account both the parameters of the patients and the robot.
- Oustaloup approximation is used for implementing the set of controllers. Some frequency conditions are detailed for satisfying the fractional calculus.
- Since the performance of a genetic algorithm can be highly dependent on the choice of the fitness function, a multi-objective function for the optimization process is proposed to ameliorate the convergence to the minimum error rate and energy.
- A comparative study with a highly optimized FOPID controller is conducted. The FOPID-FLC admits better performances than this controller.
- Finally, the robustness of the proposed control strategy is tested vs random disturbances.

The mathematical model of the exoskeleton is elaborated in section 2. Section 3 focuses on the design of the control strategy: the explication of the fractional calculus background and the fuzzy strategy, and finally the implementation of the controller. Simulation results are presented and discussed in section 4. The paper is concluded in section 5.

**II. THE LAGRANGIAN DYNAMIC EQUATION OF THE SYSTEM**

The main goal of using an exoskeleton is to aid in the rehabilitation process for subjects suffering from the lower limb impairment due to a neurological injury, by strengthening their neuro-plasticity [78]. The therapist defines the activities to be performed by the exoskeleton and notes the trajectories to be performed in passive or active exercises. These trajectories are presented in Fig. 1, in which the desired movements for both the hip joint and the knee joint are recorded during the walking cycle of kids. In fact, it shows the degrees of two angles variation until the movement process.

An exoskeleton is defined as an external wearable mechanism that is worn and moves parallel to the human body. In general, exoskeletons of lower extremities have the mechanical structure shown in Fig. 2 where all the parameters of both kid and exoskeleton of the two degrees of freedom (2 DOF) model are presented. It is composed of three vectors:

- \( q = [q_1, q_2]^T \in \mathbb{R}^2 \) denotes the position vector,
- \( \dot{q} = [\dot{q}_1, \dot{q}_2]^T \in \mathbb{R}^2 \) presents the speed vector,
- \( \ddot{q} = [\ddot{q}_1, \ddot{q}_2]^T \in \mathbb{R}^2 \) corresponds to the acceleration vector.

The dynamic model relates both the manipulator and the patient leg parameters. It is presented as follows:

\[
I(q)\ddot{q} + N(q, \dot{q})\dot{q} + G(q) = u
\]  

(1)

It is composed by the following matrices and vectors:

- \( I(q) \in \mathbb{R}^{2 \times 2} \) presents the inertia matrix
- \( N(q, \dot{q}) \) presents the Coriolis and centrifugal forces
- \( G(q) \) presents the gravitational forces
- \( u \) presents the external input forces

![Fig. 1. Children gait cycle: (a) hip movement (b) knee movement](image_url)
lower and the upper terminals of the operations respectively, and \( \alpha \) is the fractional order as it is presented in [80]-[81]:

\[
aD_t^\alpha = \begin{cases} 
    \frac{d^\alpha}{dt^\alpha} & \text{if } \alpha > 0 \\
    1 & \text{if } \alpha = 0 \\
    \int_t^\infty (dt)^{-\alpha} & \text{if } \alpha < 0
\end{cases}
\]  

(2)

Oustaloup approximation, also known as the continued fraction expansion method, is a mathematical technique used to approximate the transfer function of a system in the frequency domain. This approach was developed by the French engineer Alain Oustaloup in the 1990s, and it is widely used in the control system engineering [82]-[83].

The advantage of the Oustaloup approximation is that it provides a very accurate approximation of the fractional system with only the requirement of a small number of filter coefficients. This makes it a very efficient and computationally inexpensive technique [84].

