Mathematical Model of a Robot-spider for Group Control Synthesis: Derivation and Validation

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Abstract—A six-legged spider robot is a complex object from the point of view of the problem of synthesizing a system for controlling its movement. To synthesize an advanced control system for such a robot, which must solve non-trivial problems of overcoming obstacles, functioning under conditions of external disturbances, etc., we first solve the problem of synthesizing an information model of the object, on the basis of which its control system will subsequently be built. The paper compares two methods for synthesizing the information model of a six-legged spider-robot. In the first method, an information model is automatically synthesized from a CAD model of a spider-robot in a MATLAB-based graphical programming environment Simulink. In the second method, the information model is synthesized in the environment of dynamic modeling of technical systems SimInTech on the basis of a system of differential equations in the Cauchy form. Control loops and external influences are added to the information models synthesized in each of the modeling environments. The study showed that each of the resulting models has both its own individual advantages and disadvantages. They are mainly related to taking into account the mutual influence of various blocks of models on each other. It is shown that, in the end, the two models complement each other and make it possible to obtain an advanced basis for further synthesis of the motion control system. The results obtained in this work make it possible to use information models as a basis for the development of a control system for a physical model of a six-legged spider-robot, printed on a 3D printer and assembled on the basis of the Arduino hardware platform.

Keywords—Automatic Control; Hexapod; Complex System; Motion Control; Walking Robot; Interaction Forces; Degrees of Freedom; Information Model; Programming; Simulink; Arduino.

I. INTRODUCTION

This work investigates the problem of group control of such a complex mechanical object as a walking robot with many legs. Specifically, an example with a six-legged spider robot is considered. This research is part of a larger effort to develop a walking robot for autonomous underwater operations. Since the bottom usually has a complex surface, and the nature of the soil complicates the task of moving along it due to its mobility and looseness, the task of autonomous movement control by moving multiple robot manipulators becomes extremely difficult. The same may apply to the movement of such a robot in the air.

In this work, research is continued in the field of synthesis of control of a group of autonomous walking robots. Our task was to compare the mathematical model synthesized in [1] with the automatically synthesized model of the spider-robot, as well as, if possible, to refine the previously obtained mathematical model.

Automatic synthesis of an information model by internal means of a program based on a 3D model does not always allow influencing the final result of synthesis, which leads to the concept of a black box, in which it is impossible to change some of the fundamental foundations of the model. Building a model based on a mathematical model entails overly complicating the design of the model, in which the fundamentals are set manually. Joint consideration of two information models will allow us to supplement and clarify our data on the object under study.

The paper compares and tests the mathematical model of a spider-robot simple momented in the dynamic modeling environment of technical systems SimInTech [2] and an information model of a spider-robot automatically synthesized in MATLAB-based graphical programming environment Simulink.

The article includes a description of the design of a six-legged spider robot, followed by a description of its mathematical model. Next, based on the mathematical model, a description of the physical model and a description of the computational experiment are derived. In conclusion, a comparative analysis of the results obtained and a conclusion about the quality and effectiveness of each of the considered methods is made.

II. METHOD

The method used in this study will be discussed in detail further in three sections. In the section on designing a spider robot, we describe its structure and review the main models presented today in the world scientific literature. The next section provides a description of the mathematical models of the spider robot, including a description of the specific parameters of the designed models. Then, all presented models are linked to a specific prototype created in our laboratory.

The section following this describes the results of computational experiments obtained on synthesized models. This approach allows not only to compare the quality of each
model, but also to analyze the results obtained by checking them on the physical model of a particular robot.

III. DESIGNING A SPIDER-ROBOT

A spider-robot is a multi-legged (usually four, six, or eight legs) walking robot with limbs consisting of one or more links.

Designing a spider-robot information model is challenging due to the large number of degrees of freedom. The more limbs and links, the more nonlinear equations describing the system and the more complex the resulting mathematical model. Difficulties associated with the derivation of a mathematical model of a spider-robot force researchers to look for alternative methods for synthesizing information models.

In [3], researchers simulate a robot with eight legs, which uses an eccentric cylindrical cam mechanism for movement, powered by only two actuators. In [4], the authors, using the SimMechanics tools, synthesize the informational model of the crab. In [5] in SimMechanics the fast motion of a multi-legged dynamic model is simulated.