The Oustaloup approximation is a useful tool for implementing fractional order PID controllers, as it provides a simple and systematic way to approximate fractional order transfer functions [85]. It approximates the fractional term in a frequency band \([\omega_l, \omega_h]\) generally chosen as:

\[
\frac{\omega_h}{\omega_l} = 10^3
\]

(3)

This approach offers a practical way to approximate the fractional order derivative operators using a series of simpler first order filters. It allows for more manageable implementation and analysis of complex systems obey to (4):

\[
D^\alpha = \prod_{i=-N}^{N} \frac{1 + \frac{\alpha}{\omega_i^l}}{1 + \frac{\alpha}{\omega_i^h}}
\]

(4)

For \( N = 3 \), seven successive serial rational filters are chosen in this case as it is illustrated in Fig. 3. The roots presented in this figure (zeros and poles) of each rational filter are \((-\omega_i^l)\) and \((-\omega_i^h)\) admit respectively (5) and (6):

\[
\omega_i^l = \omega_l \left( \frac{\omega_h^l}{\omega_l^l} \right)^{\frac{\omega_l^h - \omega_l^l}{2N + 1}}
\]

(5)

\[
\omega_i^h = \omega_l \left( \frac{\omega_h^l}{\omega_l^l} \right)^{-\frac{\omega_l^h - \omega_l^l}{2N + 1}}
\]

(6)

In this case, two frequency bands are considered: \((\omega_i^l)\) and \((\omega_i^h)\) for the fractional integral and \((\omega_i^l)\) and \((\omega_i^h)\) for the fractional derivative terms ensuring (7) and (8):

\[
\omega_h^l = 10^{(2^\frac{1}{2})} \omega_i^l
\]

(7)

\[
\omega_h^h = 10^{(2^\frac{1}{2})} \omega_i^l
\]

(8)

For more clarity, the repatriation of the frequency band is presented in Fig. 4 where the behavior of the fractional filters

III. CONTROL STRATEGY

Rehabilitation robots are designed to assist individuals with physical disabilities or injuries in improving their motor function and mobility. These robots typically use various sensors and actuators to interact with the user and provide feedback on their movements. Control systems, such as fuzzy fractional order PID controllers, can be used to improve the accuracy and stability of the robot’s movements and to ensure a safe and an effective rehabilitation process.

A. Overview about Fractional Calculus

In terms of sensitivity and process improvement in response to variations dealing with perturbations or parameters, fractional order controllers exceed integer-order controllers [79]. The generalization of the integration and differentiation operations to the non-integer order operator \( aD_t^\alpha \) where \( a \) and \( t \) denote the

\[
\alpha = \{ 0, 1 \}
\]

Fig. 2. Dynamic presentation of the proposed exoskeleton

- \( N(\dot{q}, q) \in \mathbb{R}^{2 \times 2} \) corresponds to the coriolis, centrifugal forces and torques
- \( G(q) \in \mathbb{R}^2 \) represents the gravity torque vector
- \( u \in \mathbb{R}^2 \) is the vector of torques

The system characteristics used for the dynamic representation are designed as follows:

- \( m_1, m_2, l_1, l_2 \) are respectively the masses and the lengths dealing with thigh and shank segments of the exoskeleton respectively.
- \( m_t, m_s, l_t, l_s \) represent the thigh and the shank masses and lengths of the human limb respectively. These parameters depend on the kid’s age.
- \( g \) is the gravity acceleration.

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is described. It shows the location of the frequency parameters of the used FOPID controller. By this way, the FOPID behavior helps in designing and tuning an efficient controller in order to achieve the desired performance and stability.

B. The Fuzzy Fractional Order Controller

In the last decade, the extension of the application of the fractional order systems (FOS) determined a relevant increase in the control techniques [86]. Conventional controllers, including the fractional-order controllers, operate on error inputs with fixed parameters value of proportional, integral, and derivative terms. As a result, the controller performance is insufficient for a nonlinear and complex system. Instead of a fixed controller, the idea can be made to incorporate dynamic system switch between fractional order controllers with optimal parameters values. This dynamic behaviour improves the system’s structure, performs the control process and ensures a rapid convergence of the system output [87].

The basic structure of a fractional order PID controller consists of three components: the proportional, the integral, and the derivative components. The proportional component produces an output that is proportional to the error signal, while the integral component produces an output that is proportional to the fractional integral of the error signal. The derivative component produces an output proportional to the error signal’s with fractional derivative action [88].

In 1994, I. Podlubny presented the Fractional Order PID (FOPID) for controlling automatic systems. In fact, this controller is more flexible, offers a better chance to adjust the dynamic properties and provides robustness against variations of the parameters of the non-linear systems [89].

A fuzzy fractional order PID controller that combines the advantages of fuzzy logic and fractional calculus is used in this control system. It is designed to improve the performance of the control system by adjusting the output of the controller based on the error signal and the input age of the child. It allows us to obtain an adequate and optimal torque depending on the morphology of the patients (kids) in order to achieve a good control approach.

The variable masses of the thigh and the shank of the kid’s lower limb, are presented in Table I depending on the age. The mean values of the masses are taken at seven years old and the others are calculated with analogy [90]. The fuzzy logic rules define the relationship between the input variables (the age of the patient) and the output variables (the adequate torque generated for the robot). These rules are based on an expert knowledge of the rehabilitation process and the specific characteristics of the children with different ages.

Multiple Gaussian membership functions are used to define the fuzzy variables, each of them represents a different linguistic term (s: small, sm: small-medium, m:medium, mh: medium-high, h: high). Five rules are concluded for the fuzzy control process:

- **Rule 1**: If (patient is s) Then \( u = u_s \)
- **Rule 2**: If (patient is sm) Then \( u = u_{sm} \)
- **Rule 3**: If (patient is m) Then \( u = u_m \)
- **Rule 4**: If (patient is mh) Then \( u = u_{mh} \)
- **Rule 5**: If (patient is h) Then \( u = u_h \)

Accordingly, for each age \( j \) of \( s, sm, m, mh, h \) and for every joint \( k \) of 1, 2, each FOPID computes the torque \( u(t) \) through the following differential (9):

\[
U_{k_j}(t) = (K_{pk_j}e_k(t) + K_{ik_j}I_{\lambda_j}e_k(t) + K_{dk_j}D_{\mu_j}e_k(t)) \quad (9)
\]

with

- \( e_k(t) = q_{dk}(t) - q_k(t) \) represents the error between the reference and the actual position.
- \( \lambda_j \) and \( \mu_j \in [0,1] \) are respectively the fractional integrator and deribrator orders.
- \( K_{pk_j}, K_{ik_j} \) and \( K_{dk_j} \) are the gains of the FOPID controllers.
- \( I \) and \( D \) are respectively the fractional integrator and deribrator operators.
TABLE I
DIFFERENT MASS MEAN VALUES OF KIDS LIMB PROPERTIES

<table>
<thead>
<tr>
<th>age</th>
<th>2 years (s)</th>
<th>5 years (sm)</th>
<th>7 years (m)</th>
<th>9 years (mh)</th>
<th>12 years (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shank mass (kg)</td>
<td>0.541</td>
<td>0.812</td>
<td>1.082</td>
<td>1.623</td>
<td>2.164</td>
</tr>
<tr>
<td>Thigh mass (kg)</td>
<td>1.7675</td>
<td>2.65125</td>
<td>3.535</td>
<td>5.3025</td>
<td>7.07</td>
</tr>
</tbody>
</table>

To improve the control signal of the FOPID$_{kj}$, the transfer function can be presented by the following equation:

$$u_{kj}(s) = \left( K_{pkj} + K_{ikj} \left( \frac{\omega_{j}}{s} \right)^{\lambda} + K_{dkj} \left( \frac{s}{\omega_{Dj}} \right)^{\mu} \right) e_k(s)$$  \hspace{1cm} (10)

with

$$\omega_j \in [\omega_j', \omega_j'']$$  \hspace{1cm} (11)

and

$$\omega_{Dj} \in [\omega_D', \omega_D'']$$  \hspace{1cm} (12)

All the parameters presented in (10) are optimized via the genetic algorithm process. Generating these flexible parameters, the fuzzy logic system is used to handle the linguistic variables and uncertainties. The general control structure of the proposed system is illustrated in Fig. 5. It presents the closed loop of the control strategy, with the non-linear dynamics, the input and the output of the FOPID controllers, the optimization method, and the principles of FLC. It is a general presentation of the system working process.