There are works devoted to the study of individual limbs of a spider-robot. In [6] the adhesion of the limb to the surface is studied; in [7], [8], [9] and [10] they model a limb based on data obtained from living spiders; in [11], researchers use a spring-loaded inverted pendulum model as a limb mechanism, which allowed them to imitate walking, 43 running and jumping; and in [12] the results of the development of the limb of the arachnid monkey are given. The articles [13] and [14] provide solutions to the inverse kinematic problem. A detailed analysis of the spider limb using the results is given in article [15].

There are works devoted to the study of motion algorithms. In [5], the authors studied the movements of a real spider to determine a suitable movement pattern. In [16] and [17], using neural networks of a robot-spider, they teach how to adapt to different types of surfaces. In [18] and [19], a method for moving a spider-robot in tunnels is given. In works [20], [21] and [22] the problem of finding a set of admissible coordinates of extremities, ensuring the stability of a spider-robot, is solved. Physical models of spider-robots capable of working in extreme conditions are described in [23], [24] and [25]. In [26], a robot with hinged limbs is described, which allows servicing complex pipe-wire systems. In [27], the authors present the design of a robot capable of moving in a wide range of two-dimensional tunnels. In [28], a "smart" spider-robot is presented, capable of wireless monitoring of the environment.

IV. CREATING MODELS

It is necessary to synthesize and test information models of a six-legged spider-robot with three-link limbs (see Fig. 1) in two environments for dynamic modeling of technical systems: Simulink and SimInTech. A spider-robot is described by a vector of phase coordinates [25-34]:

\[ q = (\xi, \eta, \psi, \theta, \gamma, \varphi_{11}, \varphi_{12}, \varphi_{13}, \ldots)^T \]  \hspace{1cm} (1)

where \( i = \{1 – 6\} \), \( \xi, \eta \) and \( \xi \) – coordinates of the center of mass \( O \) spider-robot body, \( \psi, \theta, \gamma \) – angles of rotation of the body relative to its center of mass \( O \), \( \varphi_{11} \) – the angle of 62 rotation of the \( i \)-th limb relative to the body (\( i = 1 \), \( \varphi_{12} \) and \( \varphi_{13} \) – angles of position of its joints (\( i = 2 \) and \( i = 3 \)).

Fig. 1. Spider-robot schematic

The model in the Simulink environment is synthesized on the basis of the CAD model of the spider-robot [1], imported from the CAD environment (see Fig. 2).

Fig. 2. 3D model of a spider-robot

This information model of a spider-robot consists of blocks describing the mechanical elements of the spider-robot (body and individual links of each limb) and the connections between them. General parameters of the model (mass, dimensions, moments of inertia, etc.) and the world in which it is located (the force of gravity). The physical parameters of the spider-robot model, automatically calculated by CAD tools, are given in Table I and Table II.

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<tr>
<th>Parameter1</th>
<th>Body</th>
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<th>Joint 2</th>
<th>Joint 3</th>
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<td>0.0848</td>
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<table>
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<tr>
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<th>3. MR</th>
<th>4. ML</th>
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<td>101</td>
<td>70</td>
<td>70</td>
<td>101</td>
<td>101</td>
</tr>
</tbody>
</table>

1 \( m \) – weight, \( l \) – the length of the articulation of the limb, \( I \) – moment of inertia.

TABLE II. PARAMETERS OF THE LOCATION OF ATTACHMENTS OF LIMBS TO THE BODY

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1 \( \varphi_i \) – angle from the positive semiaxis \( O_x \), on which the attachment point of the \( i \)-th limb is located, i.e. first hinge, \( l_i \) – distance from the point \( O \) to the attachment point of the \( i \)-th limb.

2 AR – anterior right, AL – anterior left, MR – middle right,
ML – middle left, RR – rear right, RL – rear left.

To control the position of the joints of the limbs, a control loop implemented on PID controllers is added to the information model synthesized in Simulink (see Fig. 3 and Fig. 4). The parameters of the PID controllers were selected using the built-in Simulink tools [35–39].