The membership function defines the degree to which each input variable belongs to a particular fuzzy set. These functions are defined based on the physical capabilities of children of different ages. Five Gaussian membership functions centered at different age values represent each linguistic term and Fig 6 illustrates the graphic presentation of these functions:

$$F_j(x) = e^{-\frac{-(x-x_j)^2}{\sigma_j^2}}$$  \hspace{1cm} (13)

With $x_j \in \{2, 5, 7, 9, 12\}$. These equations describe the fuzzy sets reflecting the degrees of membership ranging from 0 to 1.

C. Combined Optimization of Controllers Parameters

Frequently the objective function minimization based on gradient traditional methodologies confuses local minima. This can be solved by using a dynamic algorithm, such as the Genetic Algorithm (GA), which is employed in this work to obtain the optimal set of controller parameters [91].

Genetic Algorithm is one of the most important evolutionary algorithms. It was introduced by Holland in the 1960s with three important operators which are the crossover, mutation, and selection [92]. The implementation of these operators is highly dependent of the way of encoding. The structure of the genetic algorithm can be described in the pseudocode (Algorithm 1).

Three famous minimization objective functions are used to evaluate the fitness of the controllers:

- $O_1 = IAE_1 + IAE_2$
- $O_2 = IAU_1 + IAU_2$
- $O_3 = IAT_{E_1} + IAT_{E_2}$

dealing with:

- The integral of the absolute error is
  $$IAE_k = \int_0^\infty |e(t)| dt$$  \hspace{1cm} (14)

- The integral of the control signal is
  $$IAU_k = \int_0^\infty u_k(t) dt$$  \hspace{1cm} (15)

- The integral of the time absolute error is
  $$ITAE_k = \int_0^\infty t|e_k(t)| dt$$  \hspace{1cm} (16)

with $k \in \{1, 2\}$ and reffering to hip and the knee respectively.

By repeating the above process over multiple generations, the genetic algorithm explores the solution space and converges toward the optimal solution. The optimal values of the different types of FOPID controllers obtained after iterations are collected in Table II.

The flowchart of the system is described in Fig. 7 providing more details about the control strategy. Basically, the necessary parameters of each FOPID controller are optimized via GA method. The developed process uses three objective functions to manipulate three sets (mutation, selection, and crossover). These fitness functions are based on performance criteria (IAE, IAT, IAU).

In the beginning, the reference signals $(q_{d1}, q_{d2})$ are inputted into the control system. Sensors provide feedback on the actual state of the lower limb exoskeleton’s position. This feedback is used to determine the current positions and velocities of joints $(q_1, q_2, \dot{q}_1, \dot{q}_2)$.

Then, the error ($e$) between the desired and the real trajectories is loaded in the optimal FOPID controllers to generate five optimal torques. These torques are calculated using the parameter blocks presented in the diagram. On another side, the inputted kid’s (patient) age ($x_j$) is executed by the five membership functions.

Next, the collected data (the outputted torques ($U_j$)) and the outputted functions $F_j(x)$ are treated by the make-decision process based on the Fuzzy Logic (FL) rules. The result is an outputted torque ($U_{FLC}$) that is optimal, adaptive and robust.
Algorithm 1 The genetic algorithm process

Begin
1. Population initialization: start with a randomly set of values of chromosomes which are the candidate solutions of the first population formulated from 12 chromosomes (gains, frequencies and orders).
2. Innovate the limits of the roots (gains [1 500], frequencies [0 20] and orders [0 1]) which are the specific bounds or the searching spaces.
3. While (i < 20 iterations)
   3.1 Evaluate the fitness of the individuals using O1*O2*O3
   3.2 Perform the selection: choose the most fittest two chromosomes as parents
   3.3 while (j < 12)
       3.3.1 Perform crossover randomly with 0.5 probability
       3.3.2 Perform mutation randomly with 0.5 probability
   end while (next individual test)
end while (next generation test)
End

IV. NUMERICAL EXPERIMENTS

From the research results, two scenarios are discussed in further detail. The first is dealing with testing the efficiency of the controller of the nominal case with the different ages. The second study is the influence of the distributions on the preferences of the robot. In addition to the performance used in the GA process, ISE, ITSE, and MSE are also the three performance criteria that are used in control systems engineering to evaluate the performance of the feedback control systems.

ISE quantifies the cumulative error between the desired input and the actual output of the system over a specified time interval. It is calculated via (17):

$$ ISE = \int_0^\infty e^2(t)dt $$

(17)

ITSE stands for Integral of Time Squared Error and is defined by (18). It is used to evaluate the performance of a control system in terms of its ability to track a reference signal over time.

$$ ITSE = \int_0^\infty te^2(t)dt $$

(18)

While, MSE determines the Mean Squared Error presented by (19) and it is defined as the average of the squared error over a specified time period. It is used to evaluate the overall accuracy.
TABLE II
THE OPTIMAL VALUES OBTAINED WITH GA PROCESS

<table>
<thead>
<tr>
<th>Tuned parameters</th>
<th>Designation</th>
<th>( FOPID_h )</th>
<th>( FOPID_{hm} )</th>
<th>( FOPID_m )</th>
<th>( FOPID_{ms} )</th>
<th>( FOPID_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hip controller gains</td>
<td>( K_{1p} )</td>
<td>400</td>
<td>250</td>
<td>300</td>
<td>350</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>( K_{1i} )</td>
<td>450</td>
<td>100</td>
<td>300</td>
<td>200</td>
<td>250</td>
</tr>
<tr>
<td></td>
<td>( K_{1d} )</td>
<td>200</td>
<td>235</td>
<td>200</td>
<td>250</td>
<td>200</td>
</tr>
<tr>
<td>Knee controller gains</td>
<td>( K_{2p} )</td>
<td>300</td>
<td>175</td>
<td>150</td>
<td>400</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>( K_{2i} )</td>
<td>300</td>
<td>50</td>
<td>100</td>
<td>75</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>( K_{2d} )</td>
<td>200</td>
<td>100</td>
<td>70</td>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>Fractional orders</td>
<td>( \lambda )</td>
<td>0.8</td>
<td>0.5</td>
<td>0.75</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>( \mu )</td>
<td>0.8</td>
<td>0.8</td>
<td>0.75</td>
<td>0.8</td>
<td>0.7</td>
</tr>
<tr>
<td>Integrator frequency parameters</td>
<td>( \omega_l' )</td>
<td>0.8</td>
<td>0.7</td>
<td>0.860</td>
<td>0.9</td>
<td>0.826</td>
</tr>
<tr>
<td></td>
<td>( \omega_f )</td>
<td>1.102</td>
<td>1.25</td>
<td>1</td>
<td>1.5</td>
<td>1.014</td>
</tr>
<tr>
<td></td>
<td>( \omega_{\mu} )</td>
<td>25.29</td>
<td>22.135</td>
<td>27.19</td>
<td>28.460</td>
<td>26.12</td>
</tr>
<tr>
<td>Derivative frequency parameters</td>
<td>( \omega_l'' )</td>
<td>3.8</td>
<td>5.87</td>
<td>3</td>
<td>10.2</td>
<td>3.825</td>
</tr>
<tr>
<td></td>
<td>( \omega_D )</td>
<td>5.1</td>
<td>7.5</td>
<td>5</td>
<td>15.3</td>
<td>5.102</td>
</tr>
<tr>
<td></td>
<td>( \omega_{\mu} )</td>
<td>120.166</td>
<td>185.62</td>
<td>94.86</td>
<td>322</td>
<td>120.957</td>
</tr>
</tbody>
</table>

Fig. 6. Membership functions plot

of a control system in terms of its ability to maintain the desired output.

\[
MSE = \frac{1}{t} \int_0^\infty te^2(t)dt
\]  

ISE, ITSE and MSE are important performance criteria that are used to assess the effectiveness of control systems in achieving their intended goals. They provide a quantitative measure of the system’s performance and can be used to compare different control strategies or to optimize control system parameters. These measures are used to evaluate the accuracy and effectiveness of simulation results.

A. Experiments with Different Ages

The main difference between the FOPID and the FOPID-FLC controllers stands in their tuned parameters. The FOPID controller uses also the genetic optimization algorithm to determine its fixed values (including orders, gains and frequencies). On the other hand, the FOPID-FLC controller uses FLC to automatically switch between optimized fractional order controllers FOPIDs with different sets based on the input age.