The interaction of the spider-robot model with the floor is realized by adding for each limb a force that is activated upon contact of the limb with a surface acting in the opposite direction of movement of the limb (see Fig. 5) and the frictional force of sliding of the limb with a given dependence [40–43]:

\[
\vec{F} = -k\vec{v}
\]

where \( k \) – constant depending on the characteristics of the contacting bodies, \( \vec{v} \) – limb speed.

The model in the SimInTech environment was built on the basis of the previously obtained mathematical model [1]:

\[
\dot{v} = Hu + Fu = C
\]

where \( v = q^i \) – generalized velocity vector, \( u \) – control vector (moments of rotation applied to the joints of the limbs), \( H_{24 \times 24}, F_{24 \times 18} \) and \( C_{24 \times 1} \) – functional matrices of the system. The physical characteristics of SimInTech are taken from a model synthesized in the Simulink environment. In Fig. 6 is a diagram of the model.

Interaction with the floor is set using a force applied to the coordinates of the body \( \xi \) and \( \eta \), directed against the rotation of any of the coordinates \( \phi_i (\forall i \in [1,6]) \) provided that the limb is supporting. It is worth noting that an important part of calculating the force of interaction with the floor is the correct distribution of the load on the limbs of the spider-robot. A variant of the solution to this problem is shown by us in [1, 44–46]. After analyzing the Simulink model for each hinge of the limbs of the spider-robot, friction is specified, which is expressed as a moment opposite to the control moment:

\[
m = -k_{fr}N_{ij}^j,
\]

where \( k_{fr} \in R \) – rolling friction coefficient, \( N_{ij}^j \) – support reaction force acting on the \( j \)-th joint of the \( i \)-th limb.

To visualize the operation of the model, an interface was developed (see Fig. 8), which schematically shows the position of the body and limbs in space, and also contains controls for the model [47-50].

V. SPIDER ROBOT PHYSICS MODEL

As a prototype of the physical model of the spider-robot, the model of the spider robot [29] was used. Parts of the body and limbs were 3D printed. The control controller is based on
the Arduino Due. The physical model of a spider-robot is shown in the Fig. 9.

![Physical model of a spider-robot](image)

The physical model of a spider-robot is powered by a 7.2-volt battery and can stand and move in a plane $O_{xy}$.

VI. COMPUTATIONAL EXPERIMENT

As testing of both models, the programmed movement of the spider-robot from point A to point B. The spider-robot performed a turn at point A and moved in a straight line towards point B [51-55]. The movement of the spider-robot ended when the model moved to the delta – neighborhood of point B.

The main results of the experiment:
1. The spider-robot reached the delta neighborhood of point B in 130 s,
2. Time spent on turning – 35 s,
3. Time spent on movement – 95 s,
4. The Simulink model turned a little to the right, therefore, at the stage of movement, it became necessary to constantly correct the direction and switch to the "Rotate" subroutine.

The graphs for switching control commands and movement of the spider-robot are shown in Fig. 10, Fig. 11 and Fig. 12.

![Graphs of switching control commands and movement](image)

Spider robot model in SimInTech

The main results of the experiment [56-65]:
1. The spider-robot reached the delta neighborhood of point B in 71 s,
2. Time taken to turn – 27 s,
3. Time spent on movement – 43 s,
4. Turning in place and moving forward are performed uniformly, which indicates the correctness of the original mathematical model and a successful selection of the parameters of the regulators.

The graphs for switching control commands and movement of the spider-robot are shown in Fig. 13, Fig. 14 and Fig. 15.

![Graphs of switching control commands and movement](image)
of a multi-legged walking robot based on such a joint model promises to provide a new level of efficiency and control accuracy.

The simultaneous use of two information models in different ways interact with the environment, allows a more detailed study of the behavior of the spider-robot.

This will allow us to synthesize an effective robot control system. In particular, this approach will be significant when controlling a robot in difficult conditions - on surfaces with complex geometry, the presence of drifts, unsteady surfaces, and external disturbances.

Since in the future we are interested in solving the problem of managing a group of robots that make up a multi-agent system, the refined information model will be used to synthesize more complex control systems.

The data obtained in the work will be distributed to a group of walking spider-robots.

In particular, the presented results will be used in the design of a multi-agent system of walking robots for searching and moving objects on the seabed and land.

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