The large gap of 10 years (2-12) leads to a significant difference in the anatomy and morphology between children. This difference can sometimes seem between children of the same age. Therefore, rehabilitation robots should adapt to the needs of different ages and provide individualized care for patients. In this case, simulation results are presented with different ages included in the selected band to prove the performance.

For the age of eight, Fig. 8 and Fig. 9 show the positions, errors, speeds, and torques of joints 1 and 2 using respectively the FOPID-FLC and the FOPID controllers. For more clarification, Tables III, IV, V, and VI illustrate the selected performance criteria values and improvements of the two controllers for the age of three, four, eight and eleven years old respectively.

The first curves in the presented figures illustrate the path of the desired and the real trajectories of both joints. Through these curves, we can conclude that using FOPID-FLC controller, the system tracks more accurately the referenced data than using FOPID. The proposed controller allows the adaptation of control parameters and the specific requirements of the robot model. Consequently, it enables flexible movement and effective tracking of the desired trajectories.

Besides, referred to error curves depicted in Fig. 8 and Fig. 9, FOPID-FLC is able to achieve a better performance, in terms of overshoot, than FOPID. It minimizes the discrepancy between the desired and actual positions in the startup gait cycle. Thus,
\[ \ddot{q} = I(q) - (u - N(\dot{q}, q)\dot{q} - G(q)) \]

Fig. 7. The flowchart of the whole system

Fig. 8. FOPID-FLC results for the joint 1 and 2 with the age of eight years old
it reduces the IAE performance. Values collected in Table V prove this deduction.

Energy efficiency is crucial for prolonged operation and patient comfort. The FOPID-FLC optimizes the control action based on the patient’s age. Torque curves are illustrated in the given figures. By adapting the control parameters and optimizing the control effort, the proposed controller can minimize unnecessary energy and reduce the IAU value. This is demonstrated for all the selected ages in Tables III, IV, V, and VI. Thus, using the proposed controller leads to more efficient

Intissar Zaway, A Robust Fuzzy Fractional Order PID Design Based On Multi-Objective Optimization For Rehabilitation Device Control
TABLE V

<table>
<thead>
<tr>
<th>Performance criteria</th>
<th>Joint1</th>
<th>Joint2</th>
</tr>
</thead>
<tbody>
<tr>
<td>IAE</td>
<td>0.040</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>33%</td>
<td>54%</td>
</tr>
<tr>
<td>ISE</td>
<td>0.5e-3</td>
<td>1.4e-3</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>22%</td>
</tr>
<tr>
<td>IAU</td>
<td>16.6</td>
<td>11.57</td>
</tr>
<tr>
<td></td>
<td>12%</td>
<td>86%</td>
</tr>
<tr>
<td>IATE</td>
<td>0.093</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>12%</td>
<td>86%</td>
</tr>
<tr>
<td>ITSE</td>
<td>9.5 e-4</td>
<td>3 e-3</td>
</tr>
<tr>
<td></td>
<td>26%</td>
<td>33%</td>
</tr>
<tr>
<td>MSE</td>
<td>1.9 e-5</td>
<td>6.1 e-5</td>
</tr>
<tr>
<td></td>
<td>22%</td>
<td>30%</td>
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</table>

TABLE VI

<table>
<thead>
<tr>
<th>Performance criteria</th>
<th>Joint1</th>
<th>Joint2</th>
</tr>
</thead>
<tbody>
<tr>
<td>IAE</td>
<td>0.067</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>17%</td>
</tr>
<tr>
<td>ISE</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>40%</td>
</tr>
<tr>
<td>IAU</td>
<td>27.13</td>
<td>21.01</td>
</tr>
<tr>
<td></td>
<td>3%</td>
<td>2%</td>
</tr>
<tr>
<td>IATE</td>
<td>0.165</td>
<td>0.124</td>
</tr>
<tr>
<td></td>
<td>12%</td>
<td>3%</td>
</tr>
<tr>
<td>ITSE</td>
<td>0.003</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>33%</td>
<td>36%</td>
</tr>
<tr>
<td>MSE</td>
<td>6.3 e-5</td>
<td>4 e-5</td>
</tr>
<tr>
<td></td>
<td>36%</td>
<td>36%</td>
</tr>
</tbody>
</table>

TABLE VII

<table>
<thead>
<tr>
<th>Performance criteria</th>
<th>Joint1</th>
<th>Joint2</th>
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</thead>
<tbody>
<tr>
<td>IAE</td>
<td>0.120</td>
<td>0.138</td>
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<tr>
<td></td>
<td>70%</td>
<td>90%</td>
</tr>
<tr>
<td>ISE</td>
<td>0.021</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>71%</td>
<td>78%</td>
</tr>
<tr>
<td>IAU</td>
<td>37.05</td>
<td>27.13</td>
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<tr>
<td></td>
<td>10%</td>
<td>2%</td>
</tr>
<tr>
<td>IATE</td>
<td>0.154</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>93%</td>
<td>80%</td>
</tr>
<tr>
<td>ITSE</td>
<td>0.029</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>99%</td>
<td>90%</td>
</tr>
<tr>
<td>MSE</td>
<td>4e-3</td>
<td>2 e-4</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>95%</td>
</tr>
</tbody>
</table>

TABLE VIII

<table>
<thead>
<tr>
<th>Performance criteria</th>
<th>Joint1</th>
<th>Joint2</th>
</tr>
</thead>
<tbody>
<tr>
<td>IAE</td>
<td>0.138</td>
<td>0.435</td>
</tr>
<tr>
<td></td>
<td>90%</td>
<td>94%</td>
</tr>
<tr>
<td>ISE</td>
<td>0.010</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>9%</td>
<td>78%</td>
</tr>
<tr>
<td>IAU</td>
<td>27.13</td>
<td>21.01</td>
</tr>
<tr>
<td></td>
<td>1%</td>
<td>2%</td>
</tr>
<tr>
<td>IATE</td>
<td>0.323</td>
<td>0.182</td>
</tr>
<tr>
<td></td>
<td>93%</td>
<td>80%</td>
</tr>
<tr>
<td>ITSE</td>
<td>0.022</td>
<td>0.043</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>97%</td>
</tr>
<tr>
<td>MSE</td>
<td>4.5 e-3</td>
<td>8.6 e-3</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>94%</td>
</tr>
</tbody>
</table>

energy consumption and safety of mechanical system structure than the legacy FOPID.

Simulation results show that the FOPID-FLC achieves higher tracking accuracy and limited energy compared to the FOPID controller. The referenced tables record the improvements of the performance indices of joints 1 and 2 until using FOPID-FLC instead of FOPID for different chosen ages of children. It achieves 83% of improvements in terms of IAE, 50% of improvements in terms of ISE, 19% of improvements in terms of IAU, 86% of improvements in terms of IATE, 75% of improvements in terms of IATSE and 90% of improvements in terms of MSE.

The incorporation of fuzzy logic with FOPID controllers allows more precise control actions, minimizes unwanted oscillatory behavior and provides a good system response. Despite this, the fuzzy FOPID controller presents more challenges in the design and implementation compared to the FOPID. The proposed implementation requires trial and error for the membership presentation, the searching space of optimal solutions, and the rules making.
B. Robustness Against Disturbances

It is particularly important in rehabilitation settings that the system should be robust to many variations in the environment. For this purpose, FOPID-FLC is emerged as a great controller delivering more safety for children and good functioning of the robot.

A random disturbance is used to test this efficiency. Fig. 10 presents the response of the two controllers versus this disturbance for both joints 1 and 2. Regarding this response, we can conclude that the FOPID-FLC shows its robustness against noise more effectively than FOPID.

Based on the selected performance criteria, a comparison between FOPID-FLC and FOPID is established in Tables VII and VIII for the age of three and eight years respectively. As it is revealed that the system with FOPID-FLC indicates more stability and well-controlled response than FOPID.

The improvement in % is also illustrated in these tables. The FOPID-FLC achieves 90% of improvements for the error indices and 10% of improvements in terms of the control action. Using the Fuzzy FOPID controller enables the robot to be more adequate and less sensitive to disturbances and uncertainties. Thus, it potentially reduces the overshoot thanks to its dynamic behavior.

Our previous work [93] conducts an extensive study of using PID controller optimized by GA process to control the proposed dynamic model. The finding proves that the IAU performs 29.04 and 20.05 for joint 1 and joint 2 respectively. Moreover, in this research work, at the age of eight years old, the IAU performs better, it reaches 16.3 for the hip joint and 11.57 for the knee joint.

Additionally, results illustrate that the robustness in the uncertainty case of the FOPID is studied varying 5% and 10% the shank and thigh masses. There is proven that the IAE is adjusted with 6%. Referred to the given tables, this proposed FOPID-FLC is still better than the FOPID in terms of tracking and robustness although the mass variations reach the ±50%.

These findings are due to the flexibility of the FOPID and the adaptability of the FLC to enhance the used torque in the control process.

In comparison also to a fractional order combined with sliding mode controller (SMC) that uses the grey wolf optimization technique for a lower limb rehabilitation exoskeleton [94], the ISE performance criteria, for the nominal case, reaches \(2 \times 10^{-3}\) and \(7 \times 10^{-3}\) for joint 1 and joint 2 respectively. In our proposed research, the ISE is reduced to \(0.4 \times 10^{-3}\) for joint 1 and \(1.4 \times 10^{-3}\).

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for joint 2. When uncertainties are considered, the ISE criteria of the present work are of $1 \times 10^{-3}$ and $5 \times 10^{-3}$ for joint 1 and joint 2, respectively, rather than $3 \times 10^{-3}$ and $7 \times 10^{-3}$ in the precited work.

As another example in the same study [94], in the nominal and uncertainty cases, the IAE performance criteria reaches $1 \times 10^{-3}$ for both joints. Moreover, in this proposed work, it is comprised between $1.3 \times 10^{-3}$ and $5.7 \times 10^{-3}$. Thus, we can conclude that the FL associated with FO control holds significant improvements in terms of stability and smoothness compared to FO with SMC. However, the latter is used to emphasize the accurate tracking of the desired joint angles or positions.

Further, it’s important to note that the choice of such controllers depends on the specific control problem, the dynamic model and the application conditions. In summary, both FOPID and FLC controllers are effective control systems for handling complex processes. Moreover, by combining fuzzy logic and fractional calculus, the control system provides more efficiency. It produces a robust and adaptive control for the lower limb exoskeleton.

The fuzzy fractional order PID controller with dynamic switching enhances patient adaptability, improves control performance, optimizes robustness to uncertainty, offers flexibility in control design, and adapts nonlinear and time-varying systems. These benefits make this controller an attractive approach for achieving optimal control in various applications.

FOPID-FLC controller has the added benefit of automatic parameterize but it requires more challenges in designing and implementation. Such as the input variables could include joint angles, muscle activation levels, patient feedback, and unknown parameters. These control approach limitations should be considered in the next work.

V. Conclusion

Rehabilitation robots are considered great equipment for the treatment of children with cerebral palsy. In this paper, an enhanced control approach is proposed at the aim of providing targeted therapy that deals with the specific needs of each child. The proposed FOPID-FLC ensures high accuracy, through the use of both the fuzzy logic approach and fractional order PID controller. The latter is established via outstalou approximation and optimized via the GA process to improve the trajectory tracking performance. The former is employed to obtain a highly accurate estimation of the torque. Findings show that our proposal does not only ensure good performance but it also outperforms the traditional FOPID controller. In particular, at the age of four years old, for joint 2, the ITSE and the MSE are enhanced by 90%. In addition, at the age of three years old, for joint 1, the IAE enhancement is up to 60%.

In the future, we are working on the optimization of our controller through the consideration of joint angles, muscle activation levels, and unknown patient and environmental characteristics. The implementation of the optimal solutions of FOPID controllers using the Gradient-Based Optimizer (GBO) process will be also considered. Further studies and clinical trials are recommended to validate the controller’s efficacy in real-world rehabilitation settings and assess its impact on patient recovery.

References

